BLIND AND SEMI-BLIND CHANNEL IDENTIFICATION METHODS USING SECOND ORDER STATISTICS FOR OFDM SYSTEMS

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ABSTRACT

In this paper two new blind channel identification methods suited to multicarrier system (OFDM) exploiting the redundancy introduced by the adjunction of a cyclic prefix at the emitter and relying on the evaluation of the received signal autocorrelation matrix are presented. The proposed algorithms are able to identify any channel without any constraint on their zeroes location (including non-minimum phase channels) and are robust to the addition of white noise. Moreover a further enhancement of the estimation accuracy can easily be achieved by taking advantage of already present training symbols in current systems operating in a semi-blind context. Furthermore one of the two identification strategies has a very low arithmetical complexity which makes it particularly attractive in practice. Notice that these methods are not restricted to classical DFT-type modulators and still apply with any perfect reconstruction modulator.

1. INTRODUCTION

The need for high data rates motivated the search for blind identification and equalization methods since they avoid the use of training sequences [1, 2]. However blind methods can also be used in cooperation with training symbols in order to improve their performance and are in that case referred as semi-blind methods [3]. This allows a better tracking of channel variations between two reference symbols.

There has been recently a large increase in interest towards multicarrier modulations [4]: it has been adopted for terrestrial digital broadcasting (Digital Audio and Video Broadcasting DAB, DVB) and high speed modems over twisted pairs (Asymmetric Digital Subscriber Line: ADSL).

In this paper, a new (semi)-blind method suited for Orthogonal Frequency Division Multiplexing (OFDM) systems exploiting the redundancy introduced at the emitter by the adjunction of the cyclic prefix and the LossLess (LL) Perfect Reconstruction (PR) property of the modulator filterbank is presented. Instead of throwing away the samples corresponding to the Guard Interval (GI) our technique takes into account the information hidden in these samples by estimating portions of the full received vector autocorrelation matrix. Our method relies on the usual design of the guard time: the usual cyclic prefix used in DAB or xDSL systems (i.e. not the trailing zero precoder proposed in [5, 6]) and straightforwardly applies to classical standardized OFDM systems. Moreover it is not restricted to Discrete Fourier Transform (DFT) based multicarrier systems and works with any LL PR filterbanks modulator. Two identification results have been derived: a simple direct one like in [5] and a more elaborate one based on second order equation that, unlike in [5], can be easily solved by Cholesky type decompositions. The first identification result does not require any eigen value decomposition as done in [7, 6, 8] and can lead practically to easier real time implementations. Though based on the inherent cyclostationnarity induced at the emitter by OFDM systems and unlike other proposed methods [9, 10, 5], our approach does not refer directly to cyclocorrelations: that way the well-known slow convergence of cyclospectra estimators is avoided [11].

Notice that the proposed identification results do not require any assumption on the channel zeros location unlike methods based on output diversity [7, 12, 13] and is also robust to channel order overestimation.

First a general OFDM system scheme is introduced in section 2. Then section 3 presents the new blind identification methods and section 4 describes their practical implementation. Simulations results are shown in section 5.

2. NOTATIONS AND DESCRIPTION OF THE OFDM SYSTEM

This section presents a discrete model of the baseband OFDM system and intends to settle the notations used along this paper.

As illustrated in figure 1, a multicarrier system first modulates the size N input digital vector $\mathbf{S}(k)$ using an orthogonal matrix F (which is classically an Inverse DFT) and then a cyclic prefix of length D is appended between each time domain block vector $\mathbf{s}(k)$ to be sent sequentially through the channel. The channel is modeled by a linear filter $\mathbf{c}$ and the addition of a noise b_n.

In the following, let $(.)^T$ be the operator denoting transposition, $(.)^\star$ conjugation and $(.)^H = \left((.)^t\right)^\star$. Also define the operator $\tilde{(.)}$ as : $\tilde{G}(z) = G(z^{-1})^H$.

The total number of samples to be transmitted in the time domain is thus $P = N + D$. Denote by :

$$\begin{aligned}
\mathbf{S}(k) &:= (S_0(k), \dots, S_{N-1}(k))^T \\
\mathbf{s}(k) &:= (s_0(k), \dots, s_{N-1}(k))^T \\
\mathbf{s}^{\mathrm{gi}}(k) &:= (s_0^{\mathrm{gi}}(k), \dots, s_{P-1}^{\mathrm{gi}}(k))^T \\
\mathbf{v}(k) &:= (v_0(k), \dots, v_{P-1}(k))^T \\
\mathbf{r}^{\mathrm{gi}}(k) &:= (r_0^{\mathrm{gi}}(k), \dots, r_{P-1}^{\mathrm{gi}}(k))^T
\end{aligned}$$

where $s_p^{\mathrm{gi}}(k) = s_{N-D+p}(k)$ for $0 \leq p \leq D-1$, $s_p^{\mathrm{gi}}(k) = s_{p-D}(k)$ for $D \leq p \leq P-1$ and $s_p^{\mathrm{gi}}(k) = s_{p+kP}$, $b_p(k) = b_{p+kP}$, $r_p^{\mathrm{gi}}(k) = r_{p+kP}$ for $0 \leq p \leq P-1$.

A more general communication system representing jointly the modulation and the cyclic prefix adjunction by using the filterbank formalism is provided below. This model has the advantage to include both the particular case of figure 1 but also any multiple access transmission systems as explained in [14]. Note that the general OFDM system corresponds to the one depicted in figure 2 where the usual scalar modulation matrix is replaced by the more general filtering matrix $G(z)$ covering filters of length larger than the number of subbands (which enable a better frequency selectivity).

Define $H(z)$ as $H(z) := [G^{gi}(z)^T, G(z)^T]^T$ where the $D \times N$ matrix $G^{gi}(z)$ stands for the last D rows of $G(z)$. In the following, the modulator matrix $G(z)$ is assumed to verify the LL PR (orthogonality in the scalar case) property : $G(z)\tilde{G}(z) = I_{N\times N}$.

The system can obviously be represented in a more compact fashion as described figure 2 where the matrix $H(z)$ performs in the same time the modulation and the cyclic prefix adjunction.

Denoting by $\mathbf{c} := (c_0, \ldots, c_L, 0, \ldots, 0)_{1\times P}$ the channel impulse response and by $C(z)$ its z-transform, the linear convolution by the channel $r(z) = C(z)s(z)$ can be expressed in a block form as $\mathbf{r}^{gi}(z) = \mathcal{C}(z)\mathbf{s}^{gi}(z)$ where $\mathcal{C}(z)$ is the following polyphase subband channel filtering matrix :

$$\mathcal{C}(z) = \begin{bmatrix} c_0 & z^{-1}c_{P-1} & \cdots & z^{-1}c_1 \\ c_1 & c_0 & \searrow & \vdots \\ \vdots & \searrow & \searrow & z^{-1}c_{P-1} \\ c_{P-1} & \cdots & c_1 & c_0 \end{bmatrix}$$

Since the channel order L is assumed to fulfill $L \le D \le N \le P$, $\mathcal{C}(z)$ can be decomposed into $\mathcal{C}(z) = C_0 + z^{-1}C_1$ where C_0 is the $P \times P$ Toeplitz matrix with first column $(c_0, \cdots, c_L, 0, \cdots, 0)^T$ and first line $(c_0, 0, \cdots, 0)$ and C_1 is the $P \times P$ Toeplitz matrix with first column $(0, \cdots, 0)^T$ and with first line $(0, \cdots, 0, c_L, \cdots, c_1)$.

The expression of the received block vector $\mathbf{r}^{gi}(z)$ can thus be expressed as a function of the input block $\mathbf{S}(z)$:

$$\mathbf{r}^{gi}(z) = \mathcal{C}(z)H(z)\mathbf{S}(z) \qquad (1)$$

Finally, assuming a white input signal $S_n(k)$ with variance $\sigma_S^2 = 1$, defining H_i by $H(z) = \sum_{i=0}^{T} H_i z^{-i}$ and $H_{-1} = H_{T+1} = 0$ for conciseness sake, equation (1) turns into :

$$\mathbf{r}^{gi}(z) = \left[\sum_{i=0}^{T+1}(C_0H_i + C_1H_{i-1})z^{-i}\right]\mathbf{S}(z) \qquad (2)$$

3. THE NEW BLIND CHANNEL IDENTIFICATION RESULTS

Blind methods often require extensive computations [2]. This section first details the simple new identification method and a variant with a greater arithmetical complexity but achieving a more accurate estimation.

Intuitively the identification result relies on the following property: the adjunction of the cyclic prefix modifies the structure of the modulator and helps to suppress the Inter Block Interference so that a meaningful portion of the product $C_0C_0^H$ can be found intact on the upper right and lower left corners of the received signal vector autocorrelation matrix $R_{rr} := \mathcal{E}[\mathbf{r}^{gi}(k)\mathbf{r}^{gi}(k)^H]$ (thanks to the modulator LL PR property $G(z)\tilde{G}(z) = I_{N\times N}$). Based on this observation, the channel coefficients can be evaluated up to a scalar factor (intrinsic indetermination present in all blind method) by calculating some elements of the auto-correlation matrix R_{rr}.

3.1. The low arithmetical complexity algorithm

The proposed procedure is summarized in the following theorem and is further described below.

Theorem 1 *Assuming the first channel coefficient $c_0 \neq 0$, the channel order $L < D$ and a white additive noise b_n, the channel vector $\mathbf{c}$ can be directly read (up to the scalar coefficient $c_0^\star$) from the first column of the autocorrelation matrix $R_{rr} = [R_{i,j}]$:*

$$\boxed{c_0{}^\star(c_0, \ldots, c_L) = (R_{N+1,1}, \ldots, R_{N+1+L,1})}.$$

Proof: using (2) leads to the expression of the autocorrelation matrix $R_{rr} = \sum_{i=0}^{T+1}(C_0H_i + C_1H_{i-1})(C_0H_i + C_1H_{i-1})^H + \sigma_b^2 I_{P\times P}$ where $\sigma_b^2 := \mathcal{E}[\|b_n\|^2]$ and $I_{P\times P}$ denotes the $P \times P$ identity matrix.

Applying the perfect reconstruction equation, the autocorrelation matrix R_{rr} elements can be simplified:

$$R_{rr} = C_0JC_0{}^H + C_1JC_1{}^H + \sigma_b^2 I_{P\times P}$$

where $J = S_R{}^N + S_L{}^N + I_{P\times P}$ and S_R (respectively S_L) represents the $P \times P$ right (respectively left) shift matrix "with lost" defined by $S_{R\,i,j} = 1$ if $j = i+1$ and 0 otherwise and $S_L = S_R{}^H$.

Due to the particular structure of the matrix C_1, it can be shown that:

$$C_1JC_1{}^H = C_1C_1{}^H = \begin{bmatrix} \bar{C}_1\bar{C}_1^H & 0_{L\times(P-L)} \\ 0_{(P-L)\times L} & 0_{(P-L)\times(P-L)} \end{bmatrix} \qquad (3)$$

On the other hand the product $S_R{}^N C_0{}^H$ has the following form:

$$S_R{}^N C_0{}^H = \begin{bmatrix} 0_{D\times D} & \bar{C}_0 \\ 0_{N\times D} & 0_{N\times D} \end{bmatrix}$$

where the $D \times D$ matrix $\bar{C}_0$ is the Toeplitz Matrix with first row $(c_0{}^\star, \ldots, c_L{}^\star, 0, \ldots, 0)$ and with first column $(c_0{}^\star, 0, \ldots, 0)$. Now it becomes clear that the first column of $C_0S_R{}^NC_0{}^H$ is null and that its first row is equal to $c_0{}^\star \times (\underbrace{0, \ldots, 0}_{N \text{ times}}, c_0, \ldots, c_L, \underbrace{0, \ldots, 0}_{D-L \text{ times}})$.

Finally the first row and the first column of $C_0C_0{}^H$ are equal to $L_1 := c_0\,(c_0{}^\star, \ldots, c_L{}^\star, 0, \ldots, 0)$ and to $C_1 := L_1^H$ which leads, thanks to (3) and the previous remark, to theorem 1

3.2. Cholesky decomposition based method

Practically, the calculus of $R_{N+1+i,1}$ is performed iteratively by usual averaging techniques who lead to estimation errors. This observation motivated the search of a way to improve the accuracy of the channel estimation. This seems all the more feasible since for the moment only the first column of matrix R_{rr} is used: intuitively only a part of the available information of matrix R_{rr} is taken into account. One could expect that exploiting a larger portion of R_{rr} would lead to better performance in convergence rate and the method detailed below confirms our expectations.

Let $\check{R}$ and $\check{C}_0$ be respectively the submatrices of R_{rr} and C_0 defined respectively by $\check{R}_{i,j} = R_{i+N,j}$ for $1 \le i,j \le L+1$ and $\check{C}_{i,j} = C_{0\,i,j}$ for $1 \le i,j \le L+1$. Using these notations, it can be shown that the Cholesky factorization of $\check{R}$ provides directly $\check{C}_0$ up to a phase coefficient since R_{rr} has the following structure:

$$R_{rr} = \begin{bmatrix} \times & \times & \check{C}_0\check{C}_0^H \\ \times & \times & \times \\ \check{C}_0\check{C}_0^H & \times & \times \end{bmatrix}$$

This observation leads to theorem 2.

Theorem 2 *Assuming the first channel coefficient $c_0 \neq 0$, the channel order L verifying $2L < N$ and a white additive noise b_n, the channel vector $\mathbf{c}$ can be obtained up to a scalar coefficient from the Cholesky factorization of the submatrix $\check{R} = \check{C}_0\check{C}_0^H$ of the autocorrelation matrix R_{rr}.*

3.3. Comments on the results

The two previous identification theorems deserve some comments.

Arithmetical complexity: as expected the second method is more arithmetically expensive: it requires $O(L^2)$ operations to be performed for each OFDM received block of symbols whereas the first one only requires $O(L)$ operations;

Assumptions: it is worth noting that the only requirements on our two methods is that the first channel coefficient c_0 has to be different from zero (i.e. the synchronization has to be accurately achieved), which is a lighter assumption than in [6] where both c_0 and c_L have to be different from 0. A direct consequence is that our method is naturally robust to a channel order overestimation and does not require any constraint on channel zeroes location;

Robustness to the noise: surprisingly, in both method the noise has ideally no impact on our estimation result. Indeed, when dealing with stationary additive and uncorrelated noise, only the diagonal terms of matrix R_{rr} are affected which are not used in the two previous theorems.

4. PRACTICAL IMPLEMENTATIONS, ESTIMATION STRATEGIES AND PROPERTIES

4.1. Convergence rate

In practice, the autocorrelation matrix needs to be estimated. This task can be performed iteratively by classical averages techniques e.g. : $\hat{R}_{rr}^{(N)} := \frac{1}{N}\sum_{k=0}^{N-1} \mathbf{r}^{\mathrm{gi}}(k)\mathbf{r}^{\mathrm{gi}}(k)^H$.

Assuming a second order stationary input signal, since the channel order is assumed to be finite, $\hat{R}_{rr}^{(N)}$ converges in the mean square sense to R_{rr}. Therefore it is possible to show that the channel estimation is unbiased (up to a scalar factor) and the estimation mean square error (MSE) is upper bounded by $O(\frac{1}{N})$.

For the second method, the repercussion of the estimation errors in a Cholesky factorization $C_{\check{R}}$ of $\check{R}$ lead to a matrix which is not exactly Toeplitz. In that case the distance between $C_{\check{R}}$ and $\check{C}_0$ needs to be minimized in order to retrieve the channel coefficients :

$$\boxed{(\hat{c_0}, \dots, \hat{c_L}) = \underset{(c_0, \dots, c_L)}{\operatorname{argmin}} \|\check{C}_0(c_0, \dots, c_L) - C_{\check{R}}\|} \qquad (4)$$

where $\|.\|$ stands for any matrix norm.

However in practice a bad conditioning of the real matrix $\check{R}$ can occur and in that case estimation errors can lead to a completely erroneous result (far from the true Cholesky factorization). A solution is to work in a semi-blind context as detailed in the next paragraph where the autocorrelation matrix estimator is initialized thanks to reference symbols. This allows to reduce the magnitude of the estimation errors which are really critical when dealing with ill-conditionned matrices.

Notice that the identification result provided in equation (4) is not restricted to a theoretical interest since it can be shown that its resolution in the mean square sense only consists in a matrix multiplication. Here no complex solving of a non-linear system of equations is required as opposed to [10, 5].

4.2. Semi-blind identification

An inherent problem to blind identification methods is their rather slow converging rate [2]. This drawback often prevents their use in the practical context where method based on training sequence are preferred. However it is possible to merge the advantages of both approaches operating in a semi-blind context [3].

The idea is to refine the pilot based estimation along the frame blindly. This allows a better tracking of the channel variations.

The two proposed identification methods can easily be used in a semi-blind context. Indeed it is possible to use OFDM reference symbols to initialize the estimation of the channel coefficients which provides a "reference" correlation matrix. This matrix is then used in the iterative averaging process as a mean to increase the robustness of the estimator.

As illustrated in the simulation section, this procedure practically enhance the channel estimation convergence rate.

5. SIMULATIONS AND CONCLUSIONS

This section presents a comparison between the classical DAB-like channel identification method based on a reference symbol inserted at the beginning of each frame and the two identification algorithms proposed in this paper. All results are obtained running Monte Carlo simulations based on 100 trials.

The symbols to be modulated by the IDFT belong to a QPSK constellation with average energy $\sigma_S^2 = 1$. They are independent and identically distributed.

All evaluations are made for a $N = 128$ carrier OFDM system with a cyclic prefix of $D = N/4 = 32$ samples. The channel to be estimated is a typical rice channel of order $L = 30$ whose frequency impulse response is depicted in figure 3.

Robustness to noise: in figure 4, the Mean Square Error (MSE) on the estimated channel coefficients is plotted versus the number of OFDM symbols received for various Signal to Noise Ratios (SNR): $+\infty$ (o), +10 dB ($*$) and -5 dB ($\triangle$). This figure illustrates the convergence behaviour of the low arithmetical complexity identification algorithm and its ability to identify the channel even when more noise than signal is received.

Cholesky versus direct identification: a comparison of the convergence rates of the two identification methods is provided figure 5 for a fixed SNR of 10 dB: the "low cost" algorithm ($\triangle$) and the Cholesky decomposition based algorithm ($*$). In addition for reference, the "low cost" algorithm behaviour is provided in the noiseless case (o). The simulation shows the clear improvement brought by the the Cholesky based algorithm over the low cost one: it performs even better at a SNR of 10 dB than the low cost one in the ideal noiseless case.

Semi-blind context: figure 6 illustrates how the use of reference symbols at the beginning of each frame improves the accuracy of the blind channel estimation. In the scenario used 3 frames of 100 OFDM symbols are transmitted and the estimation MSE given by the classical OFDM method, by the Cholesky based algorithm used in the blind context ($\diamond$) and used in cooperation with pilot symbols ($*$) are plotted. It clearly appears that, the proposed semi-blind identification strategy allows at the end of a frame a further gain of about 3dB on the MSE compared to the classical method.

6. REFERENCES

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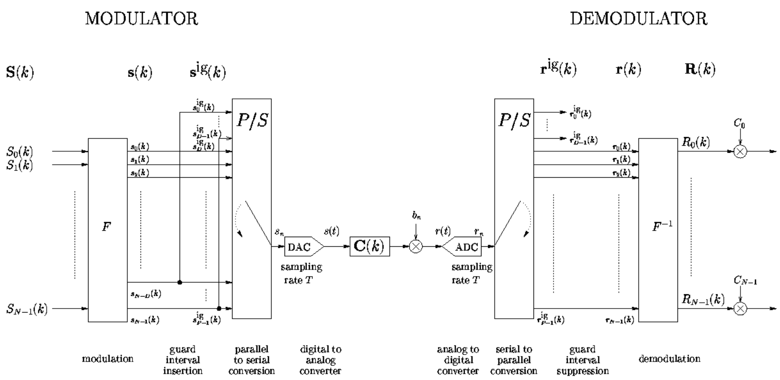

Figure 1: OFDM discrete baseband transmission system model

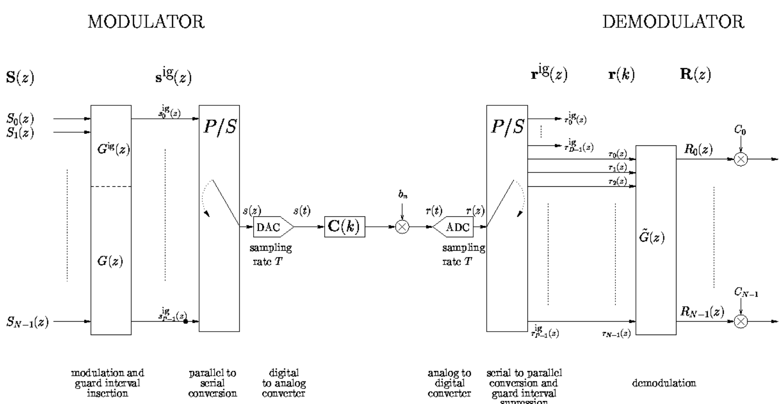

Figure 2: General OFDM transmission system

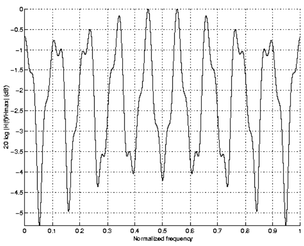

Figure 3: Rice channel frequency response

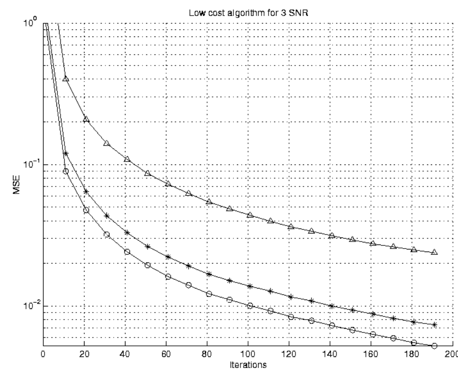

Figure 4: Channel estimation MSE for 3 SNR

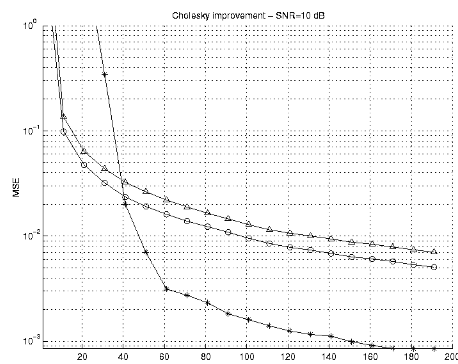

Figure 5: Cholesky improvement

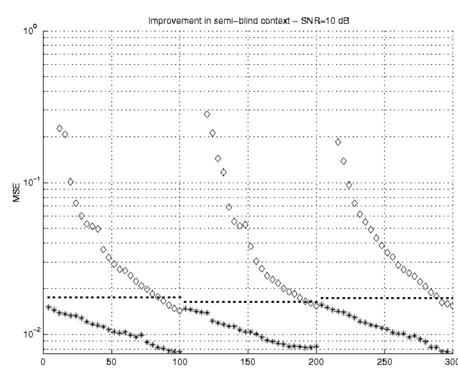

Figure 6: Classical, blind and semi-blind identification