EM-BASED ACQUISITION OF DS-CDMA WAVEFORMS FOR RADIOLOCATION

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ABSTRACT

A handshaking protocol for radiolocation is considered using DS-CDMA packet transmission. The expectation - maximization (EM) algorithm is employed for code delay acquisition. It is shown that the systematic application of EM on the flat-fading channel yields a parallel interference cancellation (PIC)-based technique for acquisition. On the frequency-selective channel, it is necessary to jointly estimate both the code delays and multipath coefficients. The EM approach in this latter case yields an algorithm that combines PIC with an eigendecomposition/eigenvalue maximization step for channel identification.

1. INTRODUCTION

A handshaking protocol (HP) based on transmission of DS-CDMA waveforms is considered for radiolocation. In the HP (e.g. [1]), a master node transmits a request-to-send (RTS) waveform to multiple reference nodes, which have knowledge of absolute and/or relative position. Upon reception of the RTS waveform, each reference node immediately transmits a DS packet. The master node measures the time-of-arrival of all packets, and thus determines range to the references. A navigation message is also superimposed on the DS packet with each reference's position.

The estimation of reference code time-of-arrival is hindered by the near-far effect – due to the design of the HP, power control is not feasible. The joint estimation of code delays has previously been considered in the CDMA literature, using algorithms such as MUSIC [2] and channel subspace identification [3]. Here, a new algorithm for multiuser code delay estimation (MCDE) based on expectation-maximization [4] is presented. It is shown that systematic application of the EM technique to MCDE leads to a parallel interference cancellation (PIC) structure.

The sequel is organized as follows. Signal and channel models for the radiolocation channel are presented in Section 2. The EM MCDE algorithm for flat fading channels is developed in Section 3 and extended to frequency-selective fading in Section 4. Simulation results and conclusions follow in Sections 5 and 6, respectively.

This work was supported in part by DARPA under grant No. MDA972-00-1-0036

2. SIGNAL AND CHANNEL MODELS FOR RADIOLOCATION

Following reception of the RTS signals, the master node receives packets of length $M+1$ symbols from K reference nodes, described by the lowpass equivalent received signal

$$r(t) = \sum_{k=1}^{K}\sum_{m=0}^{M}\sum_{p=0}^{N_f-1} f_{k,p} b_k(m) s_k(t-mT-pT_s-T_k)+n(t), \tag{1}$$

where $s_k(t)$ is the k-th reference node's DS waveform, with support $[0,T)$ s., and $\{b_k(m)\}$ is the k-th reference node's navigation message. The frequency-selective channel is modeled by a tapped delay line with spacing T_s s., where $1/(2T_s)$ is the approximate signal bandwidth, and $f_{k,p}\in\mathcal{C}$ are the path gains from reference node k to the master node. The additive thermal noise $n(t)$ is circular white Gaussian with bilateral spectral density $2N_0$.

The received waveform $r(t)$ is sampled at the Nyquist rate $1/T_s$, and symbol-length vectors of $N_s = T/T_s$ samples are formed, denoted by $\mathbf{r}(n)\in\mathcal{C}^{N_s}$ and defined by

$$\mathbf{r}(n) = [r(nN_sT_s), r((nN_s+1)T_s),\dots, r(((n+1)N_s-1)T_s)]^T.$$

The resulting vector received signal is given by

$$\mathbf{r}(n) = \sum_{k=1}^{K}\sum_{l=0}^{1} b_k(n-l)\mathbf{S}_k(T_k-lT)\mathbf{f}_k+\mathbf{n}(n), \tag{2}$$

where the signal matrix $\mathbf{S}_k(\tau)\in\mathcal{C}^{N_s\times N_f}$ is defined by

$$\mathbf{S}_k(\tau) = [\mathbf{s}_k(\tau), \mathbf{s}_k(\tau+T_s),\dots,\mathbf{s}_k(\tau+(N_f-1)T_s)],$$

and $\mathbf{f}_k = [f_{k,0},\dots f_{k,N_f-1}]^T$ is the channel vector for reference node k. The signal vectors $\mathbf{s}_k(\tau)\in\mathcal{C}^{N_s}$ are in turn defined by

$$\mathbf{s}_k(\tau) = [s_k(-\tau), s_k(T_s - \tau), \ldots, s_k((N_s-1)T_s - \tau)]^T.$$

The noise vector $\mathbf{n}(n)$ is circular Gaussian with covariance matrix $\sigma_n^2\mathbf{I}$.

3. EM MCDE ALGORITHM – FLAT FADING

The joint ML solution for the delays $\{T_k\}$ requires exhaustive search over the hypercube $[0,T)^K$. The EM algorithm replaces this exhaustive search with iterations of K independent searches over the intervals $[0,T)$. The EM algorithm is first developed for flat fading, in which case the channel vector $\mathbf{f}_k$ becomes a scalar f_k, and the product $f_k b_k(m)$ is modeled as a single complex coefficient $a_k(m) \in \mathcal{C}$. Following [5], a hidden data set at time n is defined by the signal vectors

$$\mathbf{r}_k(n) = \sum_{l=0}^{1} a_k(n-l)\mathbf{s}_k(T_k - lT) + \mathbf{n}_k(n), \qquad (3)$$

for $k = 1, \ldots, K$. The noise vectors $\mathbf{n}_k(n)$ are mutually independent with variances $\beta_k\sigma_n^2\mathbf{I}$. The β_k form a probability vector [5].

The EM algorithm [5] is compactly written as the iteration

$$\mathbf{T}^{i+1}, \mathbf{a}^{M^{i+1}} = arg \max_{\mathbf{T},\mathbf{a}^M} Q(\mathbf{T}, \mathbf{a}^M | \mathbf{T}^i, \mathbf{a}^{M^i}), \qquad (4)$$

where

$$Q(\mathbf{T}, \mathbf{a}^M | \mathbf{T}^i, \mathbf{a}^{M^i}) = \qquad (5)$$
$$E\left\{\log p(\mathbf{r}^M, \{\mathbf{r}_k^M\} | \mathbf{T}, \mathbf{a}^M) | \mathbf{r}^M, \mathbf{T}^i, \mathbf{a}^{M^i}\right\}.$$

The vectors and sequences in (5) are defined by

$$\begin{aligned} \mathbf{T} &= [T_1, \ldots, T_K]^T, \\ \mathbf{a}^M &= \{\mathbf{a}(0), \mathbf{a}(1), \ldots, \mathbf{a}(M)\}, \\ \mathbf{a}(m) &= [a_1(m), \ldots, a_K(m)]^T, \\ \mathbf{r}^M &= \{\mathbf{r}(0), \ldots, \mathbf{r}(M)\}. \end{aligned} \qquad (6)$$

The corresponding i-th EM iteration estimates are $\mathbf{T}^i$, $\mathbf{a}^{M^i}$.

In order to evaluate the objective function $Q(\bullet|\bullet)$ (5), the following conditional mean is required.

$$\overline{\mathbf{r}}_k^i(n) = E\{\mathbf{r}_k(n) | \mathbf{r}^M, \mathbf{T}^i, \mathbf{a}^{M^i}\}. \qquad (7)$$

Note that $\mathbf{r}_k(n)$ is independent of $\mathbf{r}(m)$ for $n \neq m$, when conditioned on $\mathbf{T}^i, \mathbf{a}^{M^i}$. Then

$$\overline{\mathbf{r}}_k^i(n) = (1-\beta_k)\sum_{l=0}^{1} a_k^i(n-l)\mathbf{s}_k(T_k^i - lT) + \qquad (8)$$
$$\beta_k\left[\mathbf{r}(n) - \sum_{k'\neq k}\sum_{l=0}^{1} a_{k'}^i(n-l)\mathbf{s}_{k'}(T_{k'}^i - lT)\right].$$

Note that (8) incorporates the prior estimate of the k-th signal with weight $(1-\beta_k)$, and a PIC term subtracted from $\mathbf{r}(n)$ proportional to β_k.

The objective function (5) is readily shown to equal the following

$$Q(\mathbf{T}, \mathbf{a}^M | \mathbf{T}^i, \mathbf{a}^{M^i}) \propto \qquad (9)$$
$$\sum_{k=1}^{K}\sum_{n=0}^{M} 2Re\left\{\sum_{l=0}^{1} a_k^*(n-l)\mathbf{s}_k(T_k - lT)^H\overline{\mathbf{r}}_k^i(n)\right\}$$
$$-||\sum_{l=0}^{1} a_k(n-l)\mathbf{s}_k(T_k - lT)||^2.$$

A similar objective function was considered in [6]. Here, the *unconstrained* estimate of $\mathbf{a}^M$ is used, which leads to a simpler update for the delays $\mathbf{T}$ as follows. Differentiation of (9) w.r.t. $a_k(n)$ yields the following solution, noting that $\mathbf{s}_k(T_k)^H\mathbf{s}_k(T_k - T) = 0$.

$$a_k^{i+1} = \frac{1}{E_k}\left[\mathbf{s}_k(T_k)^H\overline{\mathbf{r}}_k^i(n) + \mathbf{s}_k(T_k - T)^H\overline{\mathbf{r}}_k^i(n+1)\right], \qquad (10)$$

where $E_k = ||\mathbf{s}_k(T_k)||^2 + ||\mathbf{s}_k(T_k - T)||^2$ is the signal energy. Substitution of the amplitude estimate a_k^{i+1}, which is implicitly a function of $\mathbf{T}$, into the objective function (9), finally yields the following separable update for the delays.

$$T_k^{i+1} = \qquad (11)$$
$$\arg\max_{T_k}\sum_{n=0}^{M}|\mathbf{s}_k(T_k)^H\overline{\mathbf{r}}_k^i(n) + \mathbf{s}_k(T_k - T)^H\overline{\mathbf{r}}_k^i(n+1)|^2,$$

The update (11) can be interpreted as computing the maximum of the envelope of the output of a filter matched to $\mathbf{s}_k(T_k)$, over $T_k \in [0,T)$, where the filter input is obtained via parallel interference cancellation.

4. EM MCDE ALGORITHM – FREQUENCY-SELECTIVE FADING

When frequency-selective fading is present, the signal model (2) is employed, where $\mathbf{f}_k$ is the vector of N_f channel coefficients. The multipath spread $(N_f - 1)T_s$ is assumed to be much smaller than the symbol duration T, thus intersymbol interference (ISI) can be neglected. The hidden data is then given by

$$\mathbf{r}_k(n) = \sum_{l=0}^{1} a_k(n-l)\mathbf{S}_k(T_k - lT)\mathbf{f}_k + \mathbf{n}_k(n), \qquad (12)$$

for $k = 1, \ldots, K$. Following a similar analysis leading to (9), the EM objective function can be shown to equal

$$Q(\mathbf{T}, \mathbf{a}^M \mathbf{f} | \mathbf{T}^i, \mathbf{a}^{M^i}, \mathbf{f}^i) \propto \sum_{k=1}^{K} \frac{1}{\beta_k \sigma_n^2} \tag{13}$$

$$2Re\left\{ \sum_{n=0}^{M} \sum_{l=0}^{1} a_k(n-l)^* \mathbf{f}_k^H \mathbf{S}_k(T_k - lT)^H \overline{\mathbf{r}}_k^i(n) \right\}$$

$$- \sum_{n=0}^{M} \sum_{l=0}^{1} \sum_{l'=0}^{1} \{ a_k(n-l)^* a_k(n-l') \times$$

$$\mathbf{f}_k^H \mathbf{S}_k(T_k - lT)^H \mathbf{S}_k(T_k - l'T) \mathbf{f}_k \} .$$

The estimated hidden data, $\overline{\mathbf{r}}_k^i(n)$ is derived similarly to (8), and also has an embedded PIC step.

$$\overline{\mathbf{r}}_k^i(n) = (1-\beta_k) \sum_{l=0}^{1} a_k^i(n-l) \mathbf{S}_k(T_k^i - lT) \mathbf{f}_k^i + \tag{14}$$

$$\beta_k \left[\mathbf{r}(n) - \sum_{k' \neq k} \sum_{l=0}^{1} a_{k'}^i(n-l) \mathbf{S}_{k'}(T_{k'}^i - lT) \mathbf{f}_{k'}^i \right] .$$

The unconstrained estimates of the data $\mathbf{a}^M$ are computed, and inserted into the objective function (13) to obtain a maximization step in terms of the channel vectors $\mathbf{f}_k$. These estimates are in turn,

$$a_k^{i+1}(m) = \tag{15}$$

$$\frac{\mathbf{f}_k^H \left(\mathbf{S}_k(T_k)^H \overline{\mathbf{r}}_k^i(m) + \mathbf{S}_k(T_k - T)^H \overline{\mathbf{r}}_k^i(m+1) \right)}{\mathbf{f}_k^H \left[\mathbf{S}_k(T_k)^H \mathbf{S}_k(T_k) + \mathbf{S}_k(T_k - T)^H \mathbf{S}_k(T_k - T) \right] \mathbf{f}_k}$$

$$\mathbf{f}_k^{i+1} = arg \max_{\mathbf{f}_k}$$

$$\frac{\sum_{n=0}^{M} \left| \mathbf{f}_k^H \left(\mathbf{S}_k(T_k)^H \overline{\mathbf{r}}_k^i(n) + \mathbf{S}_k(T_k - T)^H \overline{\mathbf{r}}_k^i(n+1) \right) \right|^2}{\mathbf{f}_k^H [\mathbf{S}_k(T_k)^H \mathbf{S}_k(T_k) + \mathbf{S}_k(T_k - T)^H \mathbf{S}_k(T_k - T)] \mathbf{f}_k} .$$

Straightforward analysis shows that the $\mathbf{f}_k^{i+1}$ is the eigenvector corresponding to the maximum eigenvalue of the objective matrix $\mathbf{M}(T_k)$, defined by

$$\mathbf{M}(T_k) = \tag{16}$$

$$\left[\mathbf{S}_k(T_k)^H \mathbf{S}_k(T_k) + \mathbf{S}_k(T_k - T)^H \mathbf{S}_k(T_k - T) \right]^{-1} \times$$

$$\left[\mathbf{S}_k(T_k)^H \mathbf{R}(0) \mathbf{S}_k(T_k) + \mathbf{S}_k(T_k - T)^H \mathbf{R}(0) \mathbf{S}_k(T_k - T) \right.$$

$$\left. + 2Re \left\{ \mathbf{S}_k(T_k)^H \mathbf{R}(-1) \mathbf{S}_k(T_k - T) \right\} \right] ,$$

where $\mathbf{R}(l) = \sum_{n=0}^{M} \overline{\mathbf{r}}_k^i(n) \overline{\mathbf{r}}_k^i(n-l)^H$ is a sample correlation matrix.

Unfortunately, the estimates $\mathbf{T}^{i+1}$ and $\mathbf{f}^{i+1}$ are coupled through the form of (16), so that the maximizing eigenvector must be re-computed for every trial delay vector. However, the solutions for the delay vectors are separable in the users k. The final MCDE algorithm is expressed as follows, where N_i is the total number of EM iterations.

For $i = 1, \ldots N_i$
    Given $\mathbf{T}^i, \{\mathbf{a}^M\}^i, \mathbf{f}^i$
    Compute PIC estimates $\overline{\mathbf{r}}_k^i(n)$ using (14)
    Compute sample correlation matrix $\mathbf{R}(0), \mathbf{R}(1)$
    For $k = 1, \ldots, K$
        For $l = 0, 1, \ldots, T/\delta_T$
            $T_k = l\delta_T$
            Compute objective matrix $\mathbf{M}(T_k)$ (16)
            Compute $\mathbf{f}_k(T_k)$ as maximum eigenvector of $\mathbf{M}$
            Save $\lambda_k(T_k)$ (maximum eigenvalue)
        Next l
        $T_k^{i+1} = arg \max_{T_k} \lambda_k(T_k)$
        Save $\mathbf{f}_k^{i+1} = \mathbf{f}_k(T_k^{i+1})$ (maximum eigenvector.)
        Compute $a_k^{i+1}(n)$ using (15) and $\mathbf{f}_k^{i+1}$
    Next k
Next i

Table 1. EM-MCDE algorithm, frequency selective fading.

5. RESULTS

The EM-MCDE algorithm for flat fading (11) was simulated under the following conditions. Gold codes of length $N = 31$ chips were employed for a nominal SNR of 15 dB, where SNR = E_b/N_0, with E_b the energy per navigation message symbol. Note that the symbol duration is $T = NT_c$ s., where T_c is the chip duration. User 1 was set to the nominal 15 dB SNR, while users $k = 2, \ldots, K$ were set to $J/S = 10$ dB above user 1, thus representing a strong near-far condition. The packet length was set to $M = 15$ symbols.

Figure 1 plots delay estimation error in chips versus EM iterations for $K = 5$ users. The high SNR user $k > 1$ delay estimates converge on the first pass of the EM algorithm. User 1, which is subject to strong multiuser interference, converges after 5 iterations. Figure 2 is identical to 1, except that $K = 6$ users are now considered. Due to the additional multiuser interference, convergence is slower for user 1, but the delay error eventually settles to less than .1 chip.

The EM-MCDE algorithm in Table 1 was simulated for $K = 2$ users and length $N = 31$ Gold codes. User 1 SNR was set to 10 dB, and user 2 SNR was 20 dB. The channel length was set to $N_f = 2$, with equal coefficients. A normalized coefficient vector error at EM iteration i was defined by

$$E_k^i = 1 - \frac{|\mathbf{f}_k^{i\,H} \mathbf{f}|^2}{||\mathbf{f}_k^i||^2 ||\mathbf{f}_k||^2}. \tag{17}$$

Both the delay estimation error and coefficient error E_k^i are plotted versus EM iteration is Figure 3. There is a fairly significant error for weaker user 1 in both delay and channel estimation. This is due to ill-convergence issues of the EM algorithm discussed below.

6. CONCLUSIONS

New EM-based code acquisition algorithms have been presented for terrestrial radiolocation applications. The EM algorithm for flat-fading channels is particularly simple, and can be viewed as iterative matched filtering/envelope detection with parallel interference cancellation. For frequency-selective channels, an EM-MCDE algorithm was developed which jointly computes unconstrained user data estimates, multipath channel coefficients, and code delays. It is interesting to note that the algorithm for frequency-selective channels nevertheless employs PIC.

Initial results for the flat-fading case are very encouraging, in that the delay of a weak user can be successfully estimated using a short packet, in the presence of significant near-far interference. However, ill convergence is seen in the $K = 2$ user case for frequency-selective fading. This is a consequence of the basic EM algorithm limitation – the true data likelihood is a *nondecreasing* function of the EM estimates, and thus, it is possible to become trapped in local maxima of the likelihood function. Future work will focus on multiple initialization strategies for EM to reduce the frequency of ill-convergence.

7. REFERENCES

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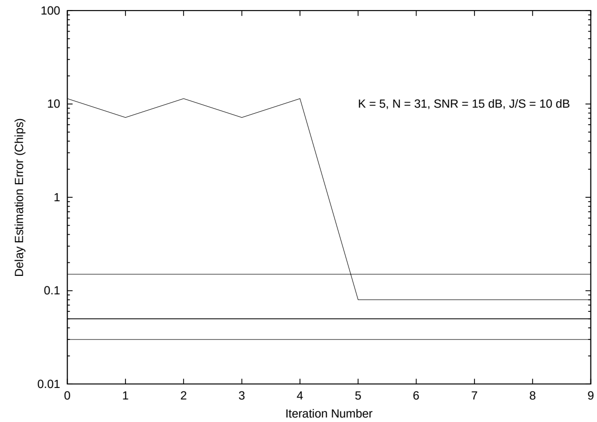

Fig. 1. Code delay estimation error, flat fading – 5 users.

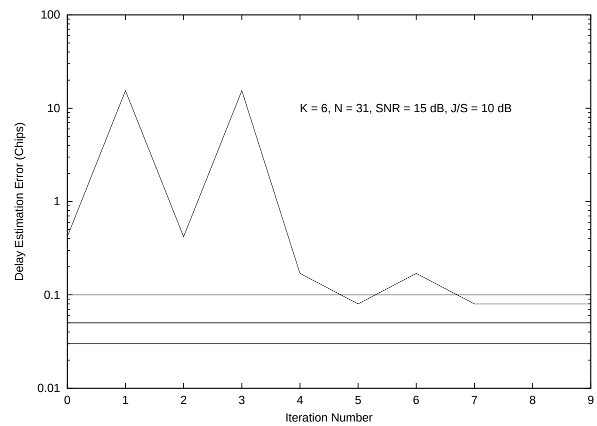

Fig. 2. Code delay estimation error, flat fading – 6 users.

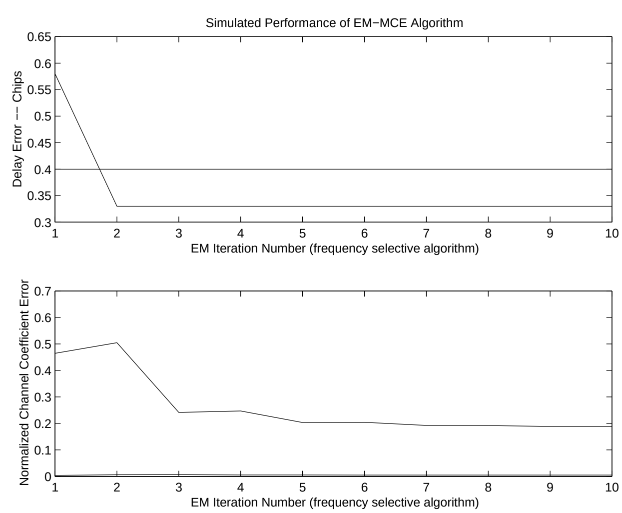

Fig. 3. Estimation errors, frequency-selective fading.