A DE-ROTATION APPROACH TO THE BLIND SEPARATION OF SYNCHRONOUS CO-CHANNEL BPSK SIGNALS

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ABSTRACT

In this paper we propose a simple algorithm for the blind separation of m synchronous co-channel BPSK signals by an array of n receiving antennas. We exploit the geometric implications of the finite alphabet property of BPSK signals. After a standard channel whitening step, the noiseless received data vectors describe an m-cube, which is a rotated version of the hypercube defined by the 2^m distinct vectors of ±1's corresponding to all the possible combinations of the states of the sources. We provide a simple procedure to estimate the vertices of this rotated hypercube in the noisy case and then show how the rotation matrix can be determined up to a sign and permutation of its columns. De-rotation of the channel-whitened data results in source separation. Simulations show the good performance of our proposed technique.

1. INTRODUCTION

The separation of noisy mixtures of co-channel sources observed at an array of receivers or sensors is a problem underlying many signal processing applications. Examples include seismic signal processing, speech processing, biomedical signal processing and wireless communications. Source separation is achieved by essentially inverting the mixing process, which is either known *a priori* or estimated. The mixing process is determined by the propagation channel and receiver characteristics. The mobile communications environment for example, is notorious for its complex multipath time-varying propagation characteristics. The mixing process in this case cannot be determined in advance and needs to be periodically estimated.

There are two approaches to estimate the mixing process; the first and more obvious one is to transmit periodically a known signal, called a training sequence, and to estimate the mixing process based on the observations. The second and more difficult approach is to perform the estimation based on the measurements and some *a priori* knowledge about the structure of the signals, for instance the constant modulus property or the finite-alphabet property of digital communications signals. This is the so-called '*blind*' approach, since it requires neither a training sequence nor knowledge of the propagation and receiver characteristics. The main attractions of blind estimation are that it not only eliminates the need for array calibration and precise knowledge of the channel characteristics but also improves the bandwidth efficiency of the channel by doing without training sequences.

In this paper, we exploit the geometric properties of finite alphabet synchronous co-channel BPSK signals, first reported by Hansen and Xu in [1]. In [2], they propose a hyperplane-based algorithm to estimate the columns of the whitened channel matrix by maximizing an objective function. This function turns out to be convex so that global convergence is not guaranteed and the algorithm has to be re-initialized if a local maximum is detected. Our proposed technique *does not involve optimization and therefore avoids those convergence problems.*

2. PROBLEM FORMULATION

We consider the problem of separating m synchronous co-channel and independent BPSK signals received by an array of n antennas ($n > m$) in the presence of uncorrelated noise. The channel and receiver characteristics are assumed unknown. Assuming matched filtering is performed over the symbol period, the k^{th} received vector can then modeled as

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s}(k) + \mathbf{v}(k) \tag{1}$$

where $\mathbf{x}(k)$ is an $n \times 1$ vector of observations, $\mathbf{s}(k)$ is an $m \times 1$ vector of ±1's corresponding to the k^{th} symbol of each source, $\mathbf{v}(k)$ is an $n \times 1$ additive noise vector and $\mathbf{A} \stackrel{\text{def}}{=} [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_m]$ is an $n \times m$ channel matrix, whose columns $\mathbf{a}_i$, known as the *source signatures*, are assumed independent . Assuming further that the channel is practically constant over say N observations, then the received data matrix can be written as

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{V} \tag{2}$$

where $\mathbf{X} \stackrel{\text{def}}{=} [\mathbf{x}(1), \cdots, \mathbf{x}(N)]$, $\mathbf{S} \stackrel{\text{def}}{=} [\mathbf{s}(1), \cdots, \mathbf{s}(N)]$ and $\mathbf{V} \stackrel{\text{def}}{=} [\mathbf{v}(1), \cdots, \mathbf{v}(N)]$.

Now, let us examine the geometric properties of the model. For simplicity, consider 2 BPSK signals received by 2 antennas in a noise-free environment where the channel matrix is just a rotation matrix. We will show later that the general problem can always be reduced to the determination of a rotation matrix by suitably preprocessing the data. In the absence of noise, the transmitted and received signal vectors describe a constellation of 2^m points. In our example, the constellation consists of only 4 points as depicted in Fig.1. The received signal constellation is simply a rotation of the transmitted signal constellation and our aim is to determine the rotation matrix from the received data.

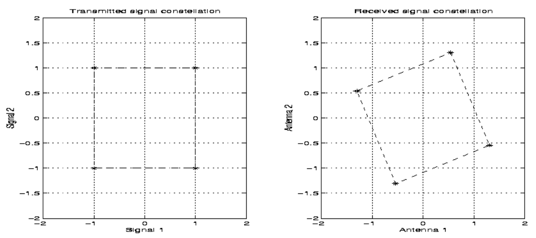

Figure 1: Transmitted and received signal constellations

3. PROPOSED ALGORITHM

Our proposed algorithm is based on the following observations:

1. the columns of a rotation matrix are just the images of the unit vectors.

2. half the vector difference between the position vectors of two adjacent vertices of a **transmitted** constellation of BPSK signals yields a unit positive or negative vector.

3. Therefore, *half the vector difference between the position vectors of two adjacent vertices of the* **received** *BPSK signals constellation should yield a column of the rotation matrix up to a sign.*

Note that although the sign ambiguity might at first appear to be a serious problem for BPSK signals, it is nevertheless easily overcome by a differential encoding scheme. All the columns of the rotation matrix can therefore be calculated, without duplication, by choosing a vertex and evaluating half the vector difference between its position vector and those of the m adjacent vertices. In the noisy case, the received signal constellation consists of "clouds" instead of points. We propose to mitigate the effects of noise by averaging over each cloud, yielding an estimate of the noiseless received signal constellation, which can then be used to estimate the rotation matrix up to a sign ambiguity on each column.

The first step of our proposed algorithm is to transform the channel matrix into a rotation matrix. This is achieved through a standard *channel whitening* procedure.

Step 1. Channel Whitening

1.Form the data covariance matrix $\mathbf{R}_x \stackrel{\text{def}}{=} \frac{1}{N}\mathbf{X}\mathbf{X}^H$

2.Perform eigendecomposition of $\mathbf{R}_x$ and select the m most significant eigenvalues $\lambda_1, \cdots, \lambda_m$ and corresponding eigenvectors $\mathbf{u}_1, \cdots, \mathbf{u}_m$

3.Estimate the noise variance σ^2 as the average of the $n-m$ least significant eigenvalues

4.Define $\mathbf{U}_s \stackrel{\text{def}}{=} [\mathbf{u}_1, \cdots, \mathbf{u}_m]$, $\alpha_i \stackrel{\text{def}}{=} \sqrt{\lambda_i - \sigma^2}$ and the whitening matrix $\mathbf{W} \stackrel{\text{def}}{=} \text{diag}(1/\alpha_1, 1/\alpha_2, \cdots, 1/\alpha_m)\mathbf{U}_s^H$

5.Form the new data matrix $\mathbf{Y} \stackrel{\text{def}}{=} \mathbf{W}\mathbf{X}$

The new data matrix can be written as $\mathbf{Y} = \mathbf{Q}\mathbf{S} + \mathbf{W}\mathbf{V}$, where $\mathbf{Q} \stackrel{\text{def}}{=} \mathbf{W}\mathbf{A}$ is an $m \times m$ *unitary or rotation* matrix. Note that the whitening procedure also reduces the data from an n-dimensional to an m-dimensional space without any loss of information.

The second step of our algorithm is to group the N received channel-whitened vectors into 2^m clusters, where $N \gg 2^m$, and to estimate the vertices of the rotated hypercube.

Step 2. Data Clustering

Select a vector to form the first group
REPEAT
  Evaluate the distance of the next vector from the average vector of each group
  IF this vector is less than one unit away from a group
  THEN add it to this group
  ELSE form another group
  END
UNTIL all vectors have been processed
  IF number of groups $> 2^m$
  THEN discard smallest groups
  END

Note that if the number of groups exceeds 2^m, we discard the smallest groups since they consist of data vectors which are badly corrupted by noise. The most significant 2^m groups are then each averaged to provide an estimate of the rotated constellation. Based on the simple observations we made at the beginning of this section, the m columns of the rotation matrix $\mathbf{Q}$ can be estimated as follows:

Step 3. Channel Estimation

1. Select a vertex of the rotated hypercube
2. Evaluate half the vector differences between the position

vector of this vertex and those of the remaining vertices
3. The columns of $\widehat{\mathbf{Q}}$ are given by m smallest-norm vector differences

Note that the m smallest-norm vector differences correspond to the differences between the position vector of the selected vertex and those of the m adjacent vertices. As we pointed out, these vector differences provide an estimate $\widehat{\mathbf{Q}}$ of the rotation matrix $\mathbf{Q}$ up to a sign and permutation of its columns. The sign ambiguity is however irrelevant to differentially encoded BPSK signals. The transmitted symbols can now be estimated by de-rotating the channel-whitened data.

Step 4. Estimation of the Symbols The data symbols can now be estimated at the receiver using

$$\widehat{\mathbf{S}} = \text{signum}\left(\widehat{\mathbf{Q}}^T \mathbf{Y}\right) \tag{3}$$

4. SIMULATIONS

In this section, we evaluate the performance of our proposed algorithm via simulations. Our simulation environment consists of 3 antennas receiving 2 synchronous co-channel BPSK signals in the presence of spatially uncorrelated Gaussian noise. We assume the signals to be of equal power with a common SNR of 10 dB and consider a typical simulation run using 1000 samples of the array output. Fig.2 illustrates the received signal constellation, each axis representing the output of an antenna. Fig.3 shows the transformed data after the channel whitening step. As can be seen, the data space has been reduced to a 2-dimensional space and the resulting signal constellation is a rotation of the transmitted constellation. The latter is recovered by de-rotation as shown in Fig. 4.

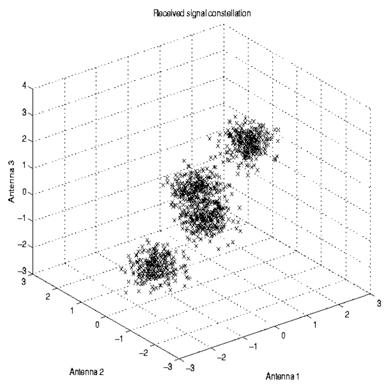

Figure 2: Received signal constellation

5. CONCLUSION

We have proposed a simple algorithm for the blind separation of synchronous co-channel BPSK signals. Based on the finite-alphabet property of such signals, the transmitted symbols can be viewed as the vertices of a hypercube. By a standard channel whitening procedure, we have shown that the received vectors can also be transformed into the vertices of a rotated hypercube. We proposed a clustering algorithm to estimate these vertices in the noisy case and showed how the rotation matrix can be easily estimated without optimization techniques and the accompanying convergence problems. Simulations support the good performance of our algorithm.

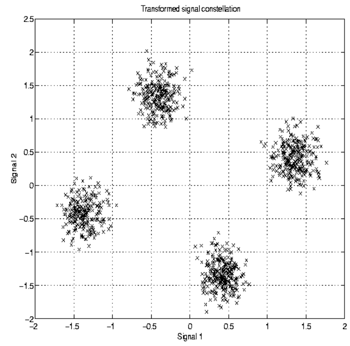

Figure 3: Transformed signal constellation

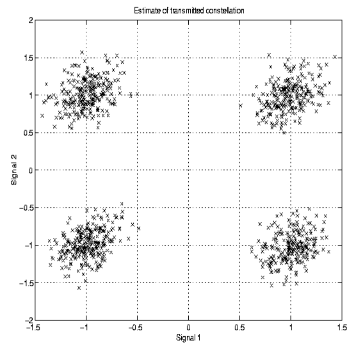

Figure 4: Estimate of transmitted constellation

6. REFERENCES

[1] L. K. Hansen and G. Xu, "Geometric Properties of the Blind Digital Co-Channel Communications Problem", *In Proc. of ICASSP'96*, pp 1085–1088.

[2] L. K. Hansen and G. Xu, "A Hyperplane-Based Algorithm for the Digital Co-Channel Communications Problem", *IEEE Transactions on Information Theory*, Vol. 43, No. 5, pp. 1536–1548, Sep. 1997.