

OPTIMUM CODEC COMPANDING FOR HIGH-SPEED PCM DATA TRANSMISSION IN TELEPHONE NETWORKS

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ABSTRACT

Codecs used in telephone networks in the United States have a μ -law companding characteristic solely designed for the purpose of transmitting voice signals [1, 2]. These μ -law codecs are not well suited for the latest generation of high-speed digital voiceband modems requiring PCM data transmission at the quantization levels of the codec [3, 4, 5, 6, 7, 8, 9].

In this paper, we have shown that using a linear codec in the telephone network is optimal for PCM transmission (in terms of a minimum symbol error rate) for the range of SNRs observed over most modern telephone lines. We demonstrate that with a linear codec we get a dramatic improvement in the performance of these PCM modems *without any* increase in the complexity of the modem. In practice, it would be necessary and relatively simple to implement an additional feature in the μ -law codec to detect a voiceband modem during the initial phase of training and switch to a linear companding characteristic.

1. INTRODUCTION

A codec is a device that interfaces an analog channel eg, local subscriber loop of a telephone network to a digital transmission trunk eg, T1 carrier. It is designed to perform the transmit encoding (A/D conversion) and receive decoding (D/A conversion) functions at the analog termination of the digital trunk. The quantization levels for A/D conversion and reconstruction points for D/A conversion are determined by the companding rule that is used. In the United States, telephone channel codecs use a μ -law companding characteristic with $\mu = 255$ [1, 2]. This was designed solely for the purpose of transmitting voice signals, that are more likely to have amplitude values near zero than at the extreme peak values allowed. In order to achieve high data rate voiceband transmission (> 33.6 Kbps) in present telephone networks, it is necessary to considerably reduce the quantization noise due to the A/D conversion process. This is achieved by transmitting PCM symbols at the quantization levels of the codec. This fact was realized early on in [3] and is the basis of the recently drafted 56K voiceband modem standard [4, 5] for PCM downstream transmission. Further, in [6, 7] techniques have been proposed to achieve high speed PCM upstream transmission at data rates > 33.6 Kbps.

With a μ -law codec in the line-card at the central office, the signal constellation must be designed according to the μ -law companding characteristic. However, as we shall show in this paper, these μ -law like signal constellations (resulting from using some of the μ -law quantization levels) are not well suited for high speed

PCM data transmission applications at the reasonably good signal-to-noise (SNR) ratios that are available on most modern telephone connections. In fact, performance tests on the recently standardized 56K (PCM) modems [4] have revealed downstream connection rates of only 40-50Kbps over most residential telephone lines. These modems are capable of achieving a maximum downstream connection rate of 53.3Kbps limited by a combination of the technology used in the modem, available network quantization levels and the signal-to-noise ratio. In this paper, we have shown that using a linear codec in the telephone network is optimal for PCM transmission (in terms of a minimum symbol error rate) for the range of SNRs observed over most modern telephone lines. We demonstrate that with a linear codec we get a dramatic improvement in the performance of these PCM modems including an increase in the maximum achievable connection rate *without any* increase in the complexity of the modem.

In section 2, we begin by calculating the maximum information transfer rate for PCM data transmission over the local subscriber loop treating it as an additive white Gaussian noise (AWGN) channel with an equi-probable distribution on the input PCM symbols [3, 5, 10, 11]. We evaluate this maximum information transfer rate with both linear and μ -law codecs for upstream (from the client modem to the central office) and downstream PCM transmission at varying SNRs. Next in section 3, we analytically solve for the "optimum" companding characteristic minimizing the raw (uncoded modulation) SER for PCM transmission with the FCC imposed average voiceband power constraint over the local subscriber loop. We show that asymptotically as the $\text{SNR} \rightarrow \infty$ (for practical purposes $\text{SNR} > 45\text{dB}$ would suffice) the optimum companding characteristic is *linear*.

2. PCM TRANSMISSION: INFORMATION TRANSFER RATE ANALYSIS

DOWNSTREAM:

The transmission link from the digital modem (i.e., the modem connected to a 4-wire digital trunk) to the client modem (i.e., the modem connected to a 2-wire twisted pair) is shown in figure 1. The subscriber loop is modeled as an AWGN channel (please see figure 2) where the transmitted symbols x_i are determined by the codec decoder (D/A converter) characteristic.

Hence, the downstream path is a discrete input - continuous output channel and the maximum information transfer rate can be

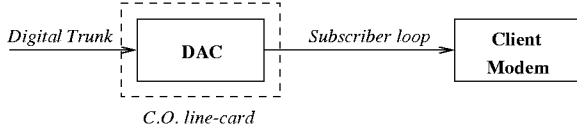


Figure 1: Block diagram of the downstream transmission path

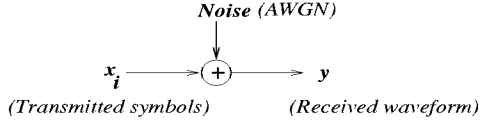


Figure 2: Model for downstream transmission path

expressed as [5, 10, 11]:

$$I = \sum_{i=1}^n \int p(y|x_i) P(x_i) \log \frac{p(y|x_i)}{p(y)} dy$$

where $x_i, i = 1, 2, \dots, n$ are the n equi-probable transmitted symbols i.e., $P(x_i) = 1/n, i = 1, 2, \dots, n$. $p(y|x_i)$ and $p(y)$ are the conditional probability density of y given x_i (Gaussian) and the probability density of y respectively.

UPSTREAM:

The transmission link from the client modem to the digital trunk is shown in figure 3. Again, the subscriber loop can be modeled as an AWGN channel but followed by a quantizer Q representing the codec encoder (A/D converter). Please see figure 4.

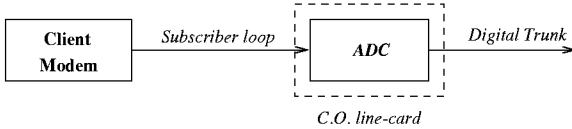


Figure 3: Block diagram of the upstream transmission path

The transmitted symbol constellation in this case is determined by the codec encoder (A/D converter) characteristic. Hence, the upstream transmission path is a discrete input - discrete output channel and the maximum information transfer rate can be expressed as [5, 10, 11]:

$$I = \sum_{i=1}^n \sum_{j=1}^m P(y_j|x_i) P(x_i) \log \frac{P(y_j|x_i)}{P(y_j)}$$

where $x_i, i = 1, 2, \dots, n$ are the n equi-probable transmitted symbols i.e., $P(x_i) = 1/n, i = 1, 2, \dots, n$. $P(y_j|x_i)$ and $P(y_j)$ are the conditional probability of y_j given x_i and the probability of y_j respectively.

We numerically evaluated this maximum information transfer rate for both upstream and downstream data transmission with a

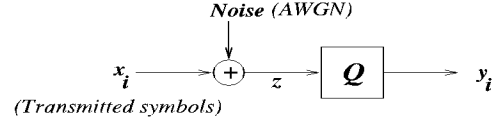


Figure 4: Model for upstream transmission path

64-point and 128-point input PCM constellation and for SNRs in the range of 25-55dB using a linear and μ -law codec. Due to the discrete number of transmitted symbols, the asymptotically maximum information transfer rate is $f_s \log_2 n = 48\text{Kbps}$ for the 64-point constellation and $= 56\text{Kbps}$ for the 128-point constellation [10] (the sampling frequency $f_s = 8\text{KHz}$). The signal constellation was chosen to satisfy the average power constraint imposed by the FCC and to have maximum separation between adjacent signal points. These results along with the Shannon capacity limit are plotted in figures 5 and 6 for downstream and upstream transmission respectively.

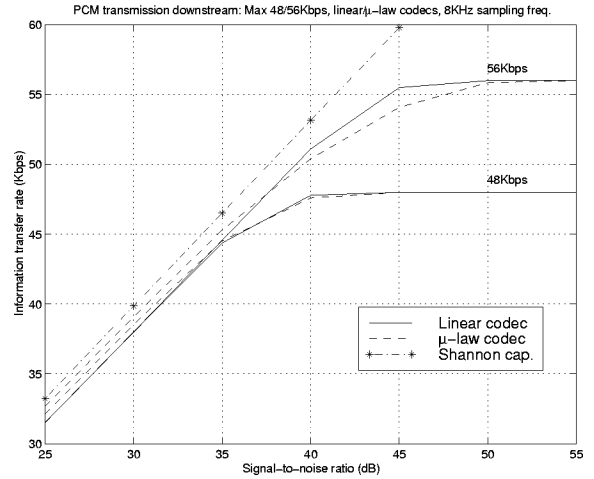


Figure 5: PCM transmission downstream: Maximum information transfer rate with linear and μ -law codecs in AWGN channel

From the figures, we see that the linear codec achieves a slightly higher maximum information transfer rate compared to the μ -law codec in the range of SNRs of interest (47-55dB). However in this range, the information transfer rate for both codecs close the maximum algebraic limit due to the discrete number of transmitted symbols.

3. "OPTIMUM" COMPANDING FOR PCM TRANSMISSION: PROBABILITY OF ERROR ANALYSIS

The information transfer rate analysis above is mainly of theoretical significance and shows us that a μ -law codec (resulting in a μ -law like signal constellation) can perform almost as well as a linear codec (with a linear signal constellation) over the range of SNRs of interest (47-55dB). However, from an implementation

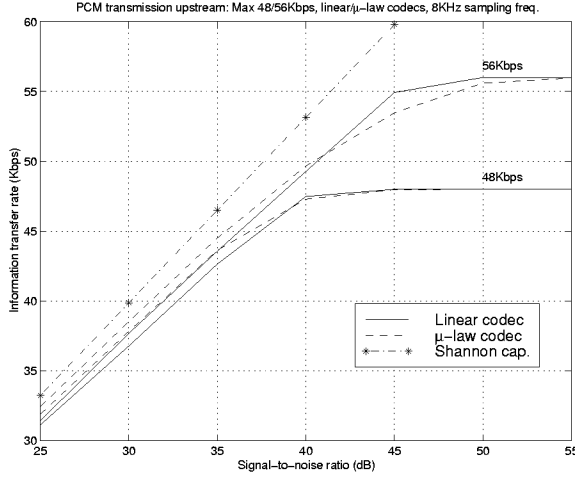


Figure 6: PCM transmission upstream: Maximum information transfer rate with linear and μ -law codecs in AWGN channel

point of view it is crucial to determine the performance (BER) that can be achieved at a particular data rate for a fixed complexity or alternatively, the complexity that must go into the modem in order to achieve a particular data rate at a specified BER. We attempt to address this issue in this section by studying the raw (uncoded modulation) SER for different signal constellations and determine the “optimum” constellation that minimizes the SER for a given average transmitted power and number of constellation points (bit rate). The codec companding law is determined by the equation for the optimum signal constellation.

Let us start by defining $\{+/-x_i, i = 1, 2, \dots, n\}$ to represent the $2n$ equi-probable transmitted PCM symbols forming the symmetric signal constellation shown in figure 7. $\{d_i, i = 1, 2, \dots, n\}$ are the corresponding distances between the adjacent signal pairs as shown in the figure.

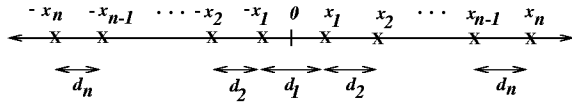


Figure 7: Transmitted PCM constellation with $2n$ symbols

We consider the AWGN channel shown in figure 2 and calculate the average SER, P_e for the above constellation. Let $P_{e|x_i}$ denote the probability of error for symbol x_i . Assuming optimal Gaussian detection (in the client modem for downstream transmission and in the ADC at the central office for upstream transmission), we can write $P_{e|x_i}$ in terms of the Q-function $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp^{-x^2/2}$ as [1],

$$\begin{aligned} P_{e|x_i} &= Q\left(\frac{d_i}{2\sigma}\right) + Q\left(\frac{d_{i+1}}{2\sigma}\right), \quad i = 1, 2, \dots, n-1 \\ &= Q\left(\frac{d_n}{2\sigma}\right), \quad i = n \end{aligned}$$

Hence, the average SER P_e is,

$$P_e = \frac{1}{n} \sum_{i=1}^n P_{e|x_i} = \frac{1}{n} Q\left(\frac{d_1}{2\sigma}\right) + \frac{2}{n} \sum_{i=2}^n Q\left(\frac{d_i}{2\sigma}\right) \quad (1)$$

Now we must design the transmitted signal constellation to satisfy the average power constraint, $\frac{1}{n} \sum_{i=1}^n x_i^2 \leq E$ where E is the FCC specified maximum average power over the local subscriber loop.

We solve the above constrained constellation design problem using the method of Lagrange multipliers:

$$\begin{aligned} J &= P_e + \lambda[E_{av} - E] \\ &= \frac{1}{n} Q\left(\frac{d_1}{2\sigma}\right) + \frac{2}{n} \sum_{i=2}^n Q\left(\frac{d_i}{2\sigma}\right) + \lambda\left[\frac{1}{n} \sum_{i=1}^n x_i^2 - E\right] \end{aligned}$$

Differentiating wrt d_i and making use of the fact that $x_i = \frac{d_1}{2} + \sum_{j=2}^i d_j$ we arrive at the following recursive equation for the d_i 's,

$$\exp -\frac{d_i^2}{8\sigma^2} = 2\sqrt{2\pi}\lambda\sigma \sum_{j=i}^n x_j, \quad i = 1, 2, \dots, n \quad (2)$$

in terms of the parameter λ which is determined from the average power constraint,

$$\frac{1}{n} \sum_{i=1}^n x_i^2 = E \quad (3)$$

Solving equations 2 and 3 will give us the optimal signal constellation and codec companding characteristic for PCM data transmission over the AWGN channel. From these equations, we make the following important observations:

- (1) For a given channel (σ^2) and maximum average power E the spacing between the adjacent signal points of the optimal constellation monotonically increases as we go away from 0.

In order to establish this fact we divide equation 2 for $i = j$ by itself for $i = j + 1$ to get,

$$\exp \frac{d_{j+1}^2 - d_j^2}{8\sigma^2} = \frac{\sum_{i=j}^n x_i}{\sum_{i=j+1}^n x_i} > 1$$

since $x_n > x_{n-1} > \dots > x_1 > 0$. This implies that $\frac{d_{j+1}^2 - d_j^2}{8\sigma^2} > 0$ or $d_{j+1} > d_j, j = 1, 2, \dots, n-1$.

- (2) The asymptotically optimal constellation as the SNR is increased is none but the linearly spaced constellation in which $d_i = d_j, \forall i, j$.

We prove this result as follows: Dividing equation 2 for $i = n$ by itself for $i = 1$ we get,

$$\exp \frac{d_1^2 - d_n^2}{8\sigma^2} = \frac{x_n}{\sum_{i=1}^n x_i} \quad (4)$$

Now since $x_n > x_{n-1} > \dots > x_1 > 0$ and using equation 3 we establish the following bounds for the RHS of equation 4:

$$\frac{1}{n} \leq \frac{x_n}{\sum_{i=1}^n x_i} < 1$$

Hence from equation 4 we have,

$$\log n \geq \frac{d_n^2 - d_1^2}{8\sigma^2} > 0$$

Therefore as $\sigma^2 \rightarrow 0$ i.e., as $\text{SNR} \rightarrow \infty$ we must have the difference $d_n^2 - d_1^2 > 0$ decreasing at least at the rate σ^2 for the term $\frac{d_n^2 - d_1^2}{8\sigma^2}$ to remain finite. In other words, the optimal constellation tends to linear with $d_1 = d_2 = \dots = d_n$ making use of observation (1) above.

- (3) *The asymptotically optimal constellation as the SNR is decreased is one in which all the energy is contained in the two outer most signal points $-x_n, x_n$.*

We prove this result by considering equation 4. Since the average power E is finite all the d_i 's must be finite as well. Therefore as $\sigma^2 \rightarrow \infty$ i.e., as $\text{SNR} \rightarrow -\infty$ dB we must have $\exp \frac{d_1^2 - d_n^2}{8\sigma^2} \rightarrow 1$ which implies that $\sum_{i=1}^n \frac{x_n}{x_i} \rightarrow$

1. The only way this can happen is if $x_i \rightarrow 0, \forall i = 1, 2, \dots, n-1$ and consequently $x_n \rightarrow \sqrt{E}$.

We notice that the μ -law constellation is somewhere "in-between" the linear constellation in observation (2) and the two-point constellation in observation (3) above.

Calculations: We evaluated the raw (uncoded modulation) SER for the linear and μ -law codecs for PCM data transmission over the AWGN channel shown in figure 2. The signal constellations were chosen to be as close to linear as possible since we have established above that a linear constellation is optimal over the range of SNRs of interest (47-55dB). In figure 8, we have plotted this raw SER as a function of the SNR for $n = 64$ i.e., maximum data rate of 48Kbps and $n = 128$ i.e., maximum data rate of 56Kbps.

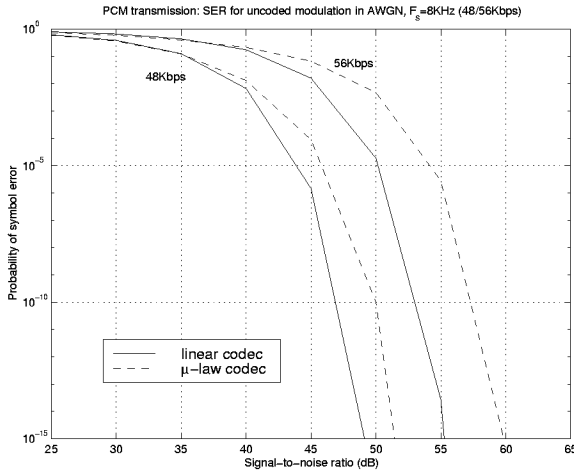


Figure 8: PCM transmission in AWGN: Raw (uncoded modulation) SER for linear and μ -law codecs

From the figure, we observe that over the range of SNRs of interest (47-55dB) the linear codec results in a considerably smaller SER compared to the μ -law codec eg, at an SNR= 50dB and for $n = 128$ the SER for the linear codec is smaller than that for the μ -law codec by a factor of about 100.

4. CONCLUSIONS

We have analytically solved for the optimum codec companding characteristic with a minimum symbol error rate (SER) for PCM data transmission at the quantization levels of the codec. We showed that for the range of SNRs found over most modern telephone lines (47-55dB) the optimal companding characteristic tends to *linear*. Further, we demonstrated that a linear codec results in a substantially smaller raw (uncoded modulation) SER compared to a μ -law codec eg, at an SNR= 50dB and for a data rate of 56Kbps, the SER for the linear codec is smaller than that for the μ -law codec by a factor of about 100. This implies that with a linear codec we can get a dramatic improvement in the performance of PCM modems including an increase in the maximum achievable connection rate *without any* increase in the complexity of the modem. In practice, it would be necessary and relatively simple to implement an additional feature in the μ -law codec to detect a specific pseudo-random code during the initial training phase of the voiceband modem and switch to a linear companding characteristic.

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