

REED-SOLOMON CODES AND THEIR PERFORMANCE FOR FCMA SYSTEMS IN FADING SATELLITE CHANNEL

Dr Mahmoud Ahmed Attia Ali, Dr. Atef Abou-El-Azm, and Eng. M. F. Marie

Department of Electrical Communication,
Faculty of Electronic Engineering,
32952-Menouf, EGYPT

ABSTRACT

Fading in mobile satellite communications severely degrades the performance of data transmission. The emphasis in this paper is on the performance of uncoded and coded FCMA in the fading environment. The channel is modeled with nonfrequency selective Rice and Rayleigh fading using noncoherent demodulation with Reed-Solomon (RS) codes and hard decision decoding. Berlekamp-Massey decoding of RS codes is used to compensate for the fading. The bit error rate (BER) after decoding is calculated for specific codes and for different values of Rician channel parameters. The results of this paper are obtained by simulation techniques. They show that substantial coding gains are obtained compared to the uncoded reference system. They are also useful as reference for validating the results of simulation studies.

1. INTRODUCTION

Many important sources of degradation in digital data systems such as impulse noise due to lightning discharges or switches, radio frequency interference from other transmitters, and multiple transmission paths in the transmission medium are non-Gaussian noise [1]. Multiple transmission paths are due to scattering over reflecting objects in mobile or refraction involved by ionosphere or troposphere in radio. This effect is the well known fading. The channels are referred to as fading when their gains, phase characteristics, or both are perturbed by randomly varying propagation conditions.

Frequency comb multiple access, FCMA, will be investigated over a slow fading channel. FCMA is basically a quasi-orthogonal frequency multiple access scheme [2]. FCMA assigns sets of carrier frequencies for each user of the system. These sets are unique for every user while each set is called 'signature'. It has been suggested to serve numerous light-traffic and long-distant points. Henceforth, multiple access interference MAI is the inherent limiting factor of performance. M -ary FCMA has been investigated where the communication channel is confined to AWGN and MAI [3]. Recently, such schemes were analysed with coding in [4]. These results seem adequate for satellite applications when all communication centres retain the line of sight propagation. In such cases non-selective frequency fading has only a slight degradation. Unfortunately, in mobile applications, or where reflection or multiple paths exist without direct propagation, fading criteria should be encountered as a must. Henceforth, M -ary FCMA is simulated assuming Rayleigh and Rician fading channels, to match the forgoing services. Rician fading suits satellite mobile channel whereas Rayleigh fading suits land mobile channel [5]. The restriction of the

applications in satellite communications depends mainly on the so called Rice factor. The Rice factor K_r is defined as the ratio of direct to reflected energy. In maritime satellite channel, it was found that K_r is typically in the range of 7-15 dB [6]. A variety of services, would refer to unshadowed channel, e.g., digital speech transmission in open areas and communication to stationary mobile users. For such applications, the Rician channel with a Rice factor is typically in the range of 3-18 dB. In communication with land mobile terminals via satellite, shadowing is caused by large obstacles in the signal path (e.g., building, and bridges). Multipath fading channel is caused by reflection of the satellite signal at a large number of points. For narrow-band communication, the channel can be assumed nonfrequency selective Rayleigh fading [7]. The received signals will be assumed detected noncoherently. Although, coherence achieves the optimum performance in terms of bit signal to noise ratio, E_b/N_0 , it is not feasible in practice. A precise frequency and phase reference must be obtained for each carrier component of the signature. This would make a coherent FCMA very difficult to implement.

In this paper, the idea of combining Reed-Solomon (RS) codes with M -ary signaling FCMA as a bandwidth efficient block technique, over the bandwidth-limited fading channels is presented. The transmission model of the FCMA system under consideration over a fading channel is depicted in Fig 1.

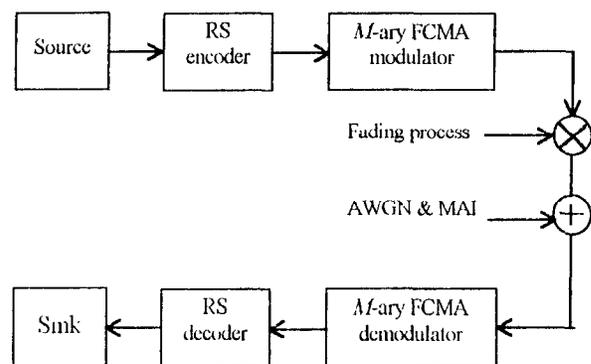


Fig 1 Transmission model of FCMA over Fading Channels.

Initially the model of communication fading channel for FCMA system is introduced. Next, the performance of coded/uncoded M -ary FCMA will present. The obtained results for such system using computer simulation are testified with the well known M -ary FSK system given in reference [8] under certain parameters and a good agreement is achieved.

2. MODELING RAYLEIGH AND RICIAN CHANNELS

Fading channel is modeled by introducing a multipath gain represented by a complex number, and a complex additive Gaussian noise AWGN. The M signaling waveforms are assumed to have equal energy and are equally likely to be transmitted. For the AWGN, the optimum receiver computes the decision variables and selects the signal corresponding to the largest decision variable. Since the signals are detected noncoherently, the phase term of the gain becomes irrelevant and can be dropped from consideration. The gain is a random variable having Rayleigh or Rician fading density functions respectively [7].

$$P(r) = 2r \exp(-r^2) \quad (1)$$

$$P(r) = \frac{r}{\sigma^2} \exp\left[-\left(\frac{r^2}{2\sigma^2} + K_r\right)\right] I_0\left(\frac{r\sqrt{2K_r}}{\sigma}\right) \quad (2)$$

where r is the channel amplitude, σ^2 is the variance of the noise at the input of the detector. The dimensions of the presented system, M -ary FCMA, were as follows: channel bandwidth (assumed to be 20 MHz) is divided into 512 bands; the centre frequencies of these bands are the carriers that could be communicated with. Every sub-band occupies about 40 kHz so that it could carry a maximum information rate of about 40 kbps. Each symbol is represented by a signature that will be sent as a set of three distinct carriers. In the simulation, the number of simultaneous users is assumed to be five.

It has been noted that as few as six sine waves with independently fluctuating random phases will give a fluctuating resultant whose envelope closely follows Rayleigh statistics and whose phase is uniformly distributed, [9,10]. The simulation statistics of Rayleigh model is achieved by considering the received carrier amplitude, of each element of a signature, as the vector sum of twenty sinusoids of equal strength and a uniform random phase. On the other hand, with Rician fading channel one of these sinusoids is considered stronger than others. The additive white Gaussian noise, AWGN, is then added to both signal models in order to construct a complete signal at the receiver. In doing so, the amplitudes of both models signals are factorised to adjust the signal to noise ratio as required. MAI is generated as an interference carriers of the same strength exactly as the transmitted signal.

3. PERFORMANCE OF UNCODED FCMA

The receiver of binary FCMA receives only two signatures or two combs of discrete frequencies, each one represents a binary 1 or 0. Meanwhile, the M symbols of M -ary FCMA are received as M different signatures. The signatures of a particular receiver have been chosen to be completely distinct whereas that of different destinations may be either nonoverlapped or overlapped in one or two elements of the signature. The overlapping criterion for different receivers are assumed random.

In this paper, it was useful to model propagation conditions in mobile satellite communications for different values of Rice

factor K_r . For instant, $K_r = 10$ dB is often used to model propagation conditions in the land mobile case in nonshadowing areas, while $K_r = 5$ dB is similarly used to model propagation conditions with slight shadowing.

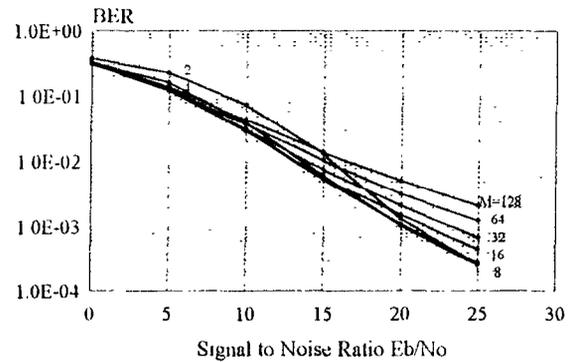


Fig 2 Performance of Uncoded FCMA over Rayleigh Channel.

Figure 2 explains the performance (BER as a function of the bit energy-to-one sided noise density ratio E_b/N_0 dB) of uncoded FCMA over Rayleigh channel. Figures 3 and 4 show the performance of uncoded FCMA for slow Rician fading channels at $K_r = 5$ and 10 dB respectively. This has been achieved for ideal noncoherent demodulation at $M = 2, 4, 8, 16, 32, 64, 128$. It is observed from these figures that there is a certain threshold region in the sense that a higher M -ary system provides poorer

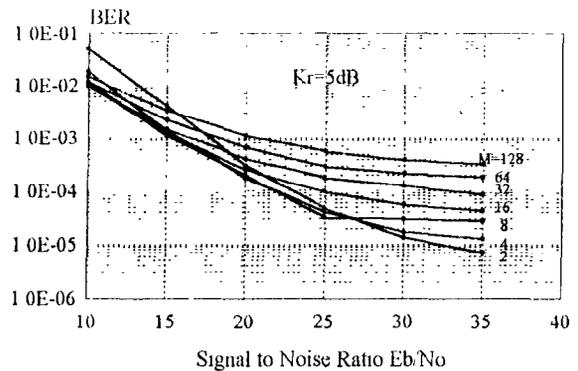


Fig 3 Performance of Uncoded FCMA over Rician Channel

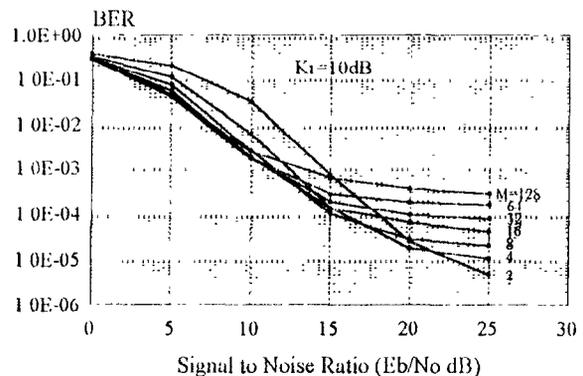


Fig 4 Performance of Uncoded FCMA over Rician Channel.

performance after that threshold as explained in Fig.5. This is due to the effect of MAI as expected. A comparison between the performance of Rayleigh, Rician, and AWGN for uncoded FCMA is depicted in Fig 6.

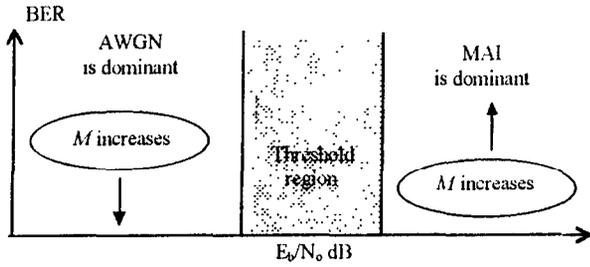


Fig.5 Regions of Error Performance of Uncoded FCMA

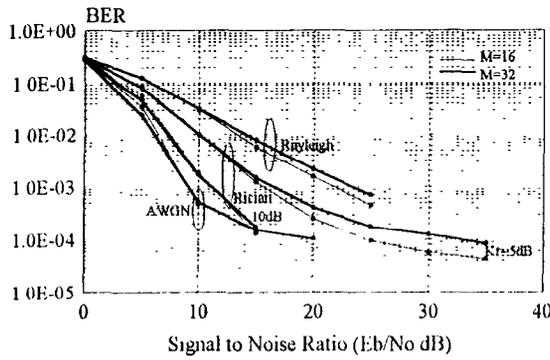


Fig 6 Performance of Uncoded FCMA in Rayleigh, Rician, and AWGN Channels

4. PERFORMANCE OF CODED FCMA

To develop an RS coded, define an RS code over $GF(2^m)$, where each code symbol is mapped to a signal set symbol. The code rate is chosen such that the rate of the coded scheme is the same as the uncoded one. The performance over fading channels using the Berlekamp decoding technique is considered. In order to evaluate the bit error probability (BER) for block-coded transmission to fading channels, the following assumptions are made (i) the data are coded by a systematic block code with blocklength n symbol = nm bits (m bits per code symbol), information length k symbols, and code rate $R=k/n$, (ii) in one code block, t symbol errors can be corrected, (iii) the fading is slow compared to the data rate, resulting in constant amplitude during one code symbol.

Berlekamp's algorithm is much more difficult to understand than other algorithms (e.g. Peterson's approach), but results in a substantially more efficient implementation [11]. The complexity of Peterson's technique increases with the square of the number of errors corrected. The complexity of Berlekamp's algorithm increases linearly allowing for construction of efficient decoder that correct dozens of errors.

Using these decoding algorithms all patterns of t symbol errors can be corrected provided that $2t \leq d_{min}-1$, where d_{min} is the minimum Hamming distance of the RS code. If for a received

code word this inequality does not hold, the decoder can not find the correct codeword. This event is called a decoding failure.

The performance has been computed using simulation techniques when the receiver uses hard decisions on the received symbols. Now the signal to noise ratio (E_b/N_0) is scaled by the code rate $R=k/n$ as

$$\frac{E_s}{N_0} = R \frac{E_b}{N_0} \text{Log}_2(M) \quad (3)$$

where E_s is the energy per symbol. Figure 7 shows E_b/N_0 as a function of the number of information symbols, k , with (31, k) RS hard decision decoding and BER = 0.001. It shows that the optimum (i.e. the minimum E_b/N_0) RS codes have relatively high code rate for AWGN channel, while the rate decreases for fading channels as fading becomes more severe.

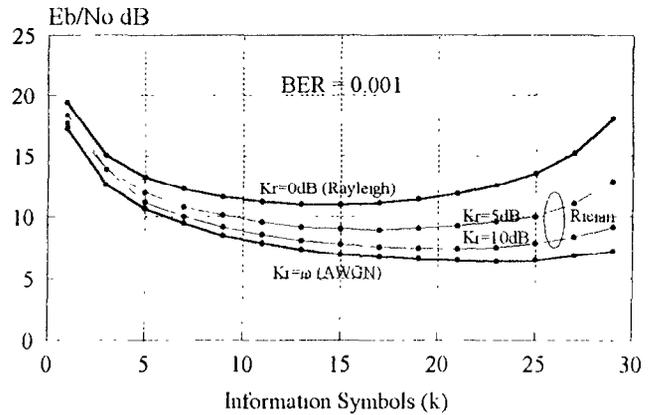


Fig 7 E_b/N_0 against Information Symbols k with RS(31,19). Hard Decision Decoding

Figure 8 shows the performance of the optimum RS codes with M -ary FCMA in Rician fading channel ($K_r = 10\text{dB}$). Table 1 lists the optimum RS codes for the different channels. A comparison between the code performance of the two codes RS(15,9) and RS(31,19) in AWGN, Rician, and Rayleigh channels is depicted in Fig 9. Figure 10 presents the signal to noise ratio E_b/N_0 as a function of the Rice factor K_r at BER=0.001. It is observed that the minimum E_b/N_0 is achieved for AWGN and RS(127,79) code while it is worse in Rayleigh and for the code RS(7,3).

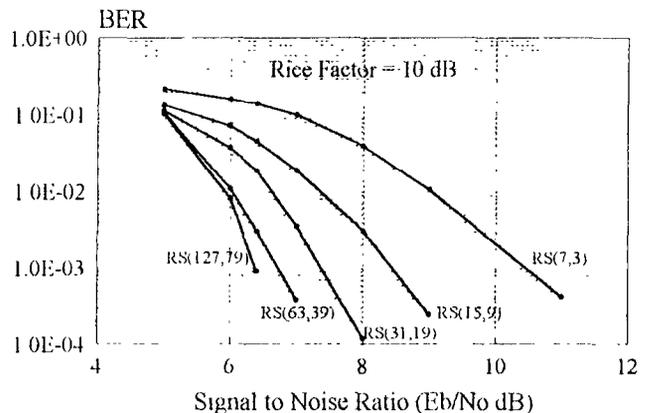


Fig 8. Optimum Codes for FCMA over Rician Channel.

Table 1 Optimum RS Codes, BER=0.001

K_r	AWGN		Rician				Rayleigh	
	∞		10dB		5dB		0dB	
M	k symb	E_b/N_0 dB	k symb	E_b/N_0 dB	k symb	E_b/N_0 dB	k symb	E_b/N_0 dB
8	5	8.88	5	10.1	3	11.9	3	14
16	11	7.61	9	8.7	9	10	7	12.2
32	23	6.44	21	7.45	17	8.94	15	10.9

5. CONCLUSION

Simulation of the uncoded and coded performance of M -ary FCMA with Reed-Solomon coding and noncoherent demodulation has been presented for slow Rician-fading channels ($0 < K_r < \infty$). The performance of coded data transmission on Rayleigh, and Rician channels is considerably improved if RS codes are used as shown in Figs.6 and 9. A simple RS code, such as RS(7,3) provides 6.1 dB coding gain at BER=0.001 over Rayleigh channel which can be increased to 19dB using RS(127,79). The worst case of fading channel (Rayleigh) compared with AWGN has shown to degrade the performance by about 13dB for uncoded 32-FCMA (see Fig.6). However, the degradation of a coded system is only valid by 5 dB (see Fig 9). Generally speaking, the gain of coding in fading channels are higher than a Gaussian channel (see Table 2). In other words, it is found that RS codes offer particularly impressive gain in fading channel. It can be noted that the results in this paper should be useful in validating the results of simulation studies, which are normally used to examine performance when other impairments are considered.

6. REFERENCES

- [1] R.E. Ziemer and W.H. Tranter, *Principle of Communications; Systems, Modulation, and Noise*, New York: John Wiley & Sons, Inc., 1995, pages 497-515.
- [2] Mahmoud A. A. Ali, *Some Investigations on the Performance of Frequency Comb Multiple Access FCMA for Communication Systems*, Egypt. University of Menoufia, PhD Thesis, 1991.
- [3] Mahmoud A. A. Ali, "Demodulation of FCMA Signal Corrupted by Multiple Access Interference, Power Change, And AWGN", *Proceeding of the Second Engineering Conf., Mansoura '97*, Egypt: El-Mansoura, April 8-10, 1997.
- [4] M. F. Marie, Atef Abou-El-Azm, and Mahmoud A. A. Ali, "Computer Simulation of Reed-Solomon Codes in FCMA System", *Proceeding of the Fifteenth National Radio Science Conference, Egypt: Helena*, Feb 24-26, C25, 1998.
- [5] Israel Korn, "The Effect of Pulse Shaping and Transmitter Filter on the Performance of FSK-DPPD and CPM-DPPD in Satellite Mobile Channel", *IEEE Journal on Selected Areas in Comm.*, Vol 13, No 2, February 1995, pages 245-249.
- [6] J. Hagenaure, and E. Lutz, "Forward error Correction Coding for Fading Compensation in Mobile Satellite Channels," *IEEE Journal on SAC.*, Vol 5, Feb 1987, pages 215-225.
- [7] John G. Proakis, *Digital Communications*, Japan: McGraw-Hill, Inc., Ch 7, 1989.
- [8] Ramon A. Khalona, "Optimum Reed-Solomon Codes for M -ary FSK Modulation with Hard Decision Decoding in Rician-fading Channels", *IEEE Trans. on Comm.*, Vol. 44, No 4, April 1996.
- [9] Mischa Schwartz, *Communication Systems and Techniques*, McGraw Hill Book Company, 1960, page 349.
- [10] George R. Cooper and Clare, *Modern Communication and Spread Spectrum*, D. McGillem, 1986, page 28.
- [11] S. Wicker, *Error Control Systems for Digital Communication and Storage*, Prentice-Hall, 1995.

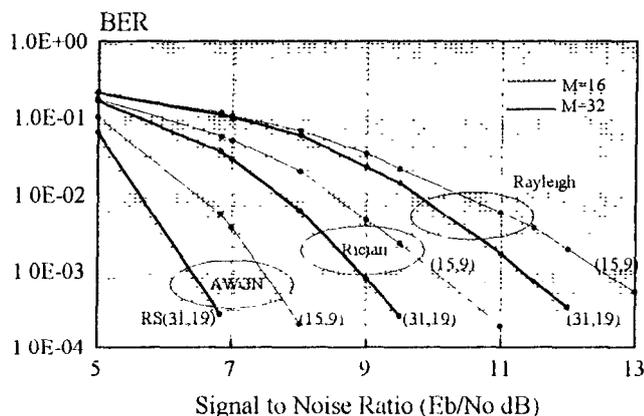


Fig 9. RS Codes for FCMA in Rayleigh, Rician, and AWGN Channels

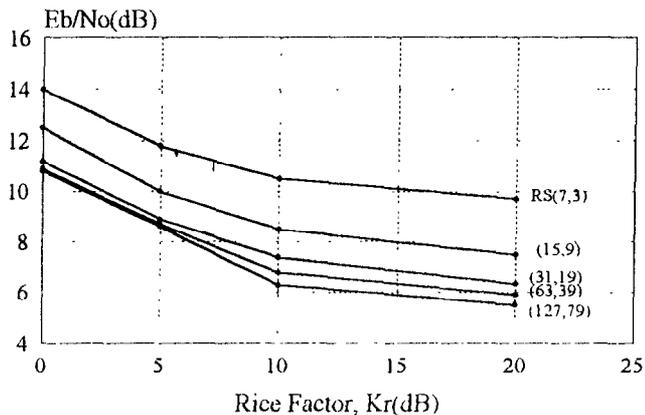


Fig.10 Eb/No versus Kr Required for BER 0.001.

Table 2 shows the results of coding gains for two of an optimum RS codes at BER=0.001 over the different channels

Table 2. Coding Gains for Optimum RS Codes, BER=0.001

RS(n,k) K_r	AWGN	Rician		Rayleigh
	∞	10dB	5dB	
RS(7, 3)	0.19 dB	1dB	3.8dB	6.1dB
RS(15,9)	1.6	3.0	5.7	9.5
RS(31,19)	2.65	4.05	7.6	11.9
RS(127,79)	4.5	6.8	12.5	19