

CLUTTER MITIGATION TECHNIQUES FOR SPACE-BASED RADAR

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ABSTRACT

The mission of a ground moving target indication (GMTI) radar, as its name implies, is to detect and classify ground-based vehicles, even ones with very low velocities. This type of radar can provide a wide area of coverage and frequent updates of a specific area of interest if the radar is placed on a satellite with a low earth orbit. However, because of the large footprint of the radar on the ground and the high satellite velocity, target signals must compete with very strong, nearby clutter. This paper describes how space-time adaptive processing (STAP) can be used for the purposes of clutter rejection in order to perform the GMTI function. In addition, we confront several important issues for a space-based radar such as pulse repetition frequency (PRF) selection, the choice of a STAP algorithm, and the number of spatial channels. These results are quantified in terms of clutter cancellation and angle accuracy.

1. INTRODUCTION

Ground surveillance radar systems are concerned with the detection of moving vehicles on the surface of the earth. This function of a radar system is known as *ground moving target indication (GMTI)*. Ideally, a GMTI radar maintains coverage of a large area which necessitates a high area coverage rate with frequent revisits to a specific area of interest. Traditionally, GMTI radars have been implemented using airborne (aircraft) platforms. However, these airborne systems are not able to maintain the desired coverage in terms of area and revisit rates. A space-based radar (SBR) system, on the other hand, can provide these necessary capabilities for large area GMTI surveillance. Since an SBR platform is at such a high altitude, it has access to a very large portion of the ground below at any point in time. This results in a much higher area coverage rate. In addition, such an SBR system has nearly unlimited access to any point on the earth, while an airborne radar may be denied access based on the current, possibly hostile environment it must operate in. For these reasons, it is desirable to implement the GMTI function aboard satellites in an SBR system.

The SBR receives the reflected returns using an array of sensor elements. The array is steered to the area of interest using a beamformer that has a beamwidth determined by its aperture. However, the projection of this beam onto the ground subtends a large area determined by the distance of the area of interest from

the radar. As a result, large clutter returns are received by the radar from the same angular region as potential targets. In the case of a pulsed radar system, clutter can be effectively mitigated using space-time adaptive processing (STAP) [1]. The STAP problem for an SBR system differs from STAP for airborne radar in several, dramatic ways. The large platform velocity of the satellite induces large Doppler frequencies on the returns, so large that it is not feasible to choose a pulse repetition frequency (PRF) that is large enough to prevent aliasing. Therefore, we must contend with this Doppler aliasing within the STAP framework and attempt to control it through proper PRF selection. In addition, the proximity of competing clutter to targets has dramatic implications on the performance of an SBR system.

This article discusses the use of STAP for SBR. We begin with a discussion of STAP in which we point out the major differences between STAP for SBR and traditional STAP for airborne radar. Next, we discuss techniques for controlling the aliasing of clutter, thereby simplifying the subsequent adaptive cancellation. Then, the performance of STAP algorithms is investigated for the SBR application. We use the criteria of the signal-to-interference-plus-noise (SINR) loss and the Cramér-Rao bound on angle accuracy to motivate our selections.

2. SPACE-TIME ADAPTIVE PROCESSING

In this section, we give a brief overview of STAP along with a discussion of two common metrics for adaptive processing: signal-to-interference-plus-noise ratio (SINR) loss and Cramér-Rao bound on angle accuracy for detected targets.

2.1. Clutter Cancellation

For the STAP processing of radar signals, we want to emphasize signals that arrive at the radar from a certain angle and at a certain Doppler frequency, while rejecting all other significant energy. We begin by defining the space-time steering vector for a signal arriving from an angle ϕ with a Doppler frequency f

$$\mathbf{v}(\phi, f) = \mathbf{b}(f) \otimes \mathbf{a}(\phi) \quad (1)$$

where

$$\mathbf{b}(f, L) = \frac{1}{\sqrt{L}} \begin{bmatrix} 1 & e^{j2\pi \frac{f}{F_s}} & \dots & e^{j2\pi(L-1) \frac{f}{F_s}} \end{bmatrix}^T \quad (2)$$

$$\mathbf{a}(\phi) = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 & e^{j2\pi \frac{d \sin \phi}{\lambda}} & \dots & e^{j2\pi(M-1) \frac{d \sin \phi}{\lambda}} \end{bmatrix}^T \quad (3)$$

This work was supported by DARPA under Air Force Contract #F19628-95-C-0002. Opinions, interpretations, conclusions and recommendations are those of the authors and are not necessarily endorsed by the United States Air Force.

are the temporal and spatial steering vectors, respectively. The radar wavelength is λ , the physical spacing between spatial phase centers is d , L is the number of pulses in a coherent processing interval (CPI), and M is the number of spatial channels. F_s is the rate at which the radar system transmits and receives pulses, known as the pulse repetition frequency (PRF). Note that in the case of an SBR, a very large aperture is required for the array in order to achieve the necessary gain on target signals. However, only a limited number of spatial channels can be digitized for adaptive processing. We assume these spatial channels are formed using non-overlapping sub-arrays of the full aperture.

A space-time snapshot containing a target signal is given by

$$\mathbf{x}(n) = \alpha_t \mathbf{v}(\phi_t, f_t) + \mathbf{x}_{i+n}(n) \quad (4)$$

where α_t , ϕ_t , and f_t are the target amplitude, angle, and Doppler frequency, respectively. $\mathbf{x}_{i+n}$ is the interference-plus-noise signal and n is the snapshot index. The interference consists of ground clutter returns of the radar's transmitted signal. The optimum space-time processor is given by [3]

$$\mathbf{w} = \mathbf{R}_{i+n}^{-1} \mathbf{v}(\phi_0, f_0) \quad (5)$$

where $\mathbf{R}_{i+n} = \mathbb{E} \{ \mathbf{x}_{i+n}(n) \mathbf{x}_{i+n}(n)^H \}$ is the interference-plus-noise covariance matrix. The space-time steering vector $\mathbf{v}(\phi_0, f_0)$ determines the angle and Doppler frequency of interest. Note that the Doppler frequency of a clutter patch is given by

$$f_c = \frac{2v_s}{\lambda} \cos \gamma_c \quad (6)$$

where v_s is the satellite velocity and $\gamma_c = \phi_c - \phi_{\text{steer}}$ is the cone angle of the clutter patch with respect to velocity axis of the satellite and ϕ_{steer} is the angle between the array axis and the velocity axis due to mechanical steering of the array.

2.2. SINR Loss

The performance of the optimum space-time processor from (5), as well as the sub-optimal processors discussed in Section 3, is measured via the output signal-to-interference-plus-noise ratio (SINR). As the name implies, this is simply the output signal power divided by the total interference-plus-noise

$$\text{SINR} = \frac{\alpha_t^2 | \mathbf{w}^H \mathbf{v}(\phi_t, f_t) |^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}} \quad (7)$$

where α_t^2 is the signal power at the output of the array. Many times, we want to compare the SINR to the maximum SINR that could possibly be achieved. This upper limit is determined by the ideal matched filter for the interference-free case, i.e., thermal noise only. Normalizing the SINR by the SNR of the ideal (interference-free) matched filter yields

$$L_{\text{sinr}} = \frac{\text{SINR}}{\text{SNR}_0} = \mathbf{v}(\phi_t, f_t)^H \mathbf{R}_{i+n}^{-1} \mathbf{v}(\phi_t, f_t) \quad (8)$$

which is known as SINR loss. Many times SINR loss is computed across angles and/or Doppler frequencies. An SINR loss of unity (0 dB) indicates perfect interference cancellation. A processor is evaluated by how closely it comes to achieving this goal. Sub-optimum processors are judged on their ability to approach the SINR loss of the optimum processor.

2.3. Angle Estimation Performance

A lower bound on the angle estimation performance of a STAP radar can be quantified using the Cramér-Rao bound (CRB) [4]. Note that the CRB gives the minimum variance of an unbiased estimator. If an estimator can achieve the CRB, then it is the maximum-likelihood estimator. The CRB is found by solving for the diagonal elements of the inverse of the Fisher information matrix. For more details, see [4]. We proceed by giving a quick, intuitive sketch of the CRB in terms of sum and difference channel beamformers for the purposes of angle estimation.

If we begin by defining sum and difference steering vectors as

$$\mathbf{v}_\Sigma = \mathbf{v} \quad \mathbf{v}_\Delta = \mathbf{D} \mathbf{v} \quad (9)$$

where $\mathbf{D} = j \text{diag} \{ \frac{1-M}{2}, \frac{3-M}{2}, \dots, \frac{M-1}{2} \}$ is the derivative matrix and the vector $\mathbf{v}_\Delta$ is technically the derivative of $\mathbf{v}_\Sigma$. Now we can form optimum sum and difference beamformers as

$$\mathbf{w}_\Sigma = c_\Sigma \mathbf{R}_{i+n}^{-1} \mathbf{v}_\Sigma \quad \mathbf{w}_\Delta = c_\Delta \mathbf{R}_{i+n}^{-1} \mathbf{v}_\Delta \quad (10)$$

where c_Σ and c_Δ normalize for unit gain on their respective steering vectors from (9). We proceed by computing the output power of the sum and difference beamformers

$$P_\Sigma = \mathbf{w}_\Sigma^H \mathbf{R}_{i+n} \mathbf{w}_\Sigma \quad P_\Delta = \mathbf{w}_\Delta^H \mathbf{R}_{i+n} \mathbf{w}_\Delta \quad (11)$$

We can also measure the normalized cross-correlation $\rho_{\Sigma\Delta}$ between the sum and difference channels, which is simply their cross-correlation normalized by their respective powers

$$\rho_{\Sigma\Delta}^2 = \frac{|\mathbf{w}_\Sigma^H \mathbf{R}_{i+n} \mathbf{w}_\Delta|^2}{P_\Sigma \cdot P_\Delta} \quad (12)$$

Using these quantities, the CRB on the angle estimation error (variance) is then given by

$$\sigma_\phi^2 \geq \frac{1}{2\pi^2 \text{SNR}_0 P_\Delta (1 - \rho_{\Sigma\Delta}^2) \cos^2 \phi} \quad (13)$$

where the term $\cos^2 \phi$ represents the non-linear degradation in performance as the steering angle moves away from broadside ($\phi = 0^\circ$). The term SNR_0 is the signal-to-noise ratio in the absence of interference (clutter) at the beamformer output.

3. SBR CLUTTER MITIGATION

The nature of the SBR clutter dictates some special considerations that are unique to the SBR problem [2, chapter 11] and have profound implications on performance. In this section, we study various options that impact SBR clutter mitigation. For this study, the satellite orbit is at an altitude of 770 km, the satellite velocity is 7160 m/sec, the average transmit power is 450 watts, the antenna has an 8×5 meter aperture, and the transmit frequency is 10 GHz. All of the analysis that follows is based on ideal covariances.

3.1. Ambiguity Control: PRF Selection

Radar clutter is distributed in angle and Doppler with a relationship given by (6). Furthermore, this distribution changes as a function of range. Thus, any attempt to cancel clutter must begin with a discussion of sampling strategies that minimize the effects of clutter aliasing in Doppler and range fold-over. The mechanism that controls these ambiguities is the PRF F_s .

By examining (6), the maximum clutter Doppler frequency is given by $\frac{2v_s}{\lambda} = 477$ kHz for our system parameters. To prevent clutter from aliasing, the Doppler sampling rate (i.e., the radar PRF) would have to be twice the maximum clutter Doppler frequency. Clearly, such a high PRF is impractical. In addition, high PRFs introduce clutter range ambiguities that occur whenever the collection time for a given pulse is shorter than the time associated with returns from the elevation footprint on the ground. This effect is accentuated at long ranges due to the increased size of the projection of the elevation beam onto the ground.

Fortunately, we are only interested in ground moving targets that have limited velocities, e.g., $v_{max} \leq 100$ km/hr. The associated Doppler spread is then limited to $-\frac{2v_{max}}{\lambda} \leq f \leq \frac{2v_{max}}{\lambda}$. Therefore, we can use a much lower PRF that still prevents target aliasing. Note that clutter aliasing will still occur, but the effect is reduced because of the attenuation of sidelobe clutter by the transmit and receive azimuthal beampatterns. If properly designed, these beampatterns can reduce most of the clutter that leaks in through the sidelobes to below the thermal noise floor power. Similarly, the elevation beampattern alleviates the problem associated with range ambiguous clutter when the PRF is too high. As a result, we need only concern ourselves with the primary aliased clutter in range and Doppler, that is, the first few fold-overs in these two dimensions. The fact that these aliased clutter points are limited in number makes the ensuing PRF selection tractable.

We can then formulate an optimization problem: choose the radar PRF such that it minimizes the maximum principal azimuthal sidelobe clutter (Doppler aliasing) and the maximum principal elevation sidelobe clutter (range ambiguities), under the constraint that the PRF should never be low enough that the radar clutter notch aliases into the region of target velocities of interest. This notch is referred to as the radar blind speed v_b . Solving this optimization problem yields the rule

$$F_s = \max \left(\frac{2v_b}{\lambda}, \min(f_{ru}, f_{dpc} \sin \phi_{az}) \right) \quad (14)$$

where

$$f_{ru} = \sqrt{\frac{2cHv_s \tan \theta_{gr} \sin \phi_{az}}{\lambda W R_s}} \quad f_{dpc} = \frac{4v_s}{W} \quad (15)$$

are the PRF that balances azimuthal and elevation sidelobe levels and the DPCA PRF, respectively. The DPCA PRF, f_{dpc} , is the PRF that causes sidelobe clutter to alias to the Doppler frequency of the mainlobe clutter. Here, θ_{gr} is the grazing angle, R_s is the range to the ground, H is the antenna height, W is the antenna width, and ϕ_{az} is the azimuth angle. The suggested PRF selection criteria is illustrated in Figure 1 as a function of range. Note the selected PRF is never allowed to fall below the blind speed PRF of $F_s = 2150$ Hz. Also the maximum PRF can never exceed $F_s = f_{dpc} = 3580$ Hz. Between these two values the PRF is chosen to be f_{ru} .

3.2. Partially Adaptive STAP

Now that we are equipped with a PRF selection rule that minimizes the aliasing of clutter in range and Doppler, we can focus our attention on methods to cancel it. First, note that the computational complexity of implementing (5) for every possible look direction depends on the cube of the number of adaptive degrees of freedom (DOF). In order to minimize the size, weight and power of the on-board signal processor, we must transform the data into a smaller

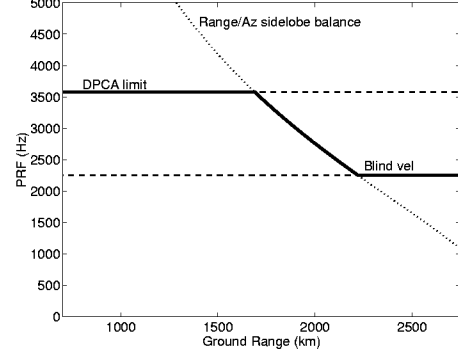


Figure 1: Radar PRF selection (solid line) to reduce effect of range and clutter ambiguities.

subspace that contains the signals and interference. Data-adaptive methods are available for estimating this subspace, e.g. using an eigendecomposition. However, such techniques require added processing complexity. Often, similar levels of performance can be achieved by exploiting a priori signal models to craft the subspace transformations. Beamspace post-Doppler STAP algorithms fall into this class and are the methods we consider for the SBR clutter mitigation. The advantage of these methods is that they can isolate the clutter in angle and Doppler because of their fixed relationship from (6) for clutter to reduce the size of the problem. Such algorithms begin by transforming the radar snapshots into beamspace

$$\mathbf{y}(n) = \mathbf{T}^H \mathbf{x}(n) \quad (16)$$

where the columns of $\mathbf{T}$ consist of beamspace steering vector given by (3). Next, the data is processed using a Doppler filter bank. This filter bank produces snapshots, $\mathbf{y}_l(n)$, each tuned to a specific Doppler frequency $-\frac{F_s}{2} < f_l < \frac{F_s}{2}$. The space-time snapshots are formed by combining Doppler bin outputs from all the beams. One common method, known as *adjacent bin STAP*, uses neighboring Doppler bins along with the Doppler bin of interest. For example, the space-time snapshot vector is

$$\mathbf{z}_l = [\mathbf{y}_l(n) \dots \mathbf{y}_{l+1}(n)] \quad (17)$$

in the case where 2 adjacent bins are desired. A "single bin" algorithm thus implies that only the Doppler bin of interest is used. Adjacent bin algorithms have been shown to exhibit very good performance for airborne radar applications due to the high degree of correlation between neighboring Doppler bins. Another common method, *PRI-staggered STAP*, uses two staggered, overlapping windows to form two different bins a and b for each Doppler frequency f_l . The two windows share all of the pulses except the first and last ones. Their corresponding snapshots are formed as

$$\mathbf{z}_i = [\mathbf{y}_i^a(n) \mathbf{y}_i^b(n)] \quad (18)$$

The PRI-staggered algorithm is illustrated in Fig 2. The DOFs used by PRI-staggered STAP are $2M$ where M is the number of beams. By comparison, adjacent bin STAP uses all of the L pulses but only the top bank of Doppler filters and its adjacent neighboring bins. The DOFs used by adjacent bin algorithms is kM where k is the number of bins used.

We compare these partially adaptive STAP algorithms in terms of SINR loss performance for a grazing angle of $\theta_{gr} = 45^\circ$ (clutter-to-noise ratio = 25 dB). The results are shown in Fig. 3. Also

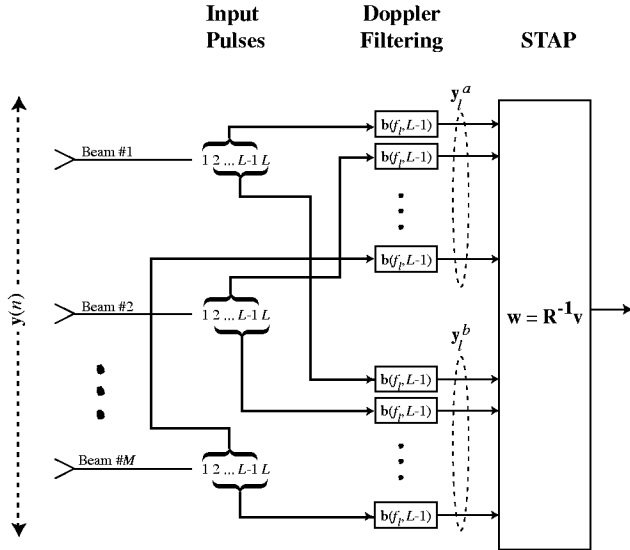


Figure 2: PRI-staggered STAP.

included is fully adaptive STAP (using all spatial and temporal DOFs) as a performance bound for the partially adaptive methods. The PRI-staggered algorithm is almost indistinguishable from fully adaptive STAP. On the other hand, the adjacent bin algorithms require multiple bins (4) just to come close to this level of performance. Note that the DOFs needed for 4 bins is $4M$ whereas PRI-staggered only required $2M$. These differences in clutter mitigation performance can be attributed to the large satellite velocity, v_s , which causes a significant Doppler spread in the clutter. As a result, multiple bins are required for the adjacent bin method. On the other hand, the PRI-staggered algorithm exploits the correlation between the two staggers to compensate for this clutter spread.

The last consideration we need to make is the number of spatial channels, i.e., beams, for the beamspace transform $\mathbf{T}$ prior to STAP. Ideally, the number of beams is minimized without sacrificing performance. We compare the performance of fully adaptive STAP for 2, 3, and 4 beams. Note that the clutter cancellation performance will be virtually indistinguishable because the aliased clutter is at a single Doppler due to the PRF selection rule. We consider the angle accuracy of the STAP algorithm using the CRB analysis from 2.3 with an output SINR = 13 dB ($\theta_{gr} = 45^\circ$). The results are shown in Fig. 4. Conceptually, clutter cancellation requires at least two spatial DOFs, one to cancel clutter and one for the look-direction constraint $\mathbf{v}(\phi_0, f_0)$. Examining Fig. 4, we see that 2 spatial channels has the worst performance. However, angle accuracy improves as the number of channels increases. In the cases of 3 and 4 channels, angle accuracy improves to better than 5:1 beamsplitting (RMSE < 0.2) for all velocities.

4. CONCLUSION

This paper has considered the mitigation of radar clutter received by an SBR. We noted that Doppler and range ambiguities play an essential role in determining performance. A simple technique was constructed for the PRF selection that minimizes the deleterious effects of these ambiguities. Furthermore, the performance

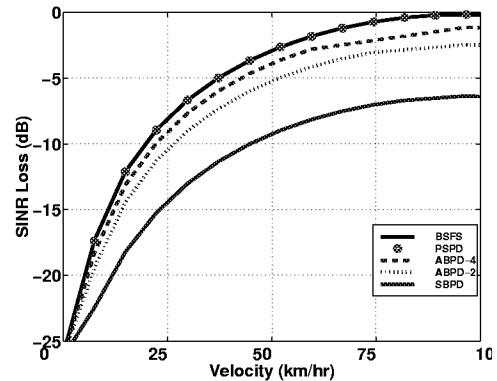


Figure 3: SINR Loss comparison for various beamspace post-Doppler STAP algorithms. Top solid line is beamspace full STAP (BSFS), circles are PRI-staggered post-Doppler (PSPD), dashed line is adjacent bin (4) post-Doppler (ABPD-4), dotted line is adjacent bin (2) post-Doppler (ABPD-2), and bottom solid line is single bin post-Doppler (SBPD).

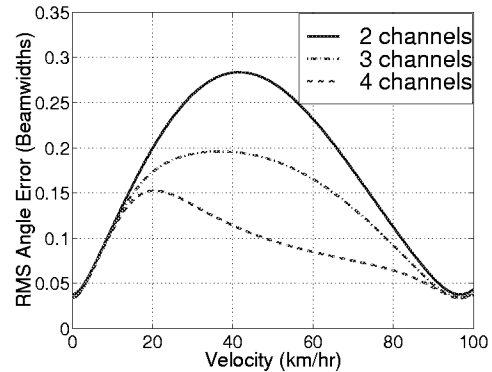


Figure 4: Root mean square error (RMSE) angle accuracy versus target velocity for 2, 3, and 4 channels.

of fully and partially adaptive STAP algorithms was analyzed. We concluded that a PRI-staggered post-Doppler STAP algorithm, operating in a beamspace consisting of 3 or more beams, has near optimal SINR as well as very good angle accuracy.

5. REFERENCES

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