NON-DATA AIDED JOINT ESTIMATION OF CARRIER FREQUENCY OFFSET AND CHANNEL USING PERIODIC MODULATION PRECODERS: PERFORMANCE ANALYSIS

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ABSTRACT

Recent results have shown that blind channel estimators, which are robust to the location of channel zeros and channel order overestimation errors, can be derived for communication channels equipped with Transmitted Induced Cyclostationarity (TIC) precoders. This paper addresses the problem of joint estimation of the unknown InterSymbol Interference (ISI) and carrier frequency offset using TIC-based set-ups. First, it is shown that the second-order cyclic statistics of the output allow recovery of the channel taps under a scaling factor ambiguity dependent on the unknown carrier offset frequency. Next, a carrier frequency estimator is proposed, and its asymptotic (large sample) performance is analyzed. It is shown that the asymptotic performance of the frequency estimator improves in the presence of a channel equalizer for high SNR's. Finally, numerical simulations are presented to colloborate the performance of proposed algorithms.

1. INTRODUCTION

It has been shown in [2], [8], that blind identification of FIR communication channels can be achieved from knowledge of the second-order statistics of the received data, without imposing any restriction on the channel zeros, color of additive noise, and channel order over-estimation errors. The underlying channel identification approach relies on the output second order cyclostationary (CS) statistics, induced by precoding the input symbol stream with a periodically time-varying precoder, referred to, as a Transmitter Induced Cyclostationarity (TIC) precoder.

Up to present, only a limited number of approaches have been proposed for blind joint estimation of the channel and the carrier frequency offset (FO). In general, these approaches rely on the iterative minimization of certain nonlinear criteria. In [5], [7], the unknown channel is first equalized using a Constant Modulus Algorithm (CMA), and then the carrier offset is tracked from the equalized output. However, in the presence of residual ISI the performance of these frequency tracking algorithms may degrade. In the present paper, estimation of the channel is obtained in a closed-form without minimizing a nonlinear criterion; hence, the potential local minima or loss of convergence associated with [5], [7], in the presence of noise, are avoided, and consistent estimates of the FO are guaranteed even in presence of residual ISI.

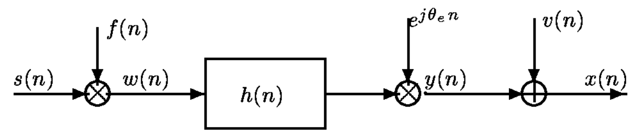

Figure 1: Communication Channel with Frequency Offset

The equivalent *baseband* discrete-time representation of a communication channel with carrier frequency offset is represented in Fig. 1, where the TIC-precoder is implemented by modulating the i.i.d. input symbol stream $s(n)$ with a strictly periodic and deterministic sequence $f(n) = f(n+P), \forall n$. The resulting sequence $w(n) := f(n)\, s(n)$ propagates through the unknown channel $h(n)$, whose output is modulated by $\exp{(j\theta_e n)}$, with θ_e standing for carrier frequency offset. The input-output relation of this communication set-up is given by

$$x(n) = y(n) + v(n) = e^{j\theta_e n}\sum_{l=0}^{L} h(l)w(n-l) + v(n)\ , \quad (1)$$

where $v(n)$ stands for additive stationary noise, independently distributed of $s(n)$. By defining the time-varying correlation of $w(n)$ at time n and lag τ as $c_{ww}(n;\tau) := Ew^*(n)w(n+\tau)$, it follows that $c_{ww}(n;\tau) = c_{ww}(n+P;\tau) = |f(n)|^2\sigma_s^2\ \delta(\tau)$, where $\sigma_s^2 := E|s(n)|^2$. Thus, $w(n)$ exhibits CS-statistics of period P. An approach for blind joint estimation of the unknown channel $\mathbf{h} := [h(0) \ldots h(L)]^T$ and the FO θ_e has been proposed in [9]. The main goal of the present paper is to analyze the asymptotic performance of the FO estimator proposed in [9]. Next, a brief overview on the channel and FO estimators from [9] are presented.

2. BLIND CHANNEL ESTIMATION

Note that the FO-model (1) can be re-written in the equivalent form of a channel without FO, but with an additional modulation superimposed on the input symbol stream $s(n)$

$$x(n) = \sum_{l=0}^{L} g(l)u(n-l) + v(n)\ , \quad (2)$$

where $g(l) := h(l) \exp(j\theta_e l)$, and $u(n) := w(n)\exp(j\theta_e n) = f(n)\exp(j\theta_e n)s(n)$. Since $c_{uu}(n;\tau) = c_{ww}(n;\tau)$, it follows that $u(n)$ exhibits CS-statistics, too. From (1), the output time-varying correlation is given by $c_{xx}(n;\tau) := Ex^*(n)x(n+\tau) = c_{xx}(n+P;\tau)$, and is periodic. Hence, it admits a Fourier Series (FS) decomposition, whose coefficients are the cyclic correlations. The cyclic correlation at cycle k and lag τ is given by $C_{xx}(k;\tau) := (1/P)\sum_{n=0}^{P-1} c_{xx}(n;\tau)\exp(-j2\pi kn/P) = \sigma_s^2 F_2(k)\exp(j\theta_e\tau)\sum_l g^*(l)\, g(l+\tau)\exp(-j2\pi kl/P) + \sigma_v^2\delta(k)$, where $F_2(k) := (1/P)\sum_{n=0}^{P-1}|f(n)|^2\exp(-j2\pi\, kn/P)$, $\sigma_s^2 := E\,|s(n)|^2$, and $\sigma_v^2 := E|v\,(n)|^2$. Considering the cyclic spectrum $S_{xx}(k;z) := \sum_\tau C_{xx}(k;\tau)z^{-\tau}$ at a cycle frequency $k \neq 0$, we obtain for $k = 1,\ldots,P-1$

$$S_{xx}(k;z) = \sigma_s^2 F_2(k)G(z)G^*(e^{-j2\pi k/P}/z^*), \quad (3)$$

where $G(z) := \sum_{l=0}^{L} g(l)z^{-L} = H(z\exp(j\theta_e))$, with $H(z) := \sum_{l=0}^{L} h(l)z^{-L}$. It can be shown that the transfer function $G(z)$ can be extracted as the gcd of the family of cyclic spectra $S_{xx}(k;z)$, $k = 1,\ldots,P-1$, provided that the period of $f(n)$ satisfies $P > L+1$ [2], [8]. Henceforth, we choose the period P so that $P > L+1$ is satisfied. Since the transfer function $G(z)$ can be estimated without any knowledge on FO θ_e, (2) can be de-convolved assuming the perfect inverse $G^{-1}(z)$. We have

$$x_1(n) := G^{-1}(z)x(n) = e^{j\theta_e n}f(n)s(n) + G^{-1}(z)v(n)\,. \quad (4)$$

Since the estimation of the true channel $h(n)$ from $g(n)$ requires knowledge of FO θ_e, we concentrate next on the frequency offset determination problem.

3. ESTIMATION OF FREQUENCY OFFSET

For real-valued input constellations (e.g., BPSK, PAM) the second order *conjugate* cyclic statistics of $x(n)$ allow the recovery of θ_e [4, 11]. Consider that $s(n)$ is a BPSK sequence, and the conjugate time-varying correlation $\tilde{c}_{xx}(n;\tau) := Ex(n)x(n+\tau) = \sigma_s^2\exp(j\theta_e(2n+\tau))\sum_l h(l)h(l+\tau) f^2(n-l) + \tilde{c}_{vv}(\tau)$, where $\tilde{c}_{vv}(\tau) := E\,v(n)\,v\,(n+\tau)$. Being periodically time-varying, the generalized Fourier Series coefficient of $\tilde{c}_{xx}\,(n;\,\tau)$, termed the conjugate cyclic correlation, is given by [c.f. (2)]

$$\tilde{C}_{xx}(\alpha;\tau) := \lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}\tilde{c}_{xx}(n;\tau)e^{-j\alpha n} = \sigma_s^2\tilde{F}_2(\alpha-2\theta_e) \times\sum_l h(l)h(l+\tau)e^{-j(\alpha-2\theta_e)l} + \tilde{c}_{vv}(\tau)\delta(\alpha), \quad (5)$$

where $\tilde{F}_2(\alpha) := \lim_{N\to\infty}(1/N)\sum_{n=0}^{N-1} f^2(n)\exp(-j\alpha n)$. Since $f(n)$ is periodic, $\tilde{F}_2(\alpha)$ consists of Kronecker deltas located at the harmonics $2\pi k/P$, with k being an integer; i.e., $\tilde{F}_2(\alpha) = \sum_k \tilde{F}_2(k)\,\delta(\alpha - 2\pi k/P)$. Due to (5), $\tilde{C}_{xx}(\alpha;\tau)$ consists also of Kronecker deltas located at frequencies $2\theta_e + 2\pi k/P$, where k is an integer. An estimate of θ_e can be obtained from the location of the kth Kronecker delta $\omega_k := 2\theta_e + 2\pi k/P$

$$\hat{\theta}_e := \frac{1}{2}(\arg\ \max_{\frac{2\pi k}{P}<\alpha<\frac{2\pi(k+1)}{P}}|\tilde{C}_{xx}(\alpha;\tau)| - \frac{2\pi k}{P}). \quad (6)$$

The condition $0 \leq 2\theta_e < 2\pi/P$, i.e., $0 \leq \omega_e T_s < \pi/P$, is adopted in order to ensure unique estimation of the offset θ_e. By choosing $\tau = 0$ in (6), the resulting statistics takes the form of the Discrete Fourier Transform (DFT)

$$\hat{\theta}_e := \frac{1}{2}(\arg\ \max_{\frac{2\pi k}{P}<\alpha<\frac{2\pi(k+1)}{P}}\left|\frac{1}{N}\sum_{n=0}^{N-1}x^2(n)e^{-j\alpha n}\right| - \frac{2\pi k}{P}). \quad (7)$$

For QAM input constellations that satisfy the moment conditions $Ew^2(n) = 0$, $Ew^3(n) = 0$, and $Ew^4(n) \neq 0$, the frequency offset is obtained from the fourth order cyclic statistics of the output. QAM(4) $\div$ QAM(32) constellations are typical input sequences for which these moment conditions hold. Defining the 4th-order time-varying correlation $\tilde{c}_{4,xx}(n;\tau) := E\;x^2(n)x^2(n+\tau)$, we deduce that $\tilde{c}_{4,xx}(n;\tau) = Es^4(n)\exp(j(4\theta_e n + 2\theta_e\tau))\sum_l h^2(l) h^2(l+\tau)\, f^4(n-l) + \tilde{c}_{4,vv}(n;\tau)$. The generalized Fourier Series coefficient of $\tilde{c}_{4,xx}(n;\tau)$, $\tilde{C}_{4,xx}(\alpha;\tau) := \lim_{N\to\infty}(1/N)\sum_{n=0}^{N-1}\tilde{c}_{4,xx}(n;\tau)\exp(-j\alpha n)$ is expressed as:

$$\tilde{C}_{4,xx}(\alpha;\tau) = Es^4(n)\tilde{F}_4(\alpha-4\theta_e)\sum_l h^2(l)h^2(l+\tau) \times e^{-j(\alpha-4\theta_e)l}e^{j2\theta_e\tau} + \delta(\alpha)Ev^2(n)v^2(n+\tau), \quad (8)$$

with $\tilde{F}_4(\alpha) := \lim_{N\to\infty}(1/N)\sum_{n=0}^{N-1} f^4(n)\exp(-j\;\alpha\;n)$. Since $\tilde{F}_4(\alpha)$ consists of a sequence of Kronecker deltas located at harmonics $2\pi k/P$, $\tilde{C}_{4,xx}(\alpha;\tau)$ consists also of a sequence of Kronecker deltas located at $4\theta_e + 2\pi k/P$, $k = 0,\ldots,P-1$. Similar to (7), we deduce the estimator

$$\hat{\theta}_e := \frac{1}{4}(\arg\max_{\frac{2\pi k}{P}<\alpha<\frac{2\pi(k+1)}{P}}\left|\frac{1}{N}\sum_{n=0}^{N-1}x^4(n)e^{-j\alpha n}\right| - \frac{2\pi k}{P}) \quad (9)$$

We note that the condition $0 \leq 4\theta_e < 2\pi/P$ is necessary for unique estimation of θ_e from (9).

4. ASYMPTOTIC PERFORMANCE

Since the asymptotic performance of estimators (7) and (9) can be established in a similar way, we next analyze the performance of (7). We assume that input $s(n)$ is BPSK (± 1), and that the zero-mean additive noise $v(n)$ satisfies the mixing condition

$$\sum_{n_1}\cdots\sum_{n_K}|c_{v\cdots v}(n_1,\ldots,n_K)| < \infty, \quad K = 1,2,\ldots \quad (10)$$

where $c_{v\cdots v}(n_1,\ldots,n_K) := \text{cum}\{v(n+n_1),\ldots,v(n+n_L),v(n)\}$ stands for $(K+1)$st-order cumulant of $v(n)$. The mixing condition (10) refers to the absolute summability of cumulants of any orders, and is a reasonable assumption in practice, since it is satisfied by all time series of weak memory, i.e., an asymptotically vanishing span of dependence on time series samples) [1, pp. 8, 25-27]. It is easy to check that the DFT-estimator (7) is equivalent to the nonlinear least-squares estimator (NLS) [6], [11]

$$\hat{\theta} := \arg\ \min_\theta J_{\text{NLS}}(\theta)\,, \quad (11)$$

$$J_{\text{NLS}}(\theta) := \frac{1}{2}\sum_{n=0}^{N-1}|x^2(n) - \sum_{k=0}^{P-1}\alpha_k e^{j(\omega_k n + \phi_k)}|^2, \quad (12)$$

where $\theta := [\alpha_0\ \phi_0\ \omega_0 \cdots \alpha_{P-1}\ \phi_{P-1}\ \omega_{P-1}]^T$. From (1), we deduce that for $n = 0, \ldots, N-1$

$$x^2(n) = e^{j2\theta_e n} \sum_{l=0}^{L} h^2(l) f^2(n-l) + \epsilon(n) , \quad (13)$$

$$\epsilon(n) := 2e^{j2\theta_e n} \sum_{0 \le l_1 < l_2 \le L} h(l_1)h(l_2)w(n-l_1)w(n-l_2) + 2e^{j\theta_e n} v(n) \sum_{l=0}^{L} h(l)w(n-l) + v^2(n). \quad (14)$$

Substituting $f^2(n) = \sum_{k=0}^{P-1} F_2(k) \exp(j2\pi kn/P)$ into (13), we obtain

$$x^2(n) = \sum_{k=0}^{P-1} \alpha_k e^{j\omega_k n} + \epsilon(n) , \quad (15)$$

$$\alpha_k := F_2(k) \left[\sum_{l=0}^{L} h^2(l) e^{-j2\pi kl/P} \right] , \quad (16)$$

$$\omega_k := \frac{2\pi k}{P} + 2\theta_e . \quad (17)$$

Thus, according to (15), the original frequency offset problem resumes to the estimation of an harmonic embedded in cyclostationary noise $\epsilon(n)$. It is of interest to know if it can be ensured that $\alpha_k \neq 0$. Due to (16), $\alpha_k = 0$ iff $F_2(k) = 0$ or $H_2(k) := H_2(\exp(j2\pi k/P)) = 0$, where $H_2(z) := \sum_{l=0}^{L} h^2(l) z^{-l}$. It can be shown that for the set of optimal precoders developed in [3], $F_2(k) \neq 0$. On the other hand, $H_2(k) = 0$ implies that $\exp(-j2\pi k/P)$ is a zero of $H_2(z)$. In practice, the condition $H_2(k) = 0$, for $\forall\ k = 0, \ldots, P-1$, is less likely to hold. However, by choosing $P = L+1$, from the Parserval identity $\sum_{l=0}^{L} |H_2(l)|^2 = \sum_{l=0}^{L} |h(l)|^4$, we conclude that there exists k, such that $H_2(k) \neq 0$. Thus, the estimation of offset θ_e is always possible, since there is at least one non-zero spectral line in the spectrum of $x^2(n)$.

The asymptotic performance of the frequency offset estimator (11) is sketched in the Appendix. From (30), the asymptotic variance of FO-estimator (11) is given by

$$\lim_{N\to\infty} N^3(\hat{\omega}_k - \omega_k)^2 = \frac{6S_{\epsilon\epsilon}(0; e^{j\omega_k})}{\alpha_k^2} = \frac{6}{\mathrm{SNR}_k} , \quad (18)$$

where $\mathrm{SNR}_k := \alpha_k^2 / S_{\epsilon\epsilon}(0; \exp(j\omega_k))$ represents the SNR corresponding to the kth harmonic $\omega_k := 2\pi k/P + 2\theta_e$. We note that if $f(n) = 1$, for $\forall\ n$, eq. (18) reduces to the known formula from the stationary case [6], [10, (32)-(40)], since the cyclic spectrum $S_{\epsilon\epsilon}(0; \exp(j\omega))$ becomes equal to the stationary spectrum $S_{\epsilon\epsilon}(\exp(j\omega))$.

FO θ_e can also be estimated by applying (7) on the equalized output $x_1(n)$ (see (4)). It is of interest to compare the asymptotic performance of this new estimator with (18). Similar to (15), we have the representation

$$x_1^2(n) = \sum_{k=0}^{P-1} \beta_k e^{j\omega_k n} + \mu(n) , \quad (19)$$

where $\beta_k := F_2(k)$ and $\mu(n) := (G^{-1}(z)v(n))^2 + 2e^{j\theta_e n} w(n)\ G^{-1}(z)v(n)$. The asymptotic performance of the FO-estimator associated to the equalized output $x_1(n)$ is given by

$$\lim_{N\to\infty} N^3(\hat{\omega}_k - \omega_k)^2 = \frac{6S_{\mu\mu}(0; e^{j\omega_k})}{\beta_k^2} , \quad (20)$$

where $S_{\mu\mu}(0; e^{j\omega_k})$ stands for the cyclic spectrum corresponding to $\mu(n)$ at zero cycle and frequency ω_k. We show next that for high SNR's, the asymptotic variance (20) is smaller than (18). For this it sufficies to note that $\lim_{\sigma_v \to 0} S_{\epsilon\epsilon}(0; e^{j\omega_k}) \neq 0$, and $\lim_{\sigma_v \to 0} S_{\mu\mu}(0; e^{j\omega_k}) = 0$.

However, a side effect of $G^{-1}(z)$ is to amplify the power of additive noise $v(n)$ at the output of equalizer. Suppose that $v(n)$ is white noise. Consider that channel $\mathbf{h}$ has unit norm, $||\mathbf{h}||_2 = 1$, and define the Signal-to-Noise Ratio SNR $:=$ 10log $\sigma_s^2/\sigma_v^2 =$ 10log σ_y^2/σ_v^2. Denote by $\mathbf{g}$ and $\mathbf{g}_1$ the impulse response vectors for $G(z)$ and $G^{-1}(z)$, respectively. Defining the scalar product $(\mathbf{g}, \mathbf{g}) := \frac{1}{2\pi} \int_0^{2\pi} G(e^{j\omega}) G^*(e^{j\omega}) d\omega$, from Schwartz's inequality $(\mathbf{g}_1^*, \mathbf{g}_1^*) \geq (\mathbf{g}, \mathbf{g}_1^*) / (\mathbf{g}, \mathbf{g})$, and $G(z)G^{-1}(z) = 1$, $(\mathbf{g}, \mathbf{g}_1^*) = 1$, it follows that $(\mathbf{g}_1^*, \mathbf{g}_1^*) \geq 1$. Since $v(n)$ is white, $\sigma_v^2(\mathbf{g}_1, \mathbf{g}_1)$ is the power of $G^{-1}(z)v(n)$, and hence the amplification of the noise at the output of equalizer $G^{-1}(z)$. Simulation experiments show that at low SNR's, the variance in (20) is larger than (18). In the next section, we perform several simulation experiments in order to illustrate this claim.

5. SIMULATION RESULTS

Throughout the simulations, we have considered the GSM channel with the baseband channel impulse response $\mathbf{h} = [0.53 + 0.07i, -0.24 - 0.23i, -0.54 - 0.32i, 0.11 + 0.44i, -0.036 - 0.099i]^T$, frequency offset $\theta_e = \pi/30$, periodic precoder $\{f(n)\}_{n=1}^{P} := [0.76\ ;\ 0.76\ ;\ 0.76\ ;\ 0.76\ ;\ 1.74]$, and AWGN $v(n)$. We have plotted the standard deviation (10log $\sqrt{E(\hat{\theta}_e - \theta_e)^2}$) of the frequency estimators (7) and (9) versus the Signal-to-Noise Ratio (SNR), defined, at the input of equalizer, as: SNR $:=$ 10log$(E|y(n)|^2/E|v(n)|^2)$ $=$ 10log$(E|s(n)|^2/E|v(n)|^2)$, $(||\mathbf{h}||_2 = 1)$. In Fig. 2-a, input $s(n)$ is BPSK and the frequency offset is estimated from the location of the first spectral line ($k = 0$) in (7), for two different scenarios: in the absence (dashed line) and presence of channel equalizer $G(z)$ ('+−' line). For both simulations $N = 100$ samples and MC $= 200$ Monte-Carlo runs have been performed. The asymptotic theoretical values (20) are represented in Fig. 2-a by the 'o−' line. We note that for moderate to high range of SNR $= 10 \div 40$ dB, the performance of estimator (7), in the presence of a channel equalizer, improves with respect to the case when no equalizer is used. But, for small SNR's (SNR $= 0 \div 10$ dB), the performance of estimator (7), in the presence of a channel equalizer, deteriorates with respect to the case when no equalizer is used. We note that the same conclusion turns out from the experimental plots presented in Fig. 2-b for a QPSK input, with the estimator (9) implemented for $k = 0$,

$N = 200$ samples, and MC$= 200$ Monte-Carlo runs. We note also that the experimental asymptotic values of the frequency estimator (7), are quite well predicted by the theoretical values for a medium to high range of SNR's and $N = 100$ samples (Fig. 2-a).

APPENDIX

It is convenient to adopt the notations from [10]:

$$y(n,\theta) \;:=\; \sum_{k=0}^{P-1} \alpha_k e^{j(\omega_k n+\phi_k)}\ , \tag{21}$$

$$\mathbf{y}(\theta) \;:=\; [y(0,\theta)\cdots y(N-1,\theta)]^T\ , \tag{22}$$

$$\mathbf{x} \;:=\; [x^2(0)\cdots x^2(N-1)]^T\ , \tag{23}$$

$$\epsilon \;:=\; [\epsilon(0)\cdots \epsilon(N-1)]^T\ , \tag{24}$$

$\mathbf{a}(n) := \partial y(n,\theta)/\partial\theta$, and $\mathbf{A} := [\mathbf{a}^T(0)\ldots\mathbf{a}^T(N-1)]^T$. We re-write (15) as: $\mathbf{x} = \mathbf{y}(\theta) + \epsilon$. In order to find the large sample performance of $\hat{\theta}$, we make use of a Taylor series expansion of the gradient $\partial\mathbf{a}/\partial\theta(\hat{\theta})$ around θ

$$\frac{\partial\mathbf{a}}{\partial\theta}(\hat{\theta}) \cong \frac{\partial\mathbf{a}}{\partial\theta}(\theta) + \frac{\partial^2\mathbf{a}(\theta)}{\partial\theta^2}(\hat{\theta}-\theta)\ . \tag{25}$$

Since $\partial\mathbf{a}/\partial\theta(\hat{\theta}) = \mathbf{0}$, from (25) we obtain

$$\hat{\theta}-\theta \cong -\left[\frac{\partial^2\mathbf{a}}{\partial\theta^2}(\theta)\right]^{-1}\frac{\partial\mathbf{a}}{\partial\theta}(\theta)\ . \tag{26}$$

Since $\frac{\partial\mathbf{a}}{\partial\theta} = \mathrm{Re}(-\frac{\partial\mathbf{y}^*(\theta)}{\partial\theta}\epsilon)$, $\frac{\partial^2\mathbf{a}}{\partial\theta^2} = \mathrm{Re}(\mathbf{Y}_N)$, where $\mathbf{Y}_N := \mathrm{Re}(\frac{\partial\mathbf{y}^*(\theta)}{\theta}\cdot\frac{\partial\mathbf{y}(\theta)}{\partial\theta^T} - \sum_{n=0}^{N-1}\frac{\partial^2\mathbf{y}^*(\theta,n)}{\partial\theta^2}\ \epsilon(n))$, (26) is equivalent to

$$\hat{\theta}-\theta = -\mathbf{Y}_N^{-1}\mathrm{Re}\left(\frac{\partial\mathbf{y}^*(\theta)}{\partial\theta}\cdot\epsilon\right)\ . \tag{27}$$

Define the $3P\times 3P$ block diagonal matrix $\mathbf{K}_N :=$ diag $([N^{1/2}\ N^{1/2}\ N^{3/2}\ ;\ldots;\ N^{1/2}\ N^{1/2}\ N^{3/2}])$. From (27), we obtain

$$\mathbf{K}_N(\hat{\theta}-\theta) = -(\mathbf{K}_N^{-1}\mathbf{Y}_N\mathbf{K}_N^{-1})^{-1}\left[\mathbf{K}_N^{-1}\mathrm{Re}\left(\frac{\partial\mathbf{y}^*(\theta)}{\partial\theta}\cdot\epsilon\right)\right]. \tag{28}$$

Since $v(n)$ satisfies (10), it can be shown that $\epsilon(n)$ satisfies the mixing condition (10), too. As a consequence of mixing assumption on $\epsilon(n)$, it follows that the second term in $\mathbf{K}_N^{-1}\mathbf{Y}_N\mathbf{K}_N^{-1}$ (i.e., $\mathbf{K}_N^{-1}\sum_{n=0}^{N-1}\frac{\partial^2\mathbf{y}^*(\theta,n)}{\partial\theta^2}\epsilon(n)\mathbf{K}_N^{-1})$) is asymptotically negligible as $N\to\infty$. Direct computations in (28) lead to

$$\lim_{N\to\infty}\mathbf{K}_N(\hat{\theta}-\theta)(\hat{\theta}-\theta)^T\mathbf{K}_N = \lim_{N\to\infty}\left[\mathbf{K}_N^{-1}\mathrm{Re}(\mathbf{A}^*\mathbf{A})\mathbf{K}_N^{-1}\right]^{-1} \times\left[\mathbf{K}_N^{-1}\mathrm{Re}(\mathbf{A}^*\epsilon)\mathrm{Re}(\mathbf{A}^*\epsilon)^T\mathbf{K}_N^{-1}\right]\left[\mathbf{K}_N^{-1}\mathrm{Re}(\mathbf{A}^*\mathbf{A})\mathbf{K}_N^{-1}\right]^{-1}. \tag{29}$$

Using the result $\lim_{N\to\infty}(1/N^{k+1})\sum_{n=0}^{N-1} n^k e^{j(\omega n+\phi)} = e^{j\phi}\ \delta(\omega)$, all the factors in (29) can be computed explicitly. We obtain the result that $\lim_{N\to\infty}\mathbf{K}_N(\hat{\theta}-\theta)(\hat{\theta}-\theta)^T\mathbf{K}_N$ is a block diagonal matrix with its $(k+1,k+1)$ diagonal block given by

$$\frac{1}{2}S_{\epsilon\epsilon}(0;e^{j\omega_k})\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{4}{\alpha_k^2} & -\frac{6}{\alpha_k^2} \\ 0 & -\frac{6}{\alpha_k^2} & \frac{12}{\alpha_k^2}\end{bmatrix}\ , \tag{30}$$

where $S_{\epsilon\epsilon}(0;e^{j\omega_k})$ stands for the cyclic spectrum of $\epsilon(n)$ corresponding to the cycle 0 and evaluated at frequency ω_k.

6. REFERENCES

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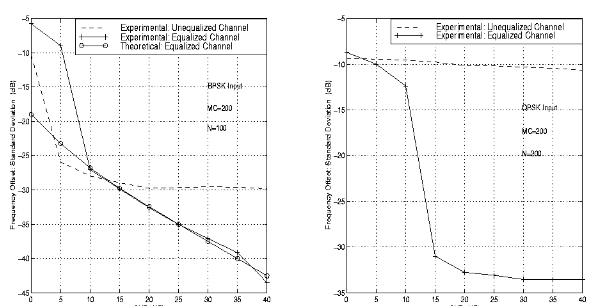

Figure 2: Std. Deviation of FO-Estimator: a) BPSK input b) QPSK input