MOTION ESTIMATION USING A VOLUME CONSERVATION HYPOTHESIS

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ABSTRACT

Nowadays, motion estimation is one of the main subjects in computer vision. Many methods developed to compute motion make use of the optical flow hypothesis. These methods usually fail to capture motion of objects with intensity evolution. We propose a new approach to solve the motion computation problem with a different type of constancy hypothesis. Because we are mainly interested in deformable moving structures, we postulate that such a structure, within a temporal image sequence, is associated with a constant volume or a constant *total intensity* over time. We call this postulate *the volume conservation* hypothesis. Results are displayed for clouds motion and deformation on meteorological satellites images.

1. INTRODUCTION

Motion computation on image sequences is currently under extensive investigation in computer vision science. We may distinguish two categories of methods: methods that are dedicated to detect regions in motion and methods that give an estimation of the velocity vector field, like correlation or optical flow methods. Correlation methods [6] consist of computing a correlation within a small window and finding the best fit between two images. This method is interesting in case of small object structures (included within the window) but is very sensitive to the window size parameter. The optical flow method, introduced by Horn and Schunck [3], assumes that a moving pixel keeps the same grey level value over time, which can be mathematically formulated by the equation:

$$\frac{dI}{dt} = 0 \qquad (1)$$

where I is the grey level value of the pixel. Using the chain rule for derivation, we have:

$$\nabla I.w + I_t = 0 \qquad (2)$$

where ∇ designs the spatial gradient operator, $I_t = \dfrac{\partial I}{\partial t}$ and $w = (\dfrac{\partial x}{\partial t}, \dfrac{\partial y}{\partial t})$ represents the velocity vector. To solve this equation, many techniques are available: a variational formulation [3], a Markov random field approach [5] or a parametric modeling [1], [4]. Optical flow methods have two main drawbacks. First, the optical flow hypothesis is not necessarily valid for some applications. Then, optical flow holds best for rigid objects. A non negligible class of objects motion cannot be studied within this framework.

In section 2 we present a new hypothesis based on a total intensity principle with a volume conservation constraint equation. Resolution and validation are discussed in section 3. In section 4 we present an example (a meteorological infrared image sequence) where the optical flow hypothesis is no more valid but where the volume conservation hypothesis does successfully apply.

2. A VOLUME CONSERVATION HYPOTHESIS

2.1. Model

As outlined in introduction, motion estimation with optical flow may be incorrect if the grey level constancy hypothesis is not verified. To overcome this difficulty, a good solution is to consider the motion of objects rather than that of the pixels, as performed by optical flow. It can be formulated in the following way: the *total intensity* of objects, *i.e.* the sum of grey level values within the object, is temporally constant. This has to be compared to the optical flow hypothesis: the intensity of each pixel is temporally constant. If one considers the intensity axis as a third dimension, our assumption becomes a *volume conservation hypothesis* as depicted on figure 1.

The definition of an elementary volume element, *i.e.*, the volume attached to a pixel (x, y) at time t of surface $ds = dxdy$ is given by:

$$d\mathcal{V} = I(x, y, t)dxdy$$

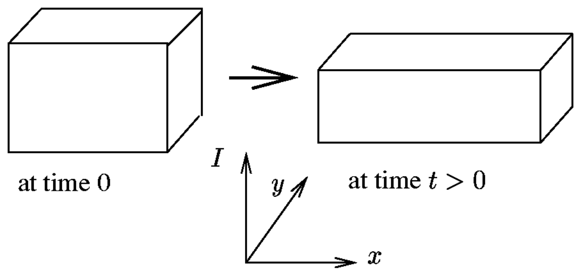

Figure 1: *Motion model: the "volume" or "total intensity" is constant.*

where (x, y) are the coordinate of the pixel and $I(x, y, t)$ its grey level value at time t.

2.2. The volume constraint equation

Lets us show that the volume conservation statement yields a mathematical constraint on pixel's velocities.

Let $\varphi(x, y, t)$ be the spatial position at time t of the pixel being at position (x, y) at time $t = 0$. In fact, φ describes the spatial evolution of a given pixel over time and therefore $\frac{\partial \varphi}{\partial t} = w$ is its velocity. Let V_t be the volume or the *total intensity* of an object $\mathcal{V}$ at time t:

$$V_t = \int\int_{\mathcal{V}} I(X, Y, t) dX dY \qquad (3)$$

It can be formulated as a function of trajectories φ by applying the change of variable (X, Y). Volume equation becomes:

$$V_t = \int\int_{\mathcal{V}} I(\varphi(x, y, t), t) J(\varphi(x, y, t)) dx dy$$

where J is the Jacobian operator. As we assume that a volume element stays constant over time, the volume conservation hypothesis is formulated by: $\frac{dV_t}{dt} = 0$ then becomes:

$$\int\int_{\mathcal{V}} \frac{d}{dt}[(I \circ \varphi) J(\varphi)] dx dy = 0$$

Using the approximation $\frac{d}{dt} J(\varphi) = \operatorname{div}(\frac{d\varphi}{dt}) J(\varphi)$, meaning that the deformation has to be small, we obtain:

$$\int\int_{\mathcal{V}} [\frac{\partial \varphi}{\partial t} \nabla I \circ \varphi + I_t \circ \varphi + I \circ \varphi \operatorname{div}(\frac{\partial \varphi}{\partial t})] J(\varphi) dx dy = 0$$

The previous equation is verified when:

$$\frac{\partial \varphi}{\partial t} \nabla I \circ \varphi + I_t \circ \varphi + I \circ \varphi \operatorname{div}(\frac{\partial \varphi}{\partial t}) = 0$$

Remember that $\frac{\partial \varphi}{\partial t} = w$, our condition can be rewritten as:

$$w \nabla I + I_t + I \operatorname{div}(w) = 0 \qquad (4)$$

that is called the volume constraint equation.

This condition is very close to the optical flow equation (2). A new term, $I \operatorname{div}(w)$, appears in it, modeling the spatial deformation of w. Wildes [7] obtains the same formulation in a different context of fluid motion modeling.

3. RESOLUTION

3.1. Solve the volume conservation equation

The problem is to determine the value of the apparent vector field w. We use a variational formulation to compute the solution of equation (4). We build a functional whose minimum value with respect to w gives the solution, w_{min}:

$$E_1(w) = \int_{\Omega} (w.\nabla I + I_t + I \operatorname{div}(w))^2 dx dy \ , \qquad (5)$$

Since equation (4) is underdetermined, as the vector w represents two unknown values, it is necessary to use an additional constraint to solve it. So we build a second functional that constrains spatial deformations of w and overconstrains equation (4):

$$E_2(w) = \int_{\Omega} (|\nabla u|^2 + |\nabla v|^2) dx dy \ , \qquad (6)$$

with $w = (u, v) = (\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t})$. Finally we minimize with respect to w the following functional:

$$E(w) = E_1(w) + \alpha E_2(w) \ , \qquad (7)$$

where α is a tuning parameter between the two terms, weighting the importance of the regularizing term E_2.

The minimization of the functional E yields a set of Euler-Lagrange equations (see [2]):

$$\begin{cases} -\frac{\partial}{\partial x}(I(w \nabla I + I_t + I \operatorname{div}(w))) - \alpha \Delta u = 0 \\ -\frac{\partial}{\partial y}(I(w \nabla I + I_t + I \operatorname{div}(w))) - \alpha \Delta v = 0 \end{cases}$$

Finally, we use the evolution equations associated to the Euler-Lagrange equations to compute w.

3.2. Validation

The volume conservation hypothesis has been applied to synthetic sequences in order to validate the model. The synthetic sequence used in figure 2 shows a growing square. It grows simultaneously in the four directions and the grey level values inside the square change over time in order to respect the volume conservation hypothesis (from clearer to darker).

As we can see in the figure 2, the method has estimated the correct vector field of the square. Notice that there is no motion estimation in the background because the grey level values are always equal to zero. Inside the square, we get non zero vectors, but the norm of these vectors is close to zero and can not be displayed because these values have no signification. This result has to be compared with the optical flow method on figure 3. This method fails because there is no grey level value conservation.

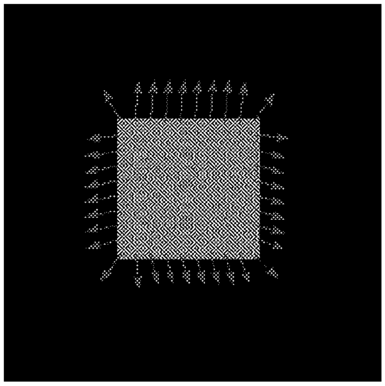

Figure 2: *Volume conservation computed on a growing square sequence.*

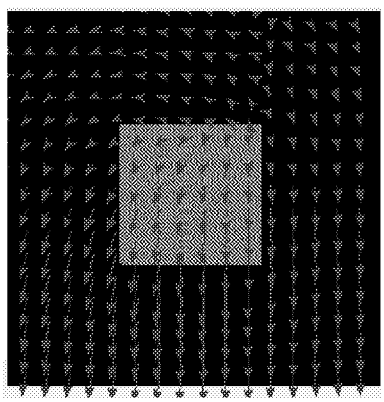

Figure 3: *Optical Flow computed on a growing square sequence.*

4. APPLICATION

The model has been applied to meteorological infrared images. On these images, grey level values measure a temperature. There are two kinds of structures: the ground and the clouds. The ground has no interest for us because there is no motion to be detected (satellites providing these data are geostationnary). The cloud temperature has an important property: it is related to its elevation. So the grey level value provides the third dimension useful in the volume conservation model. We use the volume between the top of the cloud and the ground instead of the real volume of the cloud. Figure 4 shows the volume conservation hypothesis applied to the cloud structure.

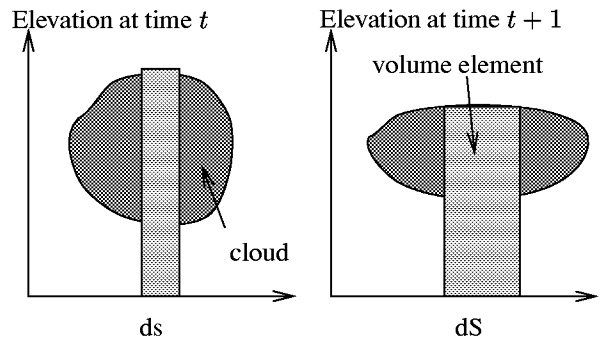

Figure 4: *Conservation of cloud between two dates.*

Figure 5 displays in background, the first image of an infrared sequence. This sequence focuses on a vortex evolution over time. Computation of optical flow and volume conservation methods are done between two successive images. The results are displayed on figures 5 and 6 and we can see that those provided by the volume conservation hypothesis are better than those obtained with the optical flow equation (figure 6):

- detection of singular points: optical flow model can not detect any singular point. The volume conservation model detects several singular points. It is very important because these points give a good estimation of the cloud motion. For example, our method locates a singular point at the center of the vortex (fig. 6), that it is very interesting in a tracking point application.
- the optical flow method detects motion in darker zones *i.e.* zones where there is no cloud and no motion. The volume conservation model does not detect motion in these zones because their grey level values are almost constant and close to zero,
- experimentation of the volume conservation method on a complete infrared sequence shows a robust result over time,
- optical flow smoothes the velocity field: one can see on figure 5, optical flow gives continuous values over the vortex and boundaries vortex and therefore detects velocity on the ground. At the opposite, the volume conservation is an object approach, and we obtain a non continuous velocity field over the vortex frontier.

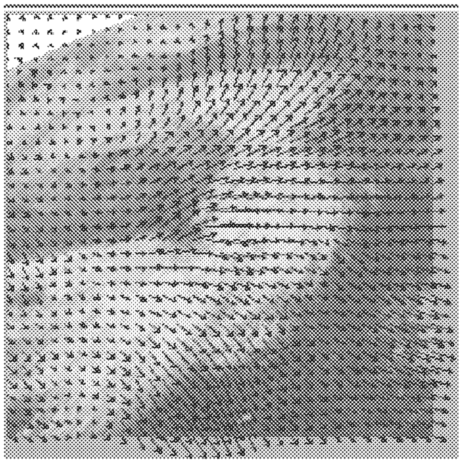

Figure 5: *Result of Optical flow method.*

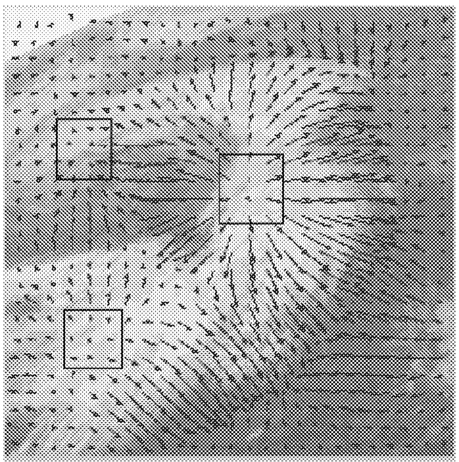

Figure 6: *Result of Volume conservation method. Squares show some singular points.*

The volume conservation method seems better adapted in the meteorological context than the optical flow method due to the object deformable modeling that optical flow cannot handle.

5. CONCLUSION

In this paper, we presented a new method for motion computation. Contrary to optical flow, that is a pixel approach, our method considers objects through the volume conservation equation (4) derivated from a total intensity principle in order to enhance the computation of deformable structures' motion. As noticed in the paper, the volume conservation method has to be applied to deformable structures which must have a total intensity constant over time. It is the case with clouds on meteorological infrared images as seen in section 4 or with synthetic scenes such as seen in subsection 3.2.

In a context like infrared data, we also get the following problem that gives us research perspectives: the images contain two type of region in which grey level values have different signification. There are structures where the volume conservation hypothesis is valid: these are deformable objects with a total intensity remaining constant. There are also structures where the optical flow hypothesis applies: these are objects where grey level value is constant over time. Thus, a mixed model should be used: first, a volume conservation model describing moving deformable structures (as cloud in meteorological data) and, for example, an optical flow model describing others structures. But it requires a pre-segmentation process in order to obtain a bi-labelled image describing zones where optical flow is valid and zones where volume conservation zone is valid. This part is currently under development.

6. REFERENCES

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