

BLIND EQUALIZERS FOR MULTIPATH CHANNELS WITH BEST EQUALIZATION DELAY

Hui Luo

Digital Video Express, LP
570 Herndon Parkway
Herndon, VA 20170, USA
Email: Hui.Luo@divx.com

Ruey-Wen Liu

Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556, USA
Email: Ruey-Wen.Liu.1@nd.edu

ABSTRACT

Recent studies show that equalizers with different equalization delays achieve different performances. The best performed equalizer is not necessarily the one with 0-delay, and often not the case when the channel is non-minimum-phase. In this paper, a blind channel equalization algorithm is presented, by which equalizers with all possible equalization delays can be calculated simultaneously from the second order statistics of received signals. A (blind) evaluation index is then presented for the purpose of selecting the best equalizer. Simulation shows that the best delayed equalizer performs much better than that for other equalization delays and those given by constant modulus algorithm and linear prediction algorithm.

1. INTRODUCTION

In high-speed wireless communication systems, equalization process is needed to suppress the intersymbol interference caused by multipath channels. Conventional equalization techniques use training signals. When wireless channels, especially mobile channels, are fast variant, training signals must be sent frequently. As such, a lot of band width has to be occupied. For example, according to 900MHz GSM standard, 26 bits out of every 148-bit frame are used as training signals [1].

Recently, blind channel identification techniques [2, 3, 4] and blind channel equalization techniques [5, 6, 7, 8, 9] have attracted a lot of interest, because they can achieve equalization without using training signals. Blind channel equalization is preferable since it bypasses the estimation of channel length and with reduced complexity.

According to recent studies, it is found that equalizers with different equalization delays [8, 9] achieve different performances. The best performed equalizer is not necessary the one with 0-delay, and often not the case when the channels are non-minimum-phase. In order to choose the best one, we can first find equalizers with all possible equalization delays and then select the best one among them according to a (blind) evaluation index which is presented here. Algorithms for a sequential estimation of such equalizers have been given in [8, 9]. Because the estimation process is sequential, it suffers from error propagation, *i.e.*, the error occurred in the earlier estimations will impact through out the latter ones.

In this paper, a new algorithm is presented, by which the equalizers with all possible equalization delays can be calculated simultaneously. In addition, a (blind) evaluation index is designed for

the purpose of selection of the best one among them. A simulation study is presented which shows the best delayed equalizer performs much better than that for other equalization delays and those given by constant modulus algorithm and linear prediction algorithm.

2. PROBLEM STATEMENT

A direct multi-channel blind equalization system is given in Fig.1:

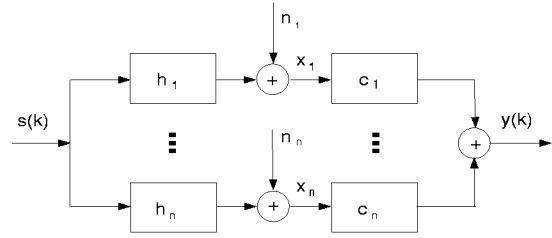


Fig.1 A model for multi-channel blind equalization

where,

- $s(\cdot)$: the (unknown) transmitted signal.
- $\{h_i(z)\}$: the (unknown) FIR channel bank.
- $\{n_i(\cdot)\}$: the additive noises.
- $\{x_i(\cdot)\}$: the received signals.
- $\{e_i(z)\}$: the FIR equalizer bank.
- $y(\cdot)$: the composite output signal.

The objective of blind channel equalization is to design an equalizer bank $\{e_i(z)\}$ based only on the second order statistics of the received signals $\{x_i(\cdot)\}$, such that the composite output signal $y(\cdot)$ is equal to the transmitted signal $s(\cdot)$ with possibly some delays. The objective for this paper is to calculate equalizer banks with all possible equalization delays and then to find the one achieving the best equalization.

In this paper, matrices are denoted by bold face uppercase letters, column vectors by bold face lowercase letters, and scalars by plain letters. All of them are defined in complex domain. The superscripts H , T , and $*$ are Hermitian operator, transpose operator, and conjugate operator, respectively.

The convolutional relation between the transmitted signal $s(\cdot)$ and the received signals $\{x_i(\cdot)\}$ is described by,

$$\mathbf{x}(k) = \mathbf{H} \mathbf{s}(k) \quad (1)$$

where, the received signals are organized in the vector form,

$$\mathbf{x}(k) = [\mathbf{x}^T(k), \mathbf{x}^T(k-1), \dots, \mathbf{x}^T(k-M+1)]^T$$

and so is the transmitted signal,

$$\mathbf{s}(k) = [s(k), s(k-1), \dots, s(k-L+1)]^T$$

(The choice of M and L will be explained later). The system response of multiple channels is represented by an $nM \times L$ Sylvester matrix \mathbf{H} ,

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_0 & \mathbf{h}_1 & \dots & \mathbf{h}_{N-1} & 0 & 0 & \dots & 0 \\ 0 & \mathbf{h}_0 & \mathbf{h}_1 & \dots & \mathbf{h}_{N-1} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 & \mathbf{h}_0 & \mathbf{h}_1 & \dots & \mathbf{h}_{N-1} \end{bmatrix}$$

which is configured by the coefficients of the channel bank $\mathbf{h}(z)$ (in vector polynomial form),

$$\begin{aligned} \mathbf{h}(z) &= [h_1(z), h_2(z), \dots, h_n(z)]^T \\ &= \mathbf{h}_0 + \mathbf{h}_1 z^{-1} + \dots + \mathbf{h}_{N-1} z^{-(N-1)} \end{aligned}$$

where, N is the length of the channel bank; $\mathbf{h}_0, \mathbf{h}_1, \dots, \mathbf{h}_{N-1}$ are all $n \times 1$ vectors, with $\mathbf{h}_0 \neq \mathbf{0}$ and $\mathbf{h}_{N-1} \neq \mathbf{0}$. From the structure of the Sylvester matrix \mathbf{H} , we have $L = N + M - 1$.

The relation between the transmitted signal $s(\cdot)$ and the composite output signal $y(\cdot)$ is described by,

$$y(k) = \mathbf{e}^H \mathbf{H} \mathbf{s}(k) \quad (2)$$

where,

$$\mathbf{e} = [\mathbf{e}_0^T, \mathbf{e}_1^T, \dots, \mathbf{e}_{M-1}^T]^H$$

which represents the equalizer bank $\mathbf{e}(z)$ (in vector polynomial form),

$$\begin{aligned} \mathbf{e}(z) &= [e_1(z), e_2(z), \dots, e_n(z)]^T \\ &= \mathbf{e}_0 + \mathbf{e}_1 z^{-1} + \dots + \mathbf{e}_{M-1} z^{-(M-1)} \end{aligned}$$

where M is the length of the equalizer bank; $\mathbf{e}_0, \mathbf{e}_1, \dots, \mathbf{e}_{M-1}$ are all $n \times 1$ vectors, with $\mathbf{e}_0 \neq \mathbf{0}$ and $\mathbf{e}_{M-1} \neq \mathbf{0}$. M is designed and must be big enough such that the Sylvester matrix \mathbf{H} is a thin matrix, i.e., $nM > L = N + M - 1$.

The relaxed zero-forcing condition for blind channel equalization is given by

$$y(k) = \alpha s(k-l) \quad (3)$$

where, α is a nonzero complex number and l is the equalization delay. Substituting Eq.(2) into Eq.(3), we have

$$\mathbf{H}^H \mathbf{e} = \alpha^* \mathbf{d}_{l+1} \quad (4)$$

where, \mathbf{d}_{l+1} is an $L \times 1$ canonical vector, in which the only nonzero element 1 locates at the $(l+1)$ th row. A solution \mathbf{e} to Eq.(4) is called an l -delayed equalizer bank, or simply, an l -delayed equalizer, denoted by \mathbf{e}_l .

The second order statistics of the received signals $\{x_i(\cdot)\}$ can be organized into many l -step autocorrelation matrices $\mathbf{R}(l)$'s, $l = 0, 1, \dots, L-1$.

$$\mathbf{R}(l) \triangleq E(\mathbf{x}(k)\mathbf{x}^H(k-l))$$

where, $E(\cdot)$ is the expectation operator. Note that $\mathbf{R}(l) = \mathbf{0}$ for $l \geq L$.

Now let us restate the blind channel equalization problem: the objective is to design the best delayed equalizer \mathbf{e}_l based on the estimated autocorrelation matrices $\hat{\mathbf{R}}(l)$'s without the knowledge of $s(\cdot)$, \mathbf{H} , and N .

3. PRELIMINARIES

The existence of such equalizer bank is not guaranteed for any given channel bank. An existence condition is given by Massey and Sain [10].

Theorem 1

An FIR equalizer bank $\{e_i(z)\}$ exists, if, and only if, the greatest common divisor (GCD) of the channel bank $\{h_i(z)\}$ satisfies the following condition:

$$\text{GCD}(h_1(z), h_2(z), \dots, h_n(z)) = z^{-l} \quad (5)$$

for some non-negative integer l .

In particular, such FIR equalizer bank doesn't exist if the channel bank $\{h_i(z)\}$ has common nonzero zeros [2]. Note that this condition can not be satisfied for a single-channel transmission except for trivial cases. This is why blind channel equalization techniques rely on multiple channels.

The following theorem due to Tong *et al* [2] is needed for our derivation.

Theorem 2

The Sylvester matrix \mathbf{H} has full column rank if, and only if, the channel bank $\{h_i(z)\}$ has no common zeros.

4. BLIND EQUALIZATION WITH DELAYS

In this paper, the following assumptions are made:

- AS1. The channel bank $\{h_i(z)\}$ doesn't have common zeros;
- AS2. The transmitted signal $s(\cdot)$ is zero-mean, stationary, and temporally uncorrelated;
- AS3. The variance of the transmitted signal $s(\cdot)$ is equal to 1;
- AS4. There is no additive noise.

AS2 is reasonable in digital communication environment, where the encoded information symbols are often interleaved before transmission for the purpose of resistance to burst errors. AS3 is made without loss of any generality. AS4 is made for the purpose of simplicity. The additive noise can be identified from $\mathbf{R}(0)$ and then removed from all $\mathbf{R}(l)$'s [2, 3].

Under above assumptions, one set of equalizers with all possible delays are given by the following theorem.

Theorem 3

If AS1-AS4 hold, then

(1). For each $l = 0, 1, \dots, L-1$, a generalized eigenvector \mathbf{c}_l of the matrix pair $\mathbf{R}^H(l+1)$ and $\mathbf{R}^H(l)$ exists, which associates with and only with the zero generalized eigenvalue;

(2). The Cholesky decomposition $\mathbf{C}^H \mathbf{R}(0) \mathbf{C} = \mathbf{U}^H \mathbf{U}$ exists and the column vectors of $\mathbf{C} \mathbf{U}^{-1} = [\mathbf{c}_0, \mathbf{c}_1, \dots, \mathbf{c}_{L-1}]$ are equalizers with all possible equalization delays from 0 to $L-1$, where $\mathbf{C} = [\mathbf{c}_0, \mathbf{c}_1, \dots, \mathbf{c}_{L-1}]$.

Proof:

According to Eq.(1) and AS2-AS4, we have

$$\mathbf{R}(l) = \mathbf{H} \mathbf{J}^l \mathbf{H}^H \quad (6)$$

where, \mathbf{J} is an $L \times L$ shifting matrix,

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}$$

Note that \mathbf{H} is a thin matrix, which means there exists some vector \mathbf{c}_l such that

$$\mathbf{H}^H \mathbf{c}_l = \mathbf{v}_l = [v_l(0), v_l(1), \dots, v_l(l), 0, \dots, 0]^T$$

where $v_l(l) \neq 0$. According to Eq.(6), it can be verified that

$$\mathbf{R}^H(l+1)\mathbf{c}_l = \mathbf{H}(\mathbf{J}^H)^{l+1}\mathbf{v}_l = \mathbf{0} \quad (7)$$

and

$$\mathbf{R}^H(l)\mathbf{c}_l = \mathbf{H}(\mathbf{J}^H)^l\mathbf{v}_l = \mathbf{H}[v_l(l), 0, \dots, 0]^T \neq \mathbf{0} \quad (8)$$

Note that Eq.(7) and Eq.(8) are equivalent to the following equation

$$\mathbf{R}^H(l+1)\mathbf{c}_l = \lambda \mathbf{R}^H(l)\mathbf{c}_l = \mathbf{0} \quad (9)$$

holds with and only with $\lambda = 0$, i.e., \mathbf{c}_l is a generalized eigenvector of the matrix pair $\mathbf{R}^H(l+1)$ and $\mathbf{R}^H(l)$, which associates with and only with the zero generalized eigenvalue. (1) is proven.

Let us now assume that \mathbf{c}_l is a such generalized eigenvector, i.e., \mathbf{c}_l is the null space of $\mathbf{R}^H(l+1)$. From AS1 and Theorem 2, we have

$$(\mathbf{J}^H)^{l+1}\mathbf{H}^H \mathbf{c}_l = \mathbf{0} \quad (10)$$

Denoting

$$\mathbf{H}^H \mathbf{c}_l = \mathbf{v}_l = [v_l(0), v_l(1), \dots, v_l(L-1)]^T$$

Eq.(10) becomes,

$$(\mathbf{J}^H)^{l+1}\mathbf{v}_l = \mathbf{0}$$

which implies that $v_l(i) = 0$ for $i = l+1, l+2, \dots, L-1$. On the other hand, since \mathbf{c}_l is not in the null space of $\mathbf{R}^H(l)$, we conclude that $v_l(l), v_l(l+1), \dots, v_l(L-1)$ can not be all zero. As such, $v_l(l) \neq 0$ and $v_l(l+1) = v_l(l+2) = \dots = v_l(L-1) = 0$. Thus,

$$\mathbf{H}^H \mathbf{C} = [\mathbf{v}_0, \mathbf{v}_1, \dots, \mathbf{v}_{L-1}] = \mathbf{V} \quad (11)$$

where \mathbf{V} is an upper triangular matrix with nonzero diagonal elements, and hence it has full rank. Since

$$\mathbf{C}^H \mathbf{R}(0)\mathbf{C} = \mathbf{C}^H \mathbf{H} \mathbf{H}^H \mathbf{C} = \mathbf{V}^H \mathbf{V}$$

we conclude that $\mathbf{C}^H \mathbf{R}(0)\mathbf{C}$ has full rank and hence it has a Cholesky decomposition,

$$\mathbf{C}^H \mathbf{R}(0)\mathbf{C} = \mathbf{U}^H \mathbf{U}$$

where \mathbf{U} is an upper triangular matrix. Note that this Cholesky decomposition is done on complex domain. Comparing the above two equations, we have

$$\mathbf{V} = \mathbf{U} \mathbf{D} \quad (12)$$

where \mathbf{D} is an arbitrary diagonal unitary matrix. Substituting Eq.(12) into Eq.(11), we have

$$\mathbf{H}^H \mathbf{C} \mathbf{U}^{-1} = \mathbf{D}$$

which shows that the column vectors of $\mathbf{C} \mathbf{U}^{-1}$ are equalizers with all possible equalization delays from 0 to $L-1$. (2) is proven.

Theorem 3 has importance in practice, where the estimated $\hat{\mathbf{R}}^H(l)$'s often have full rank because of additive noise and statistical fluctuation. In this situation, the generalized eigenvector $\hat{\mathbf{c}}_l$ associating with the generalized eigenvalue with the minimum absolute value is the best choice for \mathbf{c}_l , which minimizes $\|\hat{\mathbf{R}}^H(l+1)\mathbf{c}_l\|_2$ subject to $\|\hat{\mathbf{R}}^H(l)\mathbf{c}_l\|_2 = 1$. As such, an iterative algorithm can be designed to approach this generalized eigenvector gradually [11].

Remark 1

Theorem 3 remains valid if the order of the columns in \mathbf{C} is reversed and the autocorrelation matrices $\mathbf{R}(l)$'s are not Hermitian transposed, i.e., $\mathbf{C} = [\mathbf{c}_{L-1}, \mathbf{c}_{L-2}, \dots, \mathbf{c}_0]$, where \mathbf{c}_l is the generalized eigenvector associating with only a zero generalized eigenvalue of the matrix pair $\mathbf{R}(l+1)$ and $\mathbf{R}(l)$. Then the Cholesky decomposition $\mathbf{C}^H \mathbf{R}(0)\mathbf{C} = \mathbf{U}^H \mathbf{U}$ exists and gives equalizers with all possible equalization delays by $\mathbf{C} \mathbf{U}^{-1} = [\mathbf{e}_0, \mathbf{e}_1, \dots, \mathbf{e}_{L-1}]$.

Remark 2

Theorem 3 can be extended to the case that if the transmitted signal $s(\cdot)$ is K -step temporally uncorrelated, i.e., $E(s(t)s^*(t-K+1)) \neq 0$ and $E(s(t)s^*(t-k)) = 0$ for $k \geq K$.

Remark 3

When Theorem 3 is applied, it is not required to know the length of channel bank N , but its lower bound N_1 and upper bound N_2 . In this situation, M is chosen such that $nM > N_2 + M - 1$ and \mathbf{C} contains only $N_1 + M - 1$ columns.

5. EVALUATION INDEX

After multiple equalizers with different equalization delays are calculated, one needs an evaluation index to make a choice. The usual ISI index for an estimated l -delayed equalizer $\hat{\mathbf{e}}_l$

$$\text{ISI}(l) = 1 - \frac{\|\mathbf{H}^H \hat{\mathbf{e}}_l\|_\infty^2}{\|\mathbf{H}^H \hat{\mathbf{e}}_l\|_2^2}$$

is not very useful in this problem because \mathbf{H} is unknown.

Note that all equalizers given by Theorem 3 have normalized the amplitude variance of the equalized signal $y(\cdot)$, no matter what situation the transmitted signal $s(\cdot)$ is. Hence, if the transmitted signal has constant modulus, which often is the case in digital wireless communication, the following index can be used.

$$D(l) = \sum_k (|y(k)| - 1)^2 = \sum_k (|\hat{\mathbf{e}}_l^H \mathbf{x}(k)| - 1)^2$$

The equalizer having the smallest D value will be considered as the best delayed equalizer.

6. A COMPUTER SIMULATION

The purpose of this simulation is to show that the performance of equalizers varies as the equalization delay varies, and the variation may be significant when channels are non-minimum-phase. A comparison is made with constant modulus algorithm and linear prediction algorithm.

For the purpose of simplicity, only real numbers are used. The transmitted signal $s(\cdot)$ is a random sequence on $\{1, -1\}$. The

white Gaussian noise is added on the received signals $x_1(\cdot)$ and $x_2(\cdot)$ at the level of SNR=15dB.

The following two non-minimum-phase channels are used for the simulation.

$$\begin{cases} h_1(z) = 0.2 + 0.6z^{-1} + 0.1z^{-2} \\ h_2(z) = 0.3 - 0.8z^{-1} - 0.2z^{-2} \end{cases}$$

Based on the same received signals, five equalizer banks are iteratively calculated: one by constant modulus algorithm, one by linear prediction algorithm, and the other three by Cholesky decomposition algorithm with equalization delays 0, 1, and 2. Their ISI performance curves are shown in Fig.2a. It is shown that the best equalizer bank is for 1-delay equalization and it is much better than those given by constant modulus algorithm and linear prediction algorithm. The performance indices D for 0-delay, 1-delay and 2-delay is shown in Figure 2b. By comparison, it clearly shows that the one with 1-delay is the best.

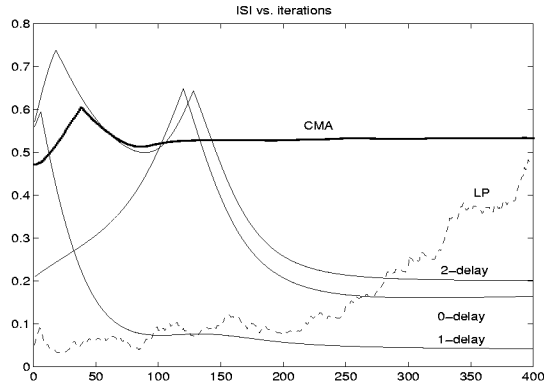


Fig.2a ISI comparison for non-minimum-phase channels

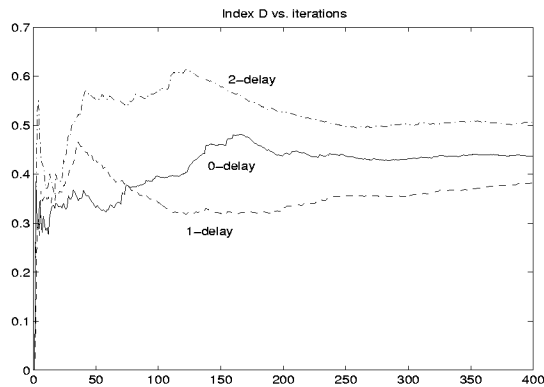


Fig.2b Index D for non-minimum-phase channels

7. CONCLUSIONS

A blind channel equalization algorithm is presented, by which a set of equalizers with all possible equalization delays can be calculated simultaneously from second order statistics of multiple received signals. As such, more choices are available for blind channel equalization under noisy environment. An evaluation index is

also defined, by which one could choose the best delayed equalizer from these candidates to achieve the best channel equalization. Simulation shows that the best delayed equalizer performs much better than that for other equalization delays and those given by constant modulus algorithm and linear prediction algorithm.

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