

# A NEW ADAPTIVE SUBBAND STRUCTURE WITH CRITICAL SAMPLING

Mariane R. Petraglia

Federal University of Rio de Janeiro  
CP 68564, CEP 21945 970  
Rio de Janeiro, RJ - Brazil  
mariane@lps.ufjf.br

Rogério G. Alves

INMETRO - National Institute of Metrology  
CP 94501, CEP 25250 020  
Duque de Caxias, RJ - Brazil  
guedes@lps.ufjf.br

## ABSTRACT

In this paper, a new adaptive subband structure with critical sampling of the subband signals, which yields exact modeling of FIR systems, is derived. An adaptation algorithm, which minimizes the sum of the subband squared-errors, is obtained for the updating of the coefficients of the new subband structure, resulting in significant convergence rate improvement for colored input signals when compared to the full-band LMS algorithm. A simplified version of the adaptation algorithm, with reduced computational complexity, is also presented. An efficient implementation of the proposed subband structure is described, with computational savings of the order of the number of subbands when compared to the full-band LMS. Computer simulations illustrate the convergence behavior of the proposed algorithms.

## 1. INTRODUCTION

Adaptive FIR filters are attractive in many applications due to their stability and unimodal performance surface properties. However, when the order of such filters is very high, a large number of operations is needed for their implementation and the adaptation algorithm presents slow convergence. Alternative structures that make use of multirate concepts have been proposed [1]-[5] with the objective of reducing the drawbacks described above. In most of these subband structures, the signals at the outputs of the analysis filter banks are down-sampled and the adaptation is performed at the reduced sampling rate [1]-[3], which leads to large computational savings for high-order filters, with the introduction of an extra input-output delay.

In order to model a finite impulse response system with small asymptotic errors, the adaptive subband structure with critical sub-sampling requires additional adaptive cross-filters between the subbands [2]. These cross-filters, however, increase the computational complexity and reduce the convergence rate of the adaptive algorithm. In this paper, a new adaptive subband structure with critical sampling of the subband signals, which also yield exact modeling of FIR systems, is obtained. The resulting structure also presents extra filters between the subbands, but such filters are identical to the direct-path adaptive filters, and do not need to be adapted separately. Therefore, the computational complexity is reduced and the adaptation speed is improved when compared to algorithms derived in [2]. An adaptation algorithm based on the normalized LMS algorithm is derived for the updating of the subfilter coefficients of the new subband structure. Then, a possible simplification in the algorithm is described, which results in computational complexity reduction, with, however, some degradation

in the convergence rate. An efficient implementation of the new subband structure, which employs cosine modulated filter banks, is discussed and its computational complexity is analyzed. The convergence behavior of the proposed algorithms is illustrated by computer simulations and compared to the behavior of the full-band LMS algorithm and of the subband algorithms presented in [2].

## 2. THE NEW ADAPTIVE SUBBAND STRUCTURE WITH CRITICAL SAMPLING

The new adaptive subband structure is derived from the filter bank structure with adaptive sparse subfilters of Fig. 1(a), which can be redrawn as in Fig. 1(b) by making use of the polyphase matrix of the analysis bank  $\mathbf{H}_p(z)$ .

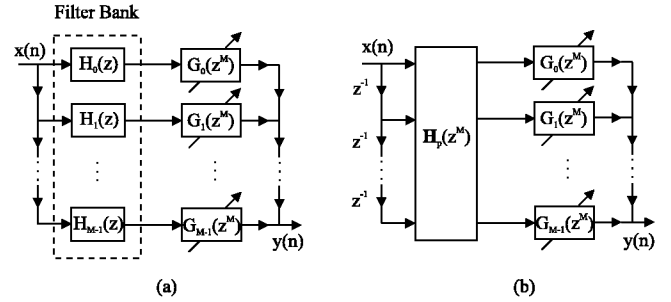


Figure 1: Adaptive structure: (a) with an analysis filter bank and sparse subfilters; (b) with polyphase representation of the analysis bank.

In a system identification application, the coefficients of the filter bank structure are adapted such as to model an unknown FIR system, denoted here by  $P(z)$ . The polyphase representation of  $P(z)$  is shown in Fig. 2(a). Including before the polyphase components  $P_k(z)$  of Fig. 2(a) the matrices  $\mathbf{H}_p(z^M)$  and  $\mathbf{F}_p(z^M)$ , as shown in Fig. 2(b), such that  $\mathbf{F}_p(z)\mathbf{H}_p(z) = z^{-\Delta}\mathbf{I}_M$ , where  $\mathbf{I}_M$  is the  $M \times M$  identity matrix, the transfer function of the system is not altered, except for the introduction of a constant delay of  $M\Delta$  samples.  $\mathbf{H}_p(z)$  and  $\mathbf{F}_p(z)$  that satisfy the above condition correspond to the analysis and synthesis polyphase matrices of a perfect reconstruction multirate system.

Figures 1(b) and 2(b) are equivalent, i.e., both structures implement the transfer function  $z^{-M\Delta}P(z)$  when

$$\begin{bmatrix} G_0(z) & G_1(z) & \cdots & G_{M-1}(z) \end{bmatrix} =$$

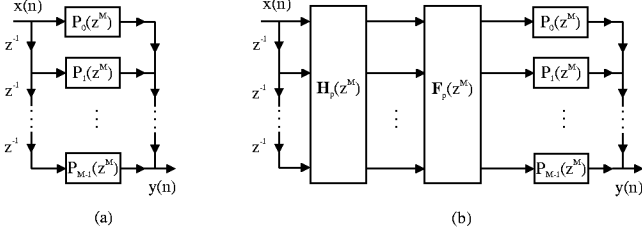


Figure 2: Equivalent polyphase decomposition representations of the unknown system  $P(z)$ .

$$\begin{bmatrix} P_0(z) & P_1(z) & \cdots & P_{M-1}(z) \end{bmatrix} \mathbf{F}_p(z). \quad (1)$$

Therefore, by using an analysis filter bank which yields perfect reconstruction and adaptive subfilters of sufficient order such that Eq. (1) can be satisfied, the structure of Fig. 1 implements exactly any FIR system.

We now derive the adaptive subband structure with critical sub-sampling by including maximally decimated perfect reconstruction  $M$ -band systems following each sparse subfilter in Fig. 1(a), as illustrated for the  $k$ -th band in Fig. 3. Then, moving the sparse

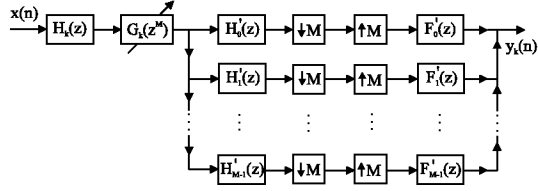


Figure 3: Illustration of the  $k$ -th band of the filter bank structure followed by perfect reconstruction multirate system.

subfilters  $G_k(z^M)$  to the right of the decimators (becoming  $G_k(z)$  by the *noble identity* [6]), and assuming that non-adjacent filters of the analysis filter bank have frequency responses which do not overlap, the structure of Fig. 4 is obtained. Observe that in the resulting structure only  $M$  subfilters need to be adapted, namely  $G_0(z), \dots, G_{M-1}(z)$ , and that they operate at a rate which is  $1/M$ -th of the input rate. From Eq. (1), the length of each subfilter  $G_k(z)$  should be  $K = (N_p + N_f)/M + 1$ , where  $N_p$  is the order of the system  $P(z)$  to be identified and  $N_f$  is the order of the synthesis filter  $F_k(z)$ .

### 3. ADAPTATION ALGORITHMS

A normalized LMS-type algorithm is used for updating the coefficients of the subfilters. Denoting  $\mathbf{X}_{i,j}(m)$  the vector containing the last  $K$  samples of the signal  $X_{i,j}(m)$  at the output of the analysis filter  $H_i(z)H_j(z)$  after down-sampling,  $\mathbf{G}_i(m)$  the vector containing the coefficients of the subfilter  $G_i(z)$  at iteration  $m$  and  $K$  the number of coefficients of each subfilter, the general form for the LMS adaptation algorithm that minimizes the sum of the instantaneous subband squared-errors, i.e.,

$$J(m) = \sum_{i=0}^{M-1} E_i^2(m), \quad (2)$$

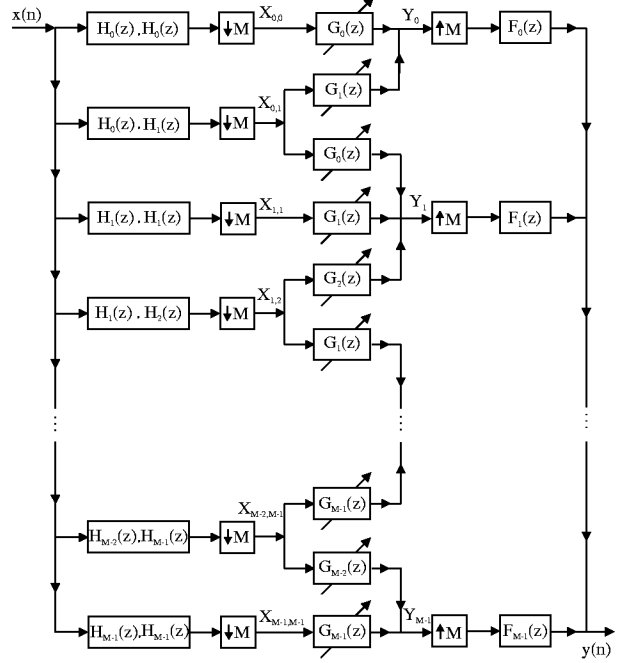


Figure 4: Adaptive subband structure with  $H'_k(z) = H_k(z)$ .

is given by <sup>1</sup>:

$$\mathbf{G}_k(m+1) = \mathbf{G}_k(m) + \mu_k [\mathbf{X}_{k,k}(m)E_k(m) + \mathbf{X}_{k-1,k}(m)E_{k-1}(m) + \mathbf{X}_{k,k+1}(m)E_{k+1}(m)], \quad (3)$$

In the above equations, the error signals  $E_k(m)$  are given by

$$E_k(m) = D_k(m - \Delta) - [\mathbf{X}_{k,k}^T(m)\mathbf{G}_k(m) + \mathbf{X}_{k-1,k}^T(m)\mathbf{G}_{k-1}(m) + \mathbf{X}_{k,k+1}^T(m)\mathbf{G}_{k+1}(m)], \quad (4)$$

where  $\Delta = (N_h + N_f)/2M$  corresponds to the delay introduced by the filter banks ( $N_h$  and  $N_f$  are the orders of the analysis and synthesis filters, respectively). In order to improve the convergence rate of the adaptation algorithm for colored noise input signals, the step-sizes are made inversely proportional to the sum of the powers of the signals involved in the adaptation of the coefficients, that is:

$$\mu_k = \frac{\mu}{P_{k,k} + P_{k-1,k} + P_{k,k+1}}, \quad (5)$$

where  $P_{i,j}$  is the power estimate of the signal  $X_{i,j}$ .

#### 3.1. Simplified Algorithm

A simplified adaptation algorithm which also converges to the optimal solution with some degradation in the convergence rate is given by

$$\mathbf{G}_k(m+1) = \mathbf{G}_k(m) + \mu_k \mathbf{X}_{k,k}(m)E_k(m), \quad (6)$$

<sup>1</sup>For the first and last subbands ( $k = 0$  and  $k = M - 1$ ) only two terms appear in the coefficient updating part of Eq. (3) and in the error expression of Eq. (4), i.e., one should consider  $\mathbf{X}_{-1,0} = \mathbf{0}$  and  $\mathbf{X}_{M-1,M} = \mathbf{0}$  in such equations.

with the error signals as in Eq. (4) and the step-sizes equal to  $\mu_k = \mu/P_{k,k}$ . The computational savings and the convergence degradation resulting from the above algorithm simplification will be discussed in the next sections.

#### 4. EFFICIENT IMPLEMENTATION AND COMPUTATIONAL COMPLEXITY

One of the major advantages of adaptive filtering in subbands is the savings in the computational complexity which can be achieved when implementing high-order filters. In this section, we describe an efficient implementation of the subband structure of Fig. 4 using cosine modulated filter banks and compare the number of multiplications required by it with those required by the full-band LMS algorithm and by the cross-filter subband algorithms of [2].

The analysis filters  $H_k(z)H_k(z)$  and  $H_k(z)H_{k+1}(z)$  applied to the input signal (see Fig. 4) can be efficiently implemented using the cosine modulation method [7], as shown below. Denoting by  $p_0(n)$  the impulse response of a prototype filter of length  $N_h + 1$  which yields perfect reconstruction when used in a cosine modulated analysis-synthesis system, the impulse response of the analysis filters  $H_k(z)$  are related to  $p_0(n)$  by [7]:

$$h_k(n) = 2p_0(n)\cos(\omega_k n + \theta_k), \quad (7)$$

with  $\omega_k = (k + \frac{1}{2})\frac{\pi}{M}$  and  $\theta_k = -(k + \frac{1}{2})(\frac{N_h}{2})\frac{\pi}{M} + (-1)^k\frac{\pi}{4}$ , or in the frequency-domain:

$$H_k(e^{j\omega}) = P_0(e^{j(\omega-\omega_k)})e^{j\theta_k} + P_0(e^{j(\omega+\omega_k)})e^{-j\theta_k}. \quad (8)$$

The impulse responses of the filters  $H_k(z)H_k(z)$  are given by  $h_k(n) * h_k(n)$ , or in the frequency-domain:

$$\begin{aligned} [H_k(e^{j\omega})]^2 &= P_0^2(e^{j(\omega-\omega_k)})e^{j2\theta_k} + P_0^2(e^{j(\omega+\omega_k)})e^{-j2\theta_k} \\ &\quad + 2P_0(e^{j(\omega-\omega_k)})P_0(e^{j(\omega+\omega_k)}). \end{aligned} \quad (9)$$

The last term of the above expression can be neglected, since it is assumed that non-adjacent analysis filters do not overlap. Thus:

$$h_k(n) * h_k(n) \approx 2[p_0(n) * p_0(n)]\cos(\omega_k n + 2\theta_k), \quad (10)$$

and the filters  $H_k(z)H_k(z)$  of Fig. 4 can be efficiently implemented by a cosine modulation technique with prototype filter  $p_0(n) * p_0(n)$  of length  $L = 2N_h + 1$ .

Similarly, the impulse responses of the filters  $H_k(z)H_{k+1}(z)$  can be approximated by:

$$h_k(n) * h_{k+1}(n) \approx 2q_0(n)\cos\left((\omega_k + \frac{\pi}{2M})n + \theta_k + \theta_{k+1}\right), \quad (11)$$

where

$$q_0(n) = p_0(n)e^{-j\frac{\pi}{2M}n} * p_0(n)e^{j\frac{\pi}{2M}n}. \quad (12)$$

Therefore, the filters  $H_k(z)H_{k+1}(z)$  can also be implemented by the cosine modulation technique with prototype filter  $q_0(n)$  given above.

The overall number of multiplications per input sample required by the proposed subband structure is:

$$\frac{2(3M - 2)(N_p + N_h)}{M^2} + \frac{2(3N_h - 1)}{M} + 4\log_2 M, \quad (13)$$

with the first term corresponding to the filtering and adaptation of the subfilters  $G_k(z)$ , the second term to the implementation of the

prototype filters, and the last term to the computation of the four DCTs required for modulation. For high-order adaptive filters, the dominant term in the above expression is  $6N_p/M$ , which is about  $M/3$  times smaller than the number of multiplications required by the full-band LMS algorithm ( $2N_p$ ).

The proposed subband structure with the simplified adaptation algorithm presented in the last section requires the following number of multiplications per sample:

$$\frac{(4M - 2)(N_p + N_h)}{M^2} + \frac{2(3N_h - 1)}{M} + 4\log_2 M, \quad (14)$$

which can be approximated by  $4N_p/M$  for high-order adaptive filters. Therefore, the number of multiplications needed to implement the simplified algorithm is about  $M/2$  times smaller than the number of multiplications required by the full-band LMS algorithm.

The complexity reduction obtained with the proposed structure for high order filters is comparable to that obtained with the cross-filter overdetermined algorithm with factorization of the cross-filters [2] (which is  $3M/5$  times smaller than that of the LMS) and much better than that obtained with the nonoverdetermined algorithm [2] (which is  $M/3\log_2 M$  smaller than that of the LMS).

#### 5. SIMULATION RESULTS

In order to compare the proposed algorithms with previously proposed ones, we performed simulations with the new subband structure, with the cross-filter structure and the overdetermined adaptation algorithms presented in [2], and with the full-band LMS algorithm. System identification problems are considered, with exact modeling of FIR systems. For the proposed structure, the length of the adaptive subfilters was  $K = (N_p + N_h)/M + 1$ , while for the cross-filter structure,  $K$  was computed as described in [2]. In all simulations, we employed the value of the step-size which resulted in the best convergence rate for each algorithm.

##### 5.1. Two-band Structure

The system to be modeled was a length  $N_p + 1 = 256$  FIR filter. The analysis and synthesis banks were cosine modulated banks with prototype filters of length  $N_h + 1 = 24$  [6]. The input signal was a white-noise gaussian sequence of unit variance. A white-noise sequence of variance  $\sigma_n^2 = 10^{-10}$  was added to the desired signal.

Figure 5 presents the MSE evolution for the LMS and subband algorithms. The proposed structure with two subbands is able of exactly modeling an FIR system and its convergence rate is similar to that of the full-band LMS when both adaptation algorithms operate with white-noise input signal and with maximum step-sizes. The cross-filter structure without cross-filter factorization takes about twice the number of iterations of the proposed structure to converge, and the factorized cross-filter algorithm presents initially the same convergence rate as the non-factorized algorithm, but the MSE reaches only  $-32$  dB.

##### 5.2. Multiband Structure

In this experiment, we performed simulations with the full-band LMS and with the proposed subband structure with  $M = 2, 4, 8$  and 16 subbands using perfect reconstruction analysis and synthesis cosine modulated filter banks [6] with prototype filter lengths

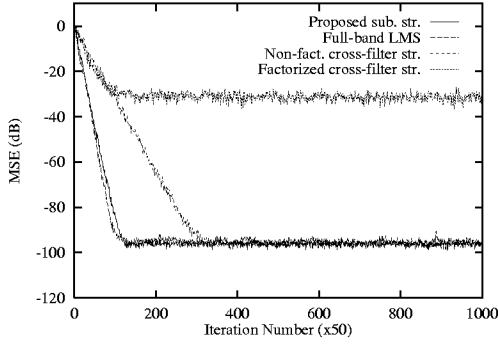


Figure 5: Simulation results for the proposed and cross-filter two-subband structures, and full-band LMS algorithm, with white noise input.

$N_h + 1 = 24, 96, 128, 256$ , respectively. An identification of a length  $N_p + 1 = 880$  FIR system was considered, and the input signal was a colored noise sequence generated by passing a white noise sequence by a first-order IIR filter with pole located at  $z = 0.9$ .

Figure 6 presents the MSE evolution of the LMS and the proposed subband algorithm (non-simplified) with different numbers of bands. This figure shows that the new subband structure presents better convergence rate than the LMS algorithm when the number of subbands  $M$  is equal or larger than 8, due to the power normalization of the step-sizes. For  $M$  larger than 2, the proposed structure converges to an MSE of the order of the stopband attenuation of the analysis filter (which is around  $-80$  dB for  $M = 8$  and 16, and  $-100$  dB for  $M = 4$ ), due to the assumption of non-overlapping non-adjacent analysis filters. For  $M = 16$ , the convergence rate of the subband structure for colored input is practically the same as for white input.

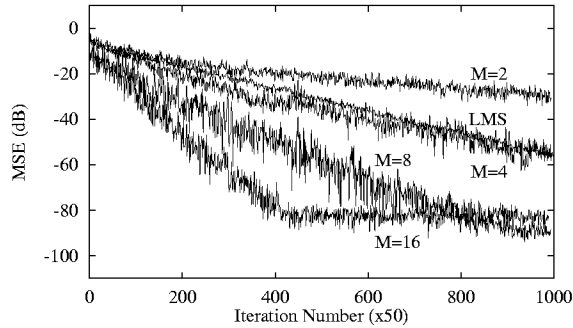


Figure 6: Simulation results for the proposed structure with  $M = 2, 4, 8$  and 16, and for the LMS algorithm, with colored input.

### 5.3. Simplified Algorithm

In this experiment, we compare the convergence behavior of the adaptation algorithms described in Section 3. The system to be identified and the filter banks used in this experiment were the same as in the multiband experiment described above. The MSE evolution results for both algorithms with  $M = 16$  subbands are

presented in Fig. 7, where we can observe the degraded convergence rate of the simplified algorithm. Such degradation corresponds to a reduction in the converge speed by a factor of 2.

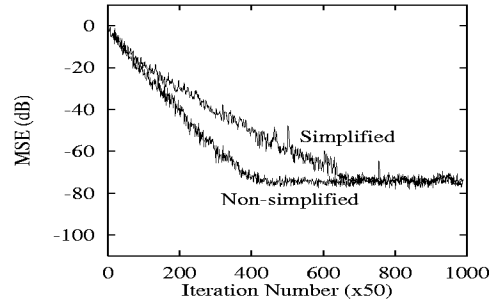


Figure 7: Simulation results for the simplified and non-simplified algorithms, with  $M = 16$  and colored input.

## 6. CONCLUSIONS

In this paper a new subband structure with critical sampling of the subband signals has been investigated. We have derived adaptation algorithms for the new subband structure, which requires the update of only  $M$  subfilters in an  $M$ -band scheme. An efficient implementation, as well as a computational complexity analysis of the new subband structure, have been presented. It has been shown that besides exactly modeling, significant convergence improvement can be achieved with the proposed structure for colored input signals. For high-order filters, savings in the computational complexity of the order of the number of subbands is obtained.

## 7. REFERENCES

- [1] A. Gilloire, "Experiments with sub-band acoustic echo cancellers for teleconferencing," *Proc. IEEE Int. Conf. Acoust., Speech, Sig. Proc.*, pp. 2141-2144, Dallas, TX, Apr. 1987.
- [2] A. Gilloire and M. Vetterli, "Adaptive filtering in subbands with critical sampling: analysis, experiments, and application to acoustic echo cancellation," *IEEE Trans. Signal Processing*, pp. 1862-1875, vol. 40, no. 8, Aug. 1992.
- [3] M. R. Petraglia and S. K. Mitra, "Performance analysis of adaptive filter structures based on subband decomposition," in *Proc. IEEE Int. Symp. Circ. Syst.*, pp. 60-63, Chicago, IL, May 1993.
- [4] M. R. Petraglia and S. K. Mitra, "Adaptive FIR filter structure based on the generalized subband decomposition of FIR filters," *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, vol. 40, pp. 354-362, Jun. 1993.
- [5] M. R. Petraglia, and R. G. Alves, "New results on adaptive filtering using filter banks," *Proc. IEEE Int. Symp. on Circuits and Systems*, pp. 2321-2324, Hong Kong, June 1997.
- [6] G. Strang and T. Nguyen, *Wavelets and Filter Banks*. Wellesley - Cambridge Press, 1996. Book web page: <http://saigon.ece.wisc.edu/~waveweb/Tutorials/book.html>.
- [7] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*. Prentice-Hall, Englewood Cliffs, New Jersey, 1993.