FADING-RESISTANT AND MUI-FREE CODES FOR CDMA SYSTEMS

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ABSTRACT

A new class of codes was proposed recently for perfect multi-user interference (MUI) suppression in Code Division Multiple Access (CDMA) systems. These so called Lagrange-Vandermonde (LV) codes offer deterministic MUI elimination without channel estimation, and convert frequency selective channels into flat fading channels. In this work, we develop dual CDMA transceivers, naturally termed Vandermonde - Lagrange (VL), with the precoder - decoder roles interchanged, in order to improve system flexibility in the code assignment. We also derive and test non-redundant and redundant techniques for combating the residual flat fading.

1. INTRODUCTION AND MODELING

Code Division Multiple Access (CDMA) is currently under consideration as the access technique for the third generation mobile communications in Europe [5]. One of the major limitations of wideband CDMA systems is multi-user interference (MUI) occurring in the presence of multipath propagation. In [2] a new class of codes, named Lagrange-Vandermonde (LV) codes, was proposed to achieve complete MUI suppression without estimating the channel or resorting to computationally intensive decoding algorithms, as in [3], [4], [7]. Using the LV coding strategy, the MUI is completely eliminated, irrespective of the channel zero locations and frequency-selective fading is converted to flat fading. In this work, we interchange the role of the precoder and decoder structures, with respect to the LV scheme, to gain flexibility in assigning user codes when slowly time-varying fading is present. We also propose redundant and non-redundant techniques for combating the residual flat fading.

The block diagram in Fig. 1 represents the uplink channel of a CDMA system, described in terms of its equivalent discrete-time baseband model, where signals, codes, and channels are represented by samples of their complex envelopes taken at the chip rate. Up-samplers and downsamplers serve the purpose of multiplexing and demultiplexing (spreading and despreading) by a factor P. Each of the M users spreads the information sequence $s_m(n)$ with the upsampler and en-
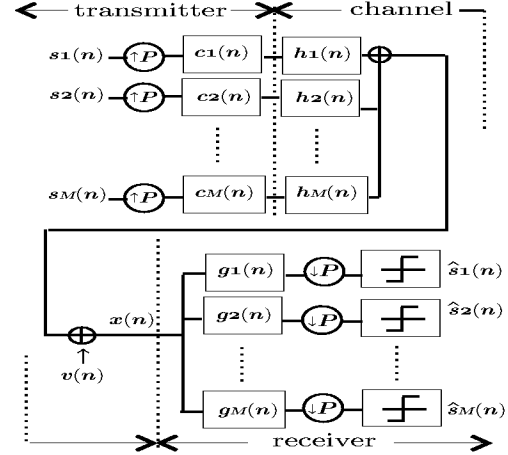

Figure 1: Multirate Discrete-Time CDMA Model.

codes it using the code $c_m(n)$ of length P, before transmission through the *unknown* Lth-order channel $h_m(l)$ which, in addition to multipath, includes the transmit spectral-shaping pulse and the mth user's asynchronism in the form of delay factors. The multiplexed data are received in AGN $v(n)$, filtered, and sampled at the chip rate. Subsequent processing by the receive-filter $g_m(k)$, is followed by despreading (downsampling) by P and decision, to obtain the estimated $\hat{s}_m(n)$. We assume that: (**a1**) $P - M \geq L$ and $M > L$, where L is the maximum expected order of all channels $\{h_m(l)\}_{m=1}^{M}$ so that there is ISI at the chip rate but not at the symbol rate, and (**a2**) the codes have $L_g \geq L$ redundant prefix bits ('guard chips').

Based on (a1)-(a2), the received samples are $x(n) = \sum_{m=1}^{M} \sum_{i=-\infty}^{\infty} s_m(i) \sum_{l=0}^{L} h_m(l) c_m(n - l - iP) + v(n)$ or, equivalently, $x(nP + k) = \sum_{m=1}^{M} \sum_{i=-\infty}^{\infty} s_m(i) \sum_{l=0}^{L} h_m(l) c_m((n - i)P + k - l) + v(nP + k)$. Hence, considering that $c_m(i) = 0$ for $i < 0$ or $i > P - 1$, the only nonzero term in the summation over i corresponds to the index $i = n$, so that $x(nP + k) = \sum_{m=1}^{M} s_m(n) \sum_{l=0}^{L} h_m(l) c_m(k - l) + v(nP + k)$. In matrix form[1], introducing the nth $P \times 1$ data vector $\boldsymbol{x}(n) := [x(nP)x(nP + 1) \cdots x(nP + P - 1)]^T$ ($\boldsymbol{v}(n)$ is

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[1]Upper (lower) boldface notation will be used for matrices (column vectors); *, ^{T}, and $^{\mathcal{H}}$ will denote complex conjugation, transposition, and Hermitian transposition, respectively.

defined similar to $\boldsymbol{x}(n)$), we get

$$\boldsymbol{x}(n) = \sum_{m=1}^{M} \boldsymbol{C}_m \boldsymbol{h}_m s_m(n) + \boldsymbol{v}(n) , \qquad (1)$$

where the convolution of the mth user's code with the channel is represented as multiplication of the mth user's $P \times (L+1)$ Toeplitz matrix $\boldsymbol{C}_m$ (with first column $[c_m(0), \cdots, c_m(M-1), 0, \cdots, 0]$ and first row $[c_m(0), 0, \cdots, 0]^T$) by the channel vector, $\boldsymbol{h}_m^T := [h_m(0), h_m(1) \cdots h_m(L)]$. If $\boldsymbol{g}_k := [g_k(P-1) \cdots g_k(1) g_k(0)]^T$ is the weight vector for receiver k, its output takes the form:

$$\boldsymbol{g}_k^T \boldsymbol{x}(n) = \boldsymbol{g}_k^T \boldsymbol{C}_k \boldsymbol{h}_k s_k(n) + \sum_{m=1, m\neq k}^{M} \boldsymbol{g}_k^T \boldsymbol{C}_m \boldsymbol{h}_m s_m(n) + \boldsymbol{g}_k^T \boldsymbol{v}(n). \quad (2)$$

Our problem is to design precoders $\{c_m(n)\}_{n=0}^{M-1}$, or equivalently their polynomials $C_m(z) := \sum_{n=0}^{M-1} c_m(n) z^{-n}$, and the corresponding decoders $\boldsymbol{g}_k$ so that the MUI (described by the sum in (2)) is eliminated completely.

2. LAGRANGE-VANDERMONDE CDMA

In [2], a simple method was derived to build sets of MUI-free codes. In particular, given a set of complex numbers $\{\rho_i\}_{i=1}^M$ (corresponding to code polynomial *roots*), the code design algorithm proceeds through the following steps [2]:

i) the transmit codes c_m, with $m = 0, \ldots M-1$, have a $\mathcal{Z}$ transform $C_m(z) := \sum_{n=0}^{M-1} c_m(n) z^{-n}$:

$$C_m(z) = \mathcal{K}_m \frac{\prod_{i=1, i\neq m}^{M}(1 - \rho_i z^{-1})}{\prod_{i=1, i\neq m}^{M}(1 - \rho_i \rho_m^{-1})}, \quad m \in [1, M]; \qquad (3)$$

ii) the M FIR receive filters are *Vandermonde* filters with: coefficients $\boldsymbol{g}_i^T := [\rho_i^{P-1}, \cdots, \rho_i, 1]$. By direct substitution, it follows that [2]:

$$\boldsymbol{g}_i^T \boldsymbol{C}_m = \mathcal{K}_m[C_m(\rho_i), \rho_i^{-1} C_m(\rho_i), \cdots, \rho_i^{-L} C_m(\rho_i)] = \boldsymbol{0}^{\mathcal{H}}; \qquad (4)$$

for $i \neq m$ and

$$\boldsymbol{g}_m^T \boldsymbol{C}_m \boldsymbol{h}_m = \mathcal{K}_m \sum_{l=0}^{L} h_m(l) \rho_m^{-l} := \mathcal{K}_m H_m(\rho_m). \quad (5)$$

Eqn.(4) proves that MUI is cancelled completely, while (5) shows that a frequency-selective channel is converted into a flat fading channel characterized by the complex factor $H_m(\rho_m)$. To cope with flat fading, we can exploit the M degrees of freedom available with the selection of the M roots ρ_i. For example, if it happens that the mth channel has a zero on ρ_m, it is evident from (5) that the mth user cannot be recovered. In such a case, using power control, as foreseen in UMTS for example [5], would not help because the received power would remain zero, irrespective of the transmitted power. However, exploiting the degrees of

freedom in selecting roots, we can avoid the problem by replacing the root ρ_m with another root $\rho'_m \neq \rho_m$, so that it is likely that ρ'_m will not be a channel root. This means that the base station (BS) must re-assign the codes to each user and change the receiver correspondingly. This operation would solve the problem of a deep fade on some channels, but it would go against the basic CDMA premise of entailing minimal user co-ordination.

3. VANDERMONDE-LAGRANGE CDMA

To simplify the procedure for switching from one code set to another and add flexibility to our system, we propose to interchange the role of the Lagrange and Vandermonde codes. In fact, the desired properties (4-5) can be achieved even if we use Vandermonde vectors as precoders and Lagrange vectors as decoders. Specifically, given a set of roots $\{\rho_i\}_{i=1}^M$, we build the code pairs as follows: i) the precoding vector for the mth user is a Vandermonde $1 \times P$ vector $\boldsymbol{c}_m := \mathcal{K}_m(1, \rho_m \ldots \rho_m^{P-1})$, where $\mathcal{K}_m$ is a proper constant introduced to fix the transmitter power; ii) the decoding vector for the kth user is the $1 \times P$ vector $\boldsymbol{g}_k^T := (0, \ldots, 0, g_k(M-1), \ldots, g_k(0))$ where the leading zeros are used to cancel the guard interval where there is superposition of two consecutive symbols. Inserting these expressions in (2), it is straightforward to verify that $\boldsymbol{g}_k^T \boldsymbol{C}_m \boldsymbol{h}_m = \rho_m^{P-1} G_k(\rho_m) H(\rho_m)$, where $G_k(z)$ is the polynomial associated to the vector $\boldsymbol{g}_k$; i.e., $G_k(z) = \sum_{l=0}^{M-1} g_k(l) z^{-l}$. Therefore, if we build the vectors $\boldsymbol{g}_k$ so that the functions $G_k(z)$ are the Lagrange polynomials

$$G_k(z) = \frac{\prod_{i=1, i\neq k}^{M}(1 - \rho_i z^{-1})}{\prod_{i=1, i\neq k}^{M}(1 - \rho_i \rho_k^{-1})}, \quad k \in [1, M], \qquad (6)$$

we can satisfy the same conditions as (4-5). We will term the transceivers adopting this section's precoder - decoder structure as Vandermonde-Lagrange (VL) transceivers. The advantage of the new formulation, with respect to [2] is that in case of a deep fade, say $H(\rho_m) = 0$, the BS needs only to warn the mth transmitter to switch from the root ρ_m to a different root, but all other users may keep working with the same roots as before. Hence, the new formulation increases the flexibility of the system in the root assignment procedure. Of course, changing one user's root requires modification of all the decoder vectors, but this operation is performed at the BS and thus it is less troublesome with respect to the scenario where all the users had to change their precoding vectors as in [2].

4. ROOT VS. POWER CONTROL

For a given set of channels, the degrees of freedom offered by the root selection could be optimally exploited by searching for the set of roots that minimize

the error probabilities of all users. However, even for known channels, such an optimization requires nonlinear programming with a large number of parameters. Furthermore, the proposed VL transceivers guarantee the zero-MUI condition, irrespective of the channel zero locations and without being necessary to know or estimate the channels. In this section, we propose a sub-optimal technique for changing the root positions when deep fades occur, while maintaining the blind feature of our VL transceivers. The method generalizes the power control technique currently foreseen by UMTS, where the BS sounds the power level of the signals received from all the active users and sends back control signals requiring users to increase or decrease their transmitted power in order to limit MUI. In principle, the VL coding strategy eliminates MUI completely, so that power control is not needed. However, some procedure must be used to recover from deep fading that arises when some channels have zeros at the corresponding users' codes.

Given the maximum number of active users, say M, the BS starts working with a set of complex roots equispaced around the unit circle. This situation corresponds to the well known Orthogonal Frequency Division Multiple Access (OFDMA) [2]. For flat fading channels, OFDMA maximizes SNR because the decoder is a filter matched to the precoder [2]. With each user's signature-root on the unit circle, the BS periodically sounds the power received from all active users. If some users are received with a power below a prescribed threshold, the BS initializes a procedure to move the corresponding root away from its initial position. If the number of active users is less than the maximum number of users, the BS may simply assign a new signature-root on the unit circle to a low-power user. However, if the unit-circle roots are all occupied because the number of active users is equal to its maximum value, the BS must undertake some different action.

Different from OFDMA, in our VL-CDMA system the BS moves the root of the deeply faded user radially on the complex plane. Specifically, the root's phase is not altered to avoid undesired superpositions with other roots, but its modulus is varied within a certain range. The user thus gets associated with a new root having the same angle as before, but with the modulus corresponding to the maximum power received from the BS. If that power level is still not sufficient for reliable decoding, the BS asks the user to increase the transmitted power.

As an example, Fig. 2 refers to the case where 32 possible users are all active and one of them (say the user corresponding to the root $\rho = 1$) transmits through a channel having a zero at $z = 1$. Fig. 2 shows the power that the BS receives from all users, as a function of the root location of the user experi-
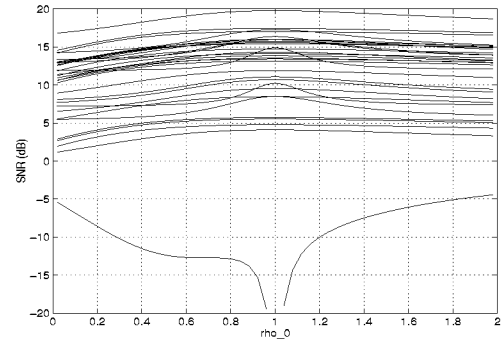

Figure 2: Power received from all 32 users vs. the modulus of the root of the user with a deep fade.
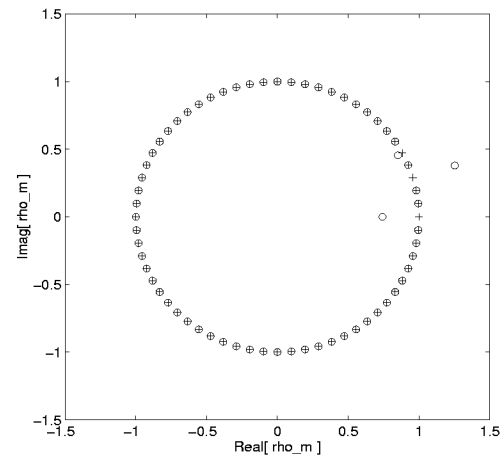

Figure 3: Power received from all users vs. the modulus of the root of the user with a deep fade.

encing the deep fade (each user transmits at a level of 0 dBW). We observe that for $\rho = 1$, the power received from one user goes to $-\infty$ dB. Changing the root location, some of the other users will suffer a loss, but the user with the deepest fade recovers. This gain is not possible by acting only on the user's transmit power. After having recovered from the deep fade, further power control may be used to balance the relative power of the different users. As an additional example, we considered the case of 64 possible users and we simulated the situation where two users' channels have zeros on the corresponding users' codes. The BS changes the position of all users such that the corresponding received power falls below a prescribed level. Fig. 3 shows the final position of the roots (the $'+'$ correspond to the initial position, whereas the $'o'$ correspond to the final position). In this particular case, the minimum ratio E_b/N_0, where E_b is the energy per bit and N_0 is the noise power spectral density, recovers from $-\infty$ dB to -7 dB.

5. REDUNDANCY AGAINST FADING

From (4-5) we recall that the VL scheme eliminates MUI and converts frequency selective fading channels into flat fading channels characterized by the complex factor $H_k(\rho_k)$. This factor may introduce an excessive attenuation if ρ_k is equal (or close) to a root of $H_k(z)$. To improve the robustness of the VL scheme, we may assign more than one signature root to each user. More specifically, if we assign Q signature roots to each user, and associate each transmitted symbol $s_k(n)$ to Q Vandermonde vectors, the signal received by the kth receiver is

$$x_k(n) = \sum_{m=1}^{M} \sum_{q=1}^{Q} C_{m,q} h_m s_m(n) + v(n) , \quad (7)$$

where $C_{m,q}$ is now the matrix corresponding to the Vandermonde vector built by the root $\rho_{m,q}$. The kth receiver is composed of a bank of Q filters, each characterized by response $g_{k,q}$ built from the Lagrange polynomial having all the roots $\rho_{i,j}$, $i = 1, \ldots, M$, $q = 1, \ldots, Q$, except $\rho_{k,q}$. Therefore, the kth receiver provides Q outputs

$$y_{k,q} = g_{k,q}^T x_k(n) = \mathcal{K}_k H_k(\rho_{k,q}) s_k(n) + g_{k,q}^T v_k(n) \quad (8)$$

which can be combined to yield an estimate of $s_k(n)$. To evaluate the potential benefits coming from the *root diversity*, we adopted the maximal ratio combining (MRC) strategy, assuming that an error-free estimate of the channel is available. In such a case, using BPSK modulation, the decided symbol is:

$$\hat{s}_k(n) = \text{sign}\left(\sum_{q=1}^{Q} H_k^*(\rho_{k,q}) y_{k,q} \right)$$

$$= \text{sign}\left(s_k(n) \mathcal{K}_k^2 \sum_{q=1}^{Q} |H_k(\rho_{k,q})|^2 + w_k(n) \right) (9)$$

where $w_k(n) := \sum_{q=1}^{Q} H_k^*(\rho_{k,q}) g_{k,q}^T v_k(n)$. The performance of MRC applied to our setup can be evaluated in terms of bit error rate (BER) by extending the classical derivations for MRC (see, e.g., [6]) to the case where the fading coefficients are expressed in terms of the channel transfer functions $H_k(\rho_{k,q})$. Specifically, modeling the channel coefficients $h(k)$ as zero mean, complex Gaussian independent random variables (rv), i.e. assuming the widely used Rayleigh fading model arising when no direct path arrives at the receiver [6], the variables $|H_k(\rho_{k,q})|^2$ have an exponential probability density function (pdf). Indeed, the exponential distribution is approximately valid also if the coefficients $h(k)$ are not Gaussian, provided that the following extra conditions are met: i) the number of paths is sufficiently high; ii) the $h(k)$'s have finite moments (which is certainly true in the practical case) and are weakly correlated. Assuming that the roots $\rho_{k,q}$ have

the same modulus, $\forall k$, the variables $|H_k(\rho_{k,q})|^2$ have the same expected value (and thus variance). In general, the rv's $|H_k(\rho_{k,q})|^2$ corresponding to different values of q, are correlated. However, if we choose the roots as $\rho_{k,q} = \rho_k \exp(j 2\pi q \Delta f)$, with $q = 0, \ldots, Q-1$ and $\Delta f > B_c$, where B_c is the channels' coherence bandwidth, the random variables $|H_k(\rho_{k,q})|^2$ are approximately uncorrelated. With this approximation, the average BER after MRC is:

$$P_e = \left(\frac{1-\mu}{2} \right)^Q \sum_{q=0}^{Q-1} \binom{Q-1+q}{q} \left(\frac{1+\mu}{2} \right)^q , \quad (10)$$

where $\mu := \sqrt{\gamma/(1+\gamma)}$, γ is the average SNR per channel, i.e. $\gamma := E\{|H_k(\rho_{k,q})|^2\} \mathcal{K}^2 / \sigma_w^2$ and σ_w^2 is the variance of $g_{k,q}^T v_k(n)$. With γ sufficiently large, say above 10 dB, P_e can be approximated as:

$$P_e \approx \left(\frac{1}{4\gamma} \right)^Q \binom{2Q-1}{Q} . \quad (11)$$

This expression is useful when it comes to choosing Q as a reasonable compromise between robustness and efficiency loss. In fact, high values of Q lead to lower BER values but, at the same time, they reduce the efficiency of the transmission system, measured as number of information bits per transmitted symbol, by exactly the factor Q.

6. ACKNOWLEDGEMENT

7. REFERENCES

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