

SCALE DETECTION BASED ON STATISTICAL CHARACTERISTICS OF EDGES IN THE SCALE SPACE

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ABSTRACT

A study of the behavior of edges in the scale space is presented in this paper. Three statistical parameters, intensity average, root-mean-square, and variance of pixel intensity in edge images are used as the basis for evaluating visual quality of edge images. A parametric family of edge images containing edge information at several scales is used to create the edge statistic (ES) curves. It is found that the number of local minima in ES curves is related to the number of scales in an image. This leads to an automated method for detecting scales in an image and a criteria for selecting optimal parameters for a multiscale edge operator.

1. INTRODUCTION

In signal and image processing, the subject of scale is of significant importance. Many well known tools in mathematics such as Fourier and wavelet transformations are related to scale analysis. These methods essentially compute the convolution of a signal with a set of basis functions that are responsive to particular scales.

In image processing, when an image consists of objects at several scales and the field of view is limited, selection of scales of interest becomes essential. Intensity changes in natural images can occur over a wide range of scales and thus, in image description, it is essential to utilize pictorial information at several scales [1]. In the last decade, a variety of edge operators possessing size-adjustable features [1]–[3] have been proposed as tools for multiscale image analysis. However, the problem for automatically detecting existing scales in an image and selecting the optimal parameters for detecting edges at particular scales is still unsolved.

Issues in multiscale data handling have been addressed in literature related to scale-space analysis [4]–[8]. In order to analyze structures at specific scales separately, a Gaussian filter of an appropriate size is used to smooth an image [1, 4]. Scale-space filtering [5] focuses on the convolution of a signal with Gaussian kernels over a continuum of the

size parameter. The behavior of differential structures of an image such as blobs, edges, and zero crossings in the scale space are used as basis for multiscale descriptions [4, 8]. Although, scale-space theory provides the framework for multiscale image analysis, there is no direct method for determining scales existing in an image. This paper proposes a new approach for scale determination and multiscale image analysis in terms of statistical characteristics of edges in the scale domain.

2. EDGES IN THE SCALE SPACE

In general, edges refer to discontinuities of visual attributes in an image. In a visual sense, the presence of edges at some scales indicates the existence of objects at those scales. Hence, information of scales existing in the image can be retrieved by studying the behavior edges in scale domain.

In natural images, most edges are often associated with abrupt changes in intensity distribution and can be approximately modeled as step edges. These edges correspond to areas of high gradient magnitudes, i.e., areas of bright pixels in a gradient image. In this paper, the gradient image is obtained using the Gaussian weighted image moment vector operator [3] with difference masks given by

$$G_x(x, y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \cdot \frac{x}{\sqrt{x^2 + y^2}}$$

and

$$G_y(x, y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \cdot \frac{y}{\sqrt{x^2 + y^2}},$$

where σ is a scale parameter. An edge image is defined as the normalized gradient image.

Generally, for a N -dimensional object in N -dimensional space, an object edge is a $N - 1$ dimensional closed hypersurface. For a 2-dimensional image, an object edge is a closed line which, by definition, has infinitesimally small

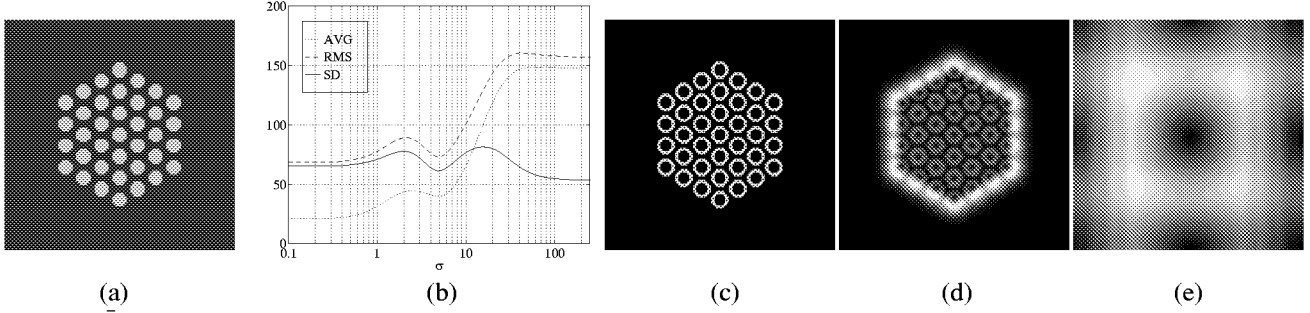


Figure 1. (a) The image consisting of 2 scales; (b) ES curves; (c)-(e) edge images with σ selected from local minima in the ES curves: (c) $\sigma = 0.1$, (d) $\sigma = 4.98$, and (e) $\sigma = \infty$.

width or area of support. This implies that supporting areas of edges in the edge image must be as small as possible. In other words, edges represented in the edge image must be as sharp as possible. Since the area of support of edges in the edge image is related to the mean, root-mean-square (RMS), and variance of edge pixel intensity, in this paper, these statistical parameters are used as quantitative measures of sharpness of edges in the edge image.

To study the behavior of edges in the scale space, a parametric family of edge images containing edges at increasing scales is created by convolving the original image with a set of difference masks of the GWIMV operators with gradually increasing σ . These images are then used to generate the edge statistic (ES) curves. Three different statistical parameters, namely, average (AVG), RMS, and standard deviation (SD) of edge pixel intensity in edge images as functions of σ are computed as

$$\begin{aligned}
 AVG(\sigma) &= \frac{1}{NM} \sum_{i=1}^N \sum_{j=1}^M e_{\sigma}(i, j), \\
 RMS(\sigma) &= \sqrt{\frac{1}{NM} \sum_{i=1}^N \sum_{j=1}^M e_{\sigma}^2(i, j)}, \\
 SD(\sigma) &= \sqrt{\left(\frac{1}{NM} \sum_{i=1}^N \sum_{j=1}^M e_{\sigma}^2(i, j) \right) - AVG(\sigma)^2},
 \end{aligned}$$

where e_{σ} is the edge image obtained using a scale parameter σ and $N \times M$ is the image size. These functions are related to the energy of the edge images. Since the sharpest edges or the smallest area of support in edge images are obtained at values of σ corresponding to scales existing in an image, the local minima in the ES curves directly indicate scales of objects in an image. The investigation of statistical characteristics of edges in the scale space is given in the next section.

3. EXPERIMENTAL RESULTS

Images consisting of objects at several scales including simulated, degraded, and real images have been analyzed. Events such as local minima in ES curves have been investigated. The original images, corresponding ES curves, and selected edge images are presented below.

3.1. Simulated images

The first example is an image consisting of objects at 2 scales, namely the small circles of diameter 8 pixels and the hexagonal cluster of 37 circles as shown in Figure 1-a. The corresponding ES curves are shown in Figure 1-b. The x -axis is displayed in logarithmic scale in accordance with exponential scales exhibited in nature. As seen, there are three local minima in the ES curves at $\sigma = 0.1$, 4.98, and 200. Edges images obtained using the GWIMV operators with values of σ selected from three local minima in the ES curves are displayed in Figures 1-c to 1-e. Clearly, while edges in Figure 1-c represent edges of circles, edges in Figure 1-d emphasize edges of the entire cluster. The edge image in Figure 1-e with $\sigma = \infty$ shows no significant information about object details.

The second example shown in Figure 2-a is an image consisting of objects at 3 scales, namely the circles, clusters of 7 circles in hexagonal shape, and the entire cluster of circles in diamond-cut shape. The corresponding ES curves are illustrated in Figure 2-b. The values of σ at local minima in the ES curves in Figure 2-b are summarized in Table 1. Although, number of local minima of the AVG curve is less than those of the RMS and SD curves, changes in slope of the AVG curve in consistent with those of RMS and SD curves can be observed. Compared to the ES curves in the previous example, there is one additional local minimum point in the RMS and SD curves. Edge images with values of σ selected from the first three local minima in the SD curve are displayed in Figures 2-c to 2-e respectively. Obviously, edges displayed in Figure 2-c with the smallest

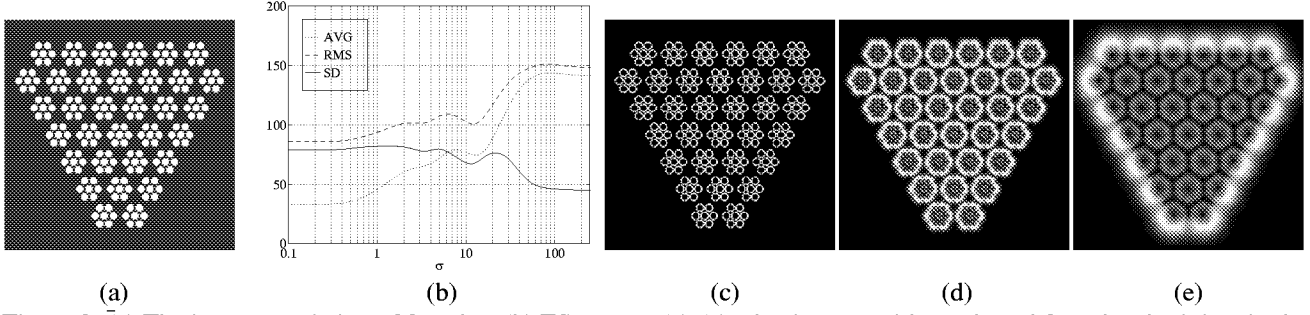


Figure 2. (a) The image consisting of 3 scales; (b) ES curves; (c)-(e) edge images with σ selected from local minima in the SD curve: (c) $\sigma = 0.1$, (d) $\sigma = 3.43$, and (e) $\sigma = 11.12$.

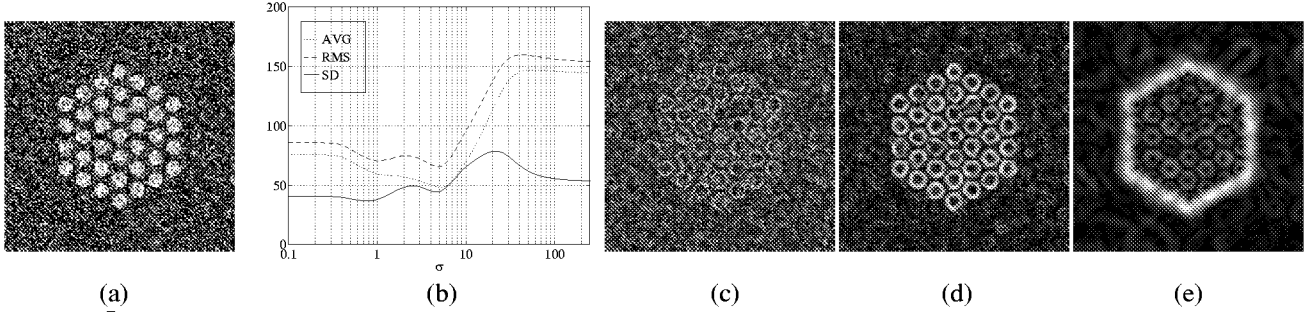


Figure 3. (a) The degraded image with SNR = 2; (b) ES curves; (c)-(e) edge images with σ selected from local minima in the AVG curve: (c) $\sigma = 0.1$, (d) $\sigma = 1.45$, and (e) $\sigma = 5.26$.

ES curve	the values of σ at local minima			
AVG	0.1	-	13.6	200
RMS	0.1	3.08	11.73	200
SD	0.1	3.43	11.12	200

Table 1: The values of σ at local minima of the ES curves in Figure 2-b.

ES curve	the values of σ at local minima			
AVG	0.1	1.45	5.2	200
RMS	0.1	0.99	4.98	200
SD	0.1	0.77	4.72	200

Table 2: The values of σ at local minima of the ES curves in Figure 3-b.

σ are edges of circles, the finest objects in the image while prominent edges displayed in Figure 2-d and 2-e with σ selected from the second and third minimum points in the SD curve are edges of groups of 7 circles in hexagonal shape and edges of the whole cluster respectively.

From these examples, it is seen that the number of local minima in the ES curves is related to the number of scales in an image. The value of σ at the local minimum point at the left extremity of the curves asymptotically converges to zero while that at the right extremity of the curves asymptotically

converges to infinity. These two values correspond to the finest and largest scales in the image, namely, the resolution and size of the image. The values of σ at local minima in the interior of ES curves correspond to particular scales of objects in the image. This relation leads to

1. the method for determining automatically scales existing in an image, and
2. the criteria for selecting the optimal scale parameter for a multiscale edge operator.

The edge operators with parameters selected from local minima in ES curves are optimal in the sense that the statistical characteristics of edge images are minimized in some ranges of scale. Transitions in ES curves can also be used to observe scale variations in the underlying image.

3.2. Noisy image

The automated scale detection method described previously can be applied to the problem of scale detection in noisy images. Although noise and object edges are both high frequency and wide bandwidth signals, scales of noise and object edges are significantly different. One solution to this problem is to select the edge operator such that it is more responsive to object edges while suppressing noise.

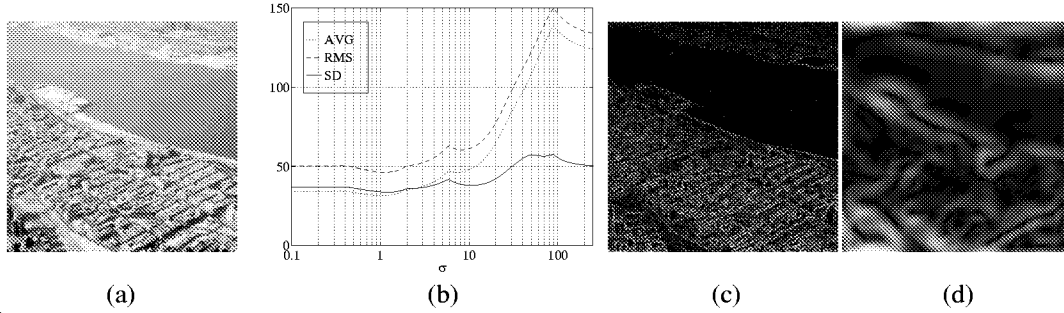


Figure 4. (a) Original image; (b) ES curves; (c)-(e) edge images with σ selected from local minima in the SD curve: (c) $\sigma = 0.1$, and (d) $\sigma = 10.54$.

This can be done using the multiscale edge operator with scale parameter selected using the proposed criteria.

The example given in Figure 3-a is the noisy version of the image in Figure 1-a degraded using an additive Gaussian noise of signal to noise ratio = 2. The corresponding ES curves are shown in Figure 3-b. The values of σ at local minima in the ES curves in Figure 3-b are summarized in Table 2. Edge images with σ selected from first three local minima in the AVG curve are displayed in Figures 3-c to 3-e respectively. In Figure 3-c with the smallest σ , edges of circles are mostly obscured by noise. In Figures 3-d and 3-e, outstanding edges are edges of circles and the entire cluster respectively. These results show that we can suppress the effect of noise in object edges by selecting the values of σ at local minima corresponding to desired scales. The smallest scale σ can be assumed to represent the noise scale.

3.3. Natural images

Unlike simulated images in previous examples, the problem of scale detection in natural image is much more complicated. In general, natural images are noisy and objects and scales in the images may not appear clearly. However, using ES analysis, scales can still be detected as illustrated in Figure 4. In this example, there are 4 local minima in the AVG and RMS curves and 6 local minima in the SD curve. Edge images with σ selected from local minima in the SD curve at $\sigma = 0.1$ and 10.54 are shown in Figures 4-c and 4-d respectively. Edges at particular scales are clearly shown in these images. For example, edges of small and middle scale objects can be seen in Figures 4-c and 4-d respectively.

4. CONCLUSIONS

A new method for automated detection of multiple scales in an image and the criteria for selecting the optimum scale parameter for a multiscale edge operator based on edge statistic-scale analysis are presented. Existence of scales in an image is indicated by local minima in the ES curves. The mul-

tiscale edge operators with scale parameters selected from local minima in the ES curves are optimal in the sense that they minimize statistical characteristics of edge images in some range of scales. Transition in ES curves can also be used to observe the scale variation in the underlying image. The multiscale edge operator used in conjunction with scale determination proposed in this paper provides a powerful methodology for multiscale image analysis.

5. REFERENCES

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