

IMPROVED EMISSION TOMOGRAPHY VIA MULTISCALE SINOGRAM ANALYSIS

K. E. Timmermann, R. D. Nowak, and K. J. Jones

Michigan State University
East Lansing, MI 48824
timmerm4@egr.msu.edu

ABSTRACT

In this paper, we extend a multiscale Bayesian approach to modeling and estimation of general Poisson processes previously developed in [1] by the first two authors, and apply it to the emission computed tomography (ECT) image reconstruction problem. We develop a practical prior model for the sinogram image, which we use to estimate the underlying sinogram intensity from the raw projection data prior reconstruction. This sinogram estimate is then used in conjunction with the standard filtered-backprojection algorithm to produce an improved image reconstruction. The impact of the new filtering approach on ECT imaging is illustrated with simulated and clinical data.

1. INTRODUCTION

Computed tomography (CT) imaging is an important imaging technique widely used in medicine, seismology, astronomy, and other fields of science and engineering. In some areas, most notably *single photon emission computed tomography* (SPECT) and *positron emission tomography* (PET), the tomographic image studies begin with photon-limited projection data, the statistics of which are well modeled by the Poisson distribution. We refer to such problems as emission computed tomography (ECT). Traditionally, classical reconstruction methods (*e.g.*, filtered-backprojection) applied to ECT produce undesirable and highly variable image reconstructions due to the “Poisson noise” in the projection data¹. It is common in practice to postprocess the raw reconstructions with a lowpass smoothing filter to improve the images. However, such postprocessing can lead to a detrimental loss in resolution and fine detail structure. More sophisticated approaches have been proposed (see Section 2 for a brief review), but these methods tend to be very computationally expensive.

In this paper we extend a multiscale-based modeling and estimation approach for general Poisson processes previously developed by the first two authors in [1], and apply it to the ECT problem. We develop a realistic and practical prior probability model for the sinogram image, and use it to compute a Bayes-optimal estimate of the intensities underlying the “data” sinogram constructed from the raw projection data. Using this optimal sinogram estimate and a standard filter-backprojection algorithm, we are able to produce significantly improved image reconstructions. Remarkably, this new method is no more computationally intensive than standard approaches based on lowpass filter postprocess-

ing, which, due to their computational simplicity, are routinely used in real clinical settings.

Projection data in ECT consists of a collection of counts which are Poisson distributed. We wish to estimate the underlying intensity and reconstruct the image. There are some important advantages to carrying out this estimation in a multiscale Bayesian framework:

- Useful Bayesian priors are easily specified in the scale-domain; thus, a computationally efficient and mathematically simple estimator can be devised.
- Coarse-scale estimators of intensities are very reliable (high SNR); thus, reliable information can be passed to finer scales to leverage their estimate.
- Bayesian estimation of scaling coefficients is optimal (in a mean square error sense); thus, the estimation process optimally adapts to local features of the underlying intensity.

We will elaborate on these points later in the paper.

To be specific throughout the paper, we consider only SPECT as an example of emission tomography. Extensions to other applications such as PET are possible. The general SPECT problem is reviewed in Section 2. In Section 3, we briefly review the multiscale multiplicative innovations (MMI) probability model for intensity functions previously introduced in [1]. The MMI model is the cornerstone for the proposed improved ECT image reconstruction process as it provides a practical prior model for the estimation of the sinogram image. In Section 4, we apply the multiscale model and estimator to SPECT imaging, and present two illustrative examples. Concluding remarks are given in Section 5.

2. EMISSION COMPUTED TOMOGRAPHY

In SPECT a patient is injected with a radiopharmaceutical which is targeted for uptake by the specific organ(s) of interest. As the nuclear decay process takes place, photons are emitted in all directions and registered upon their arrival by an array of detectors located in close proximity to the patient. The array is repositioned at many different angles ϕ_n about the subject so that counts of photons c_k^n may be obtained for each angle and each detector k . Each data projection $\mathbf{c}^n = (c_k^n)_{k=0}^{N-1}$ is Poisson distributed with an underlying intensity $\boldsymbol{\lambda}^n = (\lambda_k^n)_{k=0}^{N-1}$.

Let $\boldsymbol{\lambda}(\mathbf{x})$ denote the 2-dimensional distribution of radiopharmaceutical in the plane of interest within the patient. In nuclear medicine, this distribution can provide anatomical information or may be an indicator of functional activity [2]. The goal of ECT is to reconstruct an estimate of this distribution from the projection data.

This work was supported by the National Science Foundation, grant no. MIP-9701692

¹Throughout the paper we use the term noise as a matter of convenience.

Ignoring attenuation effects and other disturbances in the data collection process, the intensities $\{\lambda^n\}$ correspond to projections of $\lambda(x)$ along rays perpendicular to a photon detection array, which records the the location of each photon event impinging the array. The array of intensity vectors $[\lambda^0, \dots, \lambda^{(N-1)}]$ is called the *sinogram* (see Figure 3 (a)), which represents an incomplete *Radon transform* of $\lambda(x)$.

Conventional SPECT methods first reconstruct a noisy approximation to $\lambda(x)$ by computing the inverse Radon transform of the “data” sinogram $\mathbf{c} = [\mathbf{c}^0, \dots, \mathbf{c}^{N-1}]$ and then lowpass filtering the reconstruction to mitigate the noise in the image [2]. The filtered-back-projection reconstruction method [3] is the most commonly used (approximate) Radon inversion technique. Although computationally efficient, these approaches tend to produce excessive blurring and destroy fine features of the intensity. To reduce the need for an ‘aggressive’ lowpass filter in the final stage of reconstruction, a number of methods reduce the noise in the data projections by attenuating their high frequencies [4]. More advanced methods estimate the intensity of the projections $\{\lambda^n\}$ from the noisy data to account for the statistical characteristics of the noise. For example, Wiener filtering [5] approaches have been applied to the projections prior reconstruction. Furthermore, (fully) Bayesian approaches to this problem have been proposed based on Markov random field models, *e.g.*, [6, 7]. However, in comparison to those methods, our new multiscale framework is very computationally efficient, while still providing high-quality reconstructions.

3. MULTISCALE MODELING AND ESTIMATION OF POISSON PROCESSES

3.1. The Multiscale Multiplicative Model

In this subsection, we review a multiscale probability model for positive intensities developed in [1]; only 1-D intensities are considered for simplicity (see [8] for an extension to the 2-D case).

We begin by letting $\lambda_0 = (\lambda_{0,k})_{k=0}^{N-1}$ represent a sequence of integrated values of a positive function $\lambda(x)$ defined in the unit interval, *i.e.*, $\lambda_{0,k} = \int_{k/N}^{(k+1)/N} \lambda(x) dx$, for example, a spatial distribution of radioactive material. A multiscale representation of λ_0 may be obtained by iterating

$$\lambda_{j,k} = \lambda_{j-1,2k} + \lambda_{j-1,2k+1} \quad (1)$$

for $j = 1, \dots, J$ and $k = 0, \dots, N/2^j - 1$. Each coefficient $\lambda_{j,k}$ is simply the un-normalized Haar scaling coefficient at scale j and shift k of the original intensity λ_0 . Note that (1) is equivalent to defining the coefficients by $\lambda_{j,k} = \int_{2^j k/N}^{2^j (k+1)/N} \lambda(x) dx$. This shows that $\lambda_j = (\lambda_{j,k})_k$ is a lower resolution representation of the intensity than $\lambda_{j-1} = (\lambda_{j-1,k})_k$.

A very simple multiscale prior model for the intensity λ_0 can be constructed as follows. Let $\Lambda_{J,0}$ be a random variable (rv) whose sample space is the positive real line, and let $Y_{J,0}$ be an independent rv whose possible outcomes are the points in $[0, 1]$. Next, define the sequence $\Lambda_{J-1} = (\Lambda_{J-1,0}, \Lambda_{J-1,1})$ by $\Lambda_{J-1,0} = \Lambda_{J,0} Y_{J,0}$ and $\Lambda_{J-1,1} = \Lambda_{J,0} (1 - Y_{J,0})$. Now, corresponding to each element $\Lambda_{j,k}$, introduce independent rv $Y_{j,k}$ with support on $[0, 1]$, and define ‘even’ and ‘odd’ children coefficients $\Lambda_{j-1,2k}$ and $\Lambda_{j-1,2k+1}$ by

$$\begin{aligned} \Lambda_{j-1,2k} &= \Lambda_{j,k} Y_{j,k} \\ \Lambda_{j-1,2k+1} &= \Lambda_{j,k} (1 - Y_{j,k}) \end{aligned}$$

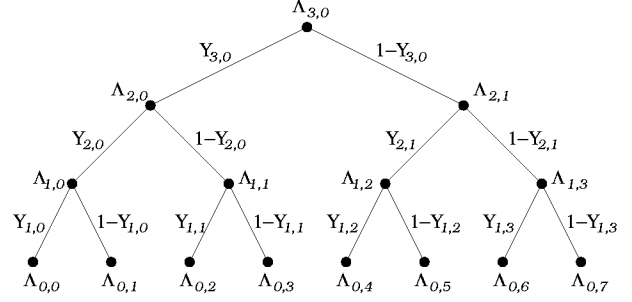


Figure 1: Multiscale multiplicative innovations model. A coarse-scale probability model of the intensity is refined via multiplicative perturbations.

for $j = J - 1$ down to $j = 1$ and for $k = 0$ to $N/2^j - 1$.

The functional relations among the variates defined in this manner are display by the binary tree of Figure 1 for $N = 8$. Since the coefficients in this structure also obey

$$\Lambda_{j,k} = \Lambda_{j-1,2k} + \Lambda_{j-1,2k+1} \quad (2)$$

we may regard each vector λ_j to be a realization of its corresponding random sequence Λ_j .

The information required to construct a representation at scale $j - 1$ from the one representation at scale j is conveyed by the rv’s $\{Y_{j,k}\}_k$ in a multiplicative fashion, and so, we call them *multiscale multiplicative innovations* (MMI).

To complete the MMI prior probability model, we restrict the set of innovations $\{Y_{j,k}\}_{j,k}$ to be independent. In addition, we impose prior distributions $f_J(\lambda)$ on $\Lambda_{J,0}$ and $f_j(y)$ on each innovation $Y_{j,k}$ for every j and k . Modeling the innovations at a given scale as identically distributed is not essential, but it leads to a practical and simpler model. One possible choice for $f_J(\lambda)$ is the Gamma distribution, and for $f_j(y)$, a mixture of beta densities of the form

$$f_j(y) = \sum_{i=1}^M p_{j,i} \frac{y^{s_i-1} (1-y)^{s_i-1}}{B(s_i, s_i)} \quad (3)$$

for $0 \leq y \leq 1$. B is the Euler beta function, $0 \leq p_{j,i} \leq 1$ is the weight of the i -th beta density with parameter s_i , and $\sum_{i=1}^M p_{j,i} = 1$. These choices are justified in [1].

3.2. Shift-Invariant MMI model

A drawback of the MMI model is its shift variant nature. In general, for an identical set of realizations $\{\lambda_{J,0}\} \cup \{y_{j,k}\}_{j,k}$, there correspond $N/2$ possible distinct outcome intensities λ_0 depending on the alignment of the intensity with respect to the binary tree; one for each distinct circular shift or displacement. One only need inspect Figure 1 to realize that not every two adjacent elements of Λ_0 have the same functional correspondence. To remove this deficiency, we take a Bayesian approach, and view each alignment of the model’s binary tree as an additional degree of freedom in the model. Then, if we regard the original model as “unshifted” (shift=0), the standard MMI model introduced above is denoted $f(\lambda|0)$. With $P(\text{shift} = m)$ denoting the probability mass func-

tion for each shift, we have

$$f(\lambda) = \sum_{m=0}^{N/2-1} f(\lambda|m)P(\text{shift} = m)$$

Since there is no reason *a priori* to favor any one possible shift, we take $P(\text{shift} = m) = 2/N$:

$$f(\lambda) = \frac{2}{N} \sum_{m=0}^{N/2-1} f(\lambda|m) \quad (4)$$

Thus, a shift-invariant MMI model is constructed by simply averaging over all possible distinct alignments of the unshifted model.

The key properties of the SI-MMI model are:

- The model includes $1/f$ -type processes [8], often used to model real-world imagery [9].
- The model provides a mathematically tractable match to the Poisson nature of the data. This will become evident in the next section.

3.3. Optimal Estimation of Poisson Processes

Now using the MMI prior model, we can derive a simple scale-domain Bayesian estimator. We observe counts $\mathbf{c}$ and wish to estimate their underlying intensity λ . We regard the data $\mathbf{c}$ to be a realization of a random sequence $\mathbf{C}$ which is Poisson distributed with parameter λ . The intensity λ itself is thought, as before, to be an unknown realization of a random sequence Λ . We seek the posterior mean estimate $\hat{\lambda} \equiv E[\Lambda|\mathbf{C} = \mathbf{c}]$.

The multiscale approach to this estimation proceeds as follows. First, a multiscale analysis on the data is obtained: We let $\mathbf{c}_0 = \mathbf{c}$ and iterate

$$\mathbf{c}_{j,k} = \mathbf{c}_{j-1,2k} + \mathbf{c}_{j-1,2k+1}$$

for $j = 1, \dots, J$ and $k = 0, \dots, N/2^j - 1$. Then, due to the typical high SNR² of the coefficient $c_{J,0}$ a robust estimate for $\lambda_{J,0}$ is given by $\hat{\lambda}_{J,0} \equiv E[\Lambda_{J,0}|\mathbf{c}] = E[\Lambda_{J,0}|\mathbf{c}_{J,0}] \approx c_{J,0}$.

We obtain higher resolution estimates of the intensity in a recursive manner as follows. In general, $\mathbf{c}_j$ is a sufficient statistic for λ_j , and $\mathbf{c}_{j-1,2k}$ and $\mathbf{c}_{j-1,2k+1}$ are sufficient statistics for $\mathbf{y}_{j,k}$ [10]. This justifies the estimate $\hat{\mathbf{y}}_{j,k} \equiv E[\mathbf{Y}_{j,k}|\mathbf{c}] = E[\mathbf{Y}_{j,k}|\mathbf{c}_{j-1,2k}, \mathbf{c}_{j-1,2k+1}]$. Using (3), it is shown in [1] that this estimate is given by

$$\hat{\mathbf{y}}_{j,k} = \frac{1}{2} \left(1 + d_{j,k} \frac{\sum_i p_{j,i} \frac{B(s_i + c_{j-1,2k}, s_i + c_{j-1,2k+1})}{B(s_i, s_i) (2s_i + c_{j,k})}}{\sum_i p_i \frac{B(s_i + c_{j-1,2k}, s_i + c_{j-1,2k+1})}{B(s_i, s_i)}} \right) \quad (5)$$

where $d_{j,k} = c_{j-1,2k} - c_{j-1,2k+1}$.

Now, exploiting the independence of the innovation variates, the estimates for all coefficients from scales $j = J - 1$ down to scale $j = 0$ are iteratively obtained by

$$\begin{aligned} \hat{\lambda}_{j-1,2k} &\equiv E[\Lambda_{j-1,2k}|\mathbf{c}] = \hat{\lambda}_{j,k} \hat{\mathbf{y}}_{j,k} \\ \hat{\lambda}_{j-1,2k+1} &\equiv E[\Lambda_{j-1,2k+1}|\mathbf{c}] = \hat{\lambda}_{j,k} (1 - \hat{\mathbf{y}}_{j,k}) \end{aligned}$$

²It is easily verified that in a Poisson process the SNR increases linearly with the underlying intensity. For example, for a 128 by 128 pixel image, the SNR at scale J is 42 dB larger as that for the average data point $c_{0,k}$.

The desired estimate is simply $\hat{\lambda} = \hat{\lambda}_0$. In general, the complexity of the proposed estimation filter is $\mathcal{O}(N)$. A shift-invariant estimate can be obtained by averaging intensity estimates resulting from different shifts of the data together, requiring $\mathcal{O}(N \log N)$ operations [8]. Furthermore, a similar multiscale analysis and estimation procedure can be easily developed for the 2-d case using a 2-d Haar-like decomposition [8, 11].

4. APPLICATION TO SPECT IMAGING

An intuitively appealing approach to the emission tomographical imaging problem would be to model each individual projection λ^n using the SI-MMI model and estimate them from the projection data; then, using the estimates $\{\hat{\lambda}^n\}$ reconstruct the image. This approach, however, has been found to produce circular artifacts in the final image due to a ‘lack of coordination’ among the otherwise highly correlated intensity projections. To correct for this deficiency, we apply a two dimensional version of the SI-MMI model and estimate the underlying ‘intensity’ sinogram. In this manner, not only will the estimation be coordinated and avoid artifacts, but also it will enhance the performance of the estimator by exploiting the highly structured correlation typical of the data sinogram. For example, consider the data sinogram of a pelvic slice image shown in Figure 3(a). Our new ECT algorithm is described as follows.

Multiscale Sinogram Estimation and ECT Reconstruction

1. Form ‘data’ sinogram from observed projection data
2. Estimate underlying ‘intensity’ sinogram using 2-d version of the estimation method described in Section 3.
3. Reconstruct image from estimated sinogram using standard filtered backprojection algorithm.

4.1. Examples

To demonstrate the performance of the new approach to ECT, we have applied it to two sets of data: the Shepp Logan head phantom (shown in Figure 2(a)), and real data from a human pelvic clinical study. The two data sets have been processed in three different ways so that comparisons can be made and a relative degree of performance can be determined. In Figure 2(b), an (unprocessed) reconstruction of the phantom data, based on Poisson data projections, is presented. Figure 2(c) show a lowpass filtered³ version of 2(b). The smoothing process removes high frequency noise at the expense of some high definition detail in the image. Finally, Figure 2(d) displays the reconstructed image using the new sinogram estimation method. In this new image, high definition features (e.g., edges) are clearly preserved despite the high degree of noise reduction.

Figures 3(b), (c), and (d) of the pelvic bone study were obtained by the same processes used in the corresponding phantom figures. Note that in Figure 3(d), the high definition image structure becomes evident after processing according to the new approach. This is in contrast with the result in (c), where the image is seen to be oversmoothed to achieve a similar degree of noise removal.

³A 2-d Butterworth filter with cut-off $\pi/3$ rad. was used in both examples in this section.

5. CONCLUDING REMARKS

We have introduced a new approach to ECT imaging based on the multiscale multiplicative model previously developed by two of the authors. We extended the MMI prior model to the shift invariant SI-MMI model, and use it to estimate the sinogram from photon-limited data. We found that by working with the projection data directly, we were able to model the true Poisson nature of the data sinogram and, thus, avoided making inaccurate assumptions about the statistics in the reconstructed image. Also, by working with the sinogram we were able to exploit the strong correlation in the data sinogram to leverage the estimation process. Examples with synthetic and clinical data demonstrated the performance of our new method.

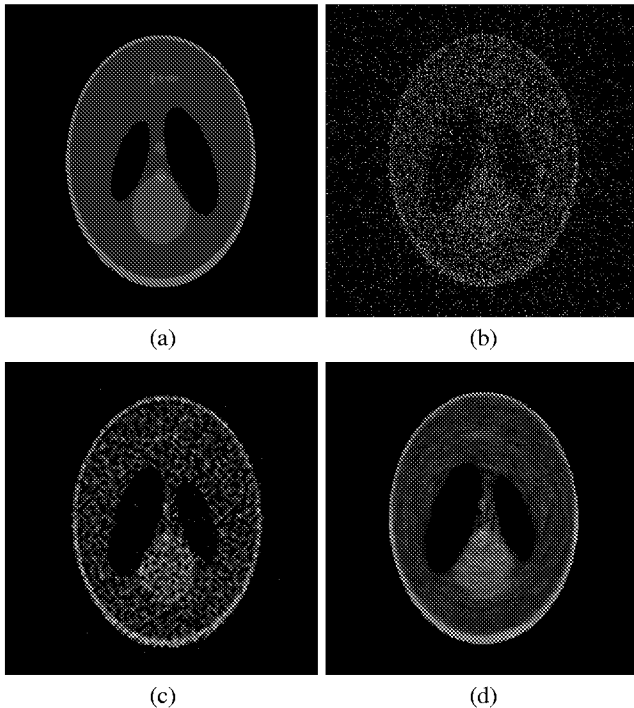


Figure 2: Shepp Logan head phantom image reconstruction. (a) Phantom. (b) Phantom reconstruction from noisy data. (c) Low-pass filtered image from (b). (d) Phantom reconstruction from MMI-processed sinogram.

6. REFERENCES

- [1] K. E. Timmermann and R. D. Nowak, "Multiscale Bayesian estimation of Poisson intensities," in *Proc. Asilomar Conf. Signals, Systems, and Comp.*, Pacific Grove, CA, IEEE Computer Society Press, 1997.
- [2] J. A. Sorenson and M. E. Phelps, *Physics in Nuclear Medicine*. New York: Grune & Stratton, 2nd ed., 1987.
- [3] A. C. Kak and M. Slaney, *Principles of Computerized Tomographic Imaging*. New York: IEEE Press, 1988.
- [4] L. Chang, "Attenuation corrections and incomplete projections in SPECT," *IEEE Trans. Nuclear Science*, vol. 26, pp. 2980–2989, 1979.

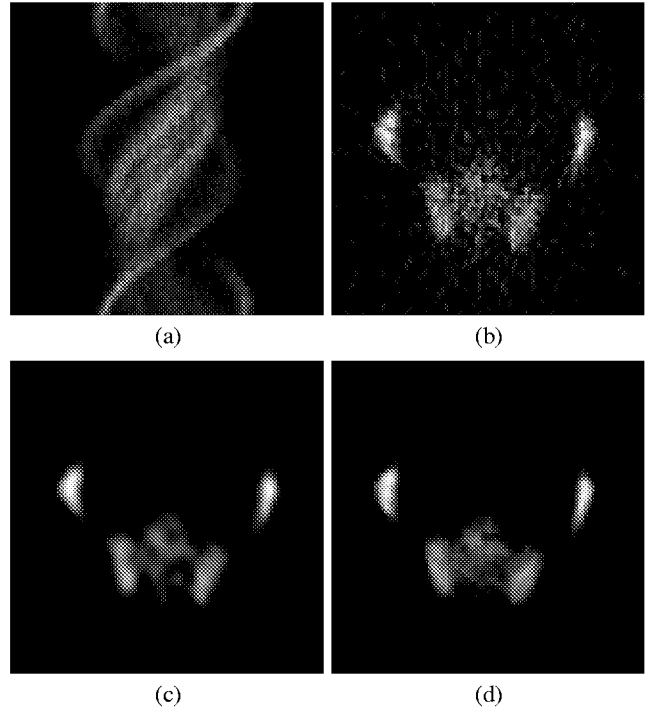


Figure 3: Pelvic bone study image reconstruction. (a) Sinogram of pelvic bone. (b) Pelvic bone reconstruction from noisy data. (c) Lowpass filtered image from (b). (d) Pelvic bone reconstruction from MMI-processed sinogram.

- [5] J. M. Links, R. W. Jeremy, S. M. Dyer, T. L. Frank, and L. C. Becker, "Weiner filtering improves quantification of regional myocardial perfusion with thallium-201 spect," *J. Nuclear Medicine*, vol. 31, pp. 1230–1236, 1990.
- [6] T. Hebert and R. Leahy, "A generalized EM algorithm for 3-d Bayesian reconstruction from Poisson data using Gibbs priors," *IEEE Trans. Medical Imaging*, vol. 8, no. 2, pp. 194–202, 1989.
- [7] S. S. Saquib, C. A. Bouman, and K. Sauer, "A non-homogeneous MRF model for multiresolution Bayesian estimation," *IEEE Int'l Conf. on Image Proc.*, 1996.
- [8] R. D. Nowak and K. E. Timmermann, "Stationary wavelet-based intensity models for photon-limited imaging," in *Proc. IEEE Int. Conf. on Image Proc.*, Chicago, 1998.
- [9] A. van der Schaaf and J. H. van Hateren, "Modelling the power spectra of natural images," *Vision Research*, vol. 36, no. 17, pp. 2759–2770, 1996.
- [10] K. E. Timmermann and R. D. Nowak, "Multiscale Bayesian analysis of Poisson data," *Michigan State University, Dept. Elec. Eng. Tech. Rep.*, October, 1997.
- [11] K. E. Timmermann and R. D. Nowak, "Multiscale modeling and estimation of Poisson processes with application to photon-limited imaging," *submitted to IEEE Transactions on Information Theory, Special Issue on Multiscale Statistical Signal Analysis*, February, 1998.