COHERENT DETECTION OF RADAR SIGNALS IN G-DISTRIBUTED CLUTTER

George Mamić, Nathan Stitt, and D. Robert Iskander[(*)]

[(*)] Communications and Information Processing Group
School of Electrical & Electronic Systems Engineering
Queensland University of Technology,
GPO Box 2434, Brisbane, QLD 4001, Australia
e-mail: d.iskander@qut.edu.au

ABSTRACT

Recently the G distribution has been proposed as a new model for extremely heterogeneous clutter in SAR returns. In this paper, we develop a technique for estimating the parameters of the G distribution, show that the G distribution represents an amplitude distribution of a spherically invariant random process for certain values of its parameters, and design coherent detectors for known and unknown signals embedded in G-distributed clutter. The performance of the detectors under specific conditions is then provided.

1. INTRODUCTION

In high resolution radars the clutter can no longer be generalised by the central limit theorem to have Gaussian statistics. Such non-Gaussian clutter returns would be passed by the detectors as being targets and hence lead to unacceptable levels of false alarm. In addition to this problem of non-optimal performance of the detectors, it was realised that if the clutter was non-Gaussian then there existed no unique specification for the joint probability density function (pdf) of the returns [1, 6].

Modeling this clutter by non-Gaussian distributions led to the requirement for classification techniques, which have the ability to choose an appropriate model from a library of distributions.

An alternative solution involves the use of generalised distributions, which contain an implicit library of more specific distributions as special cases of the parameters of the general distribution. An example of such a distribution is the Compound Gaussian or the Rayleigh Mixture [9, 5].

In synthetic aperture radar (SAR), the clutter returns from urban areas is often extremely heterogeneous. For such clutter, the G distribution has been recently proposed to model the radar return [3]. The idea for the G distribution was based on the multiplicative model, where the outcome is assumed to be the product of the terrain backscatter and the speckle noise. In this case, the distribution of the terrain backscatter is modelled by the square root of the generalised inverse Gaussian distribution and the speckle noise by the square root of the Gamma distribution. The distribution of the amplitude return is denoted by $X \sim G_A(\alpha, \gamma, \lambda, n)$ and has the following pdf [3],

$$\begin{aligned} f_X(x) &= \frac{2n^n(\lambda/\gamma)^{\alpha/2}}{\Gamma(n)K_\alpha(2\sqrt{\lambda\gamma})} x^{2n-1} \left(\frac{\gamma + nx^2}{\lambda}\right)^{(\alpha-n)/2} \\ &\times \; K_{\alpha-n}\left(2\sqrt{\lambda(\gamma+nx^2)}\right) \end{aligned} \tag{1}$$

where $\gamma \geq 0$, $\lambda \geq 0$, $-\infty < \alpha < \infty$, and $n > 0$ are the distributional parameters, $x \geq 0$, and where $K_\nu(\cdot)$ denotes the modified Bessel function of the second kind of order ν.

The ability to coherently detect signals, with unknown parameters such as amplitude and phase in a non-Gaussian clutter environment, is of significant importance in the radar scenario. This can be achieved in a reasonably simple manner if the clutter is modelled by a spherically invariant random process [1, 2].

In this paper, the use of the G distribution for coherent clutter modeling is proposed. The paper is organised in the following manner. In the next section, we present a method for estimating the parameters of the G distribution. Section 3 provides details on the spherical invariance of the G distribution. In Section 4, we design optimal detection structures in the Neyman-Pearson sense for the case where the parameters of the signal are known and the case where the amplitude and phase of the signal need to be estimated. We conclude in Section 5.

2. ESTIMATION OF THE PARAMETERS OF THE G DISTRIBUTION

In order for a distribution to be useful in practice, one must have the ability to estimate the parameters of the distribution. Initially we calculated the maximum likelihood estimators for the parameters of the G distribution. However, no closed form expressions could be derived for the solution of the four parameters and, due to the complexity of the resulting equations, numerical techniques were not practically feasible. An alternative estimation approach is to use the method of moments.

The r^{th} order moment of the G distribution is given by [3],

$$E(X^r) = \left(\frac{\gamma}{n^2\lambda}\right)^{r/4} \frac{K_{\alpha+r/2}(2\sqrt{\gamma\lambda})\Gamma(n+r/2)}{K_\alpha(2\sqrt{\gamma\lambda})\Gamma(n)} \qquad (2)$$

where $r > 0$. The simultaneous solution of four arbitrary moment equations is computationally very expensive and does not lead to good parameter estimates unless the number of available samples is large (more than a thousand). To overcome this problem, we propose the following methodology.

Let

$$R_1 = \frac{E(X^{2r+2})}{E(X^{2r})} \qquad (3)$$

$$R_2 = \frac{E(X^{2r-2})}{E(X^{2r})} \qquad (4)$$

and define

$$\begin{aligned} M_r &= R_1 - \frac{\gamma}{n^2\lambda}(n+r-1)(n+r)R_2 \\ &= \frac{(n+r)(\alpha+r)}{n\lambda} \end{aligned} \qquad (5)$$

By using four different values of the moment order parameter, r, we form four equations of the form (5) (substituting the moment ratios by their sample counterparts) and solve for the remaining parameters. These equations are far easier to manipulate and solve than those which are obtained by simply utilising the relationship provided in (2). The resulting equations can be solved for the four parameters using numerical techniques such as the multi-dimensional Newton-Raphson technique, provided the order of the moments is kept low. This approach leads to good parameters estimates for sample sizes less than a thousand.

3. SPHERICAL INVARIANCE OF THE G DISTRIBUTION

The clutter process in high resolution radars can be characterised using the theory of Spherically Invariant Random Vectors (SIRV's). Let

$$\tilde{\boldsymbol{Z}} = S\tilde{\boldsymbol{X}} \qquad (6)$$

where S is a nonnegative random variable which is independent of the zero mean complex Gaussian vector which is defined as $\boldsymbol{X} = [\boldsymbol{X}_I, \boldsymbol{X}_Q]$. The subscripts I and Q correspond to the in-phase and quadrature components, respectively. The resulting $\boldsymbol{Z} = [\boldsymbol{Z}_I, \boldsymbol{Z}_Q]$ is a SIRV and can be thought of as a complex Gaussian vector which has a modulating or smearing variable S [4]. The probability density function of $\boldsymbol{Z}$ is uniquely determined by its mean, covariance matrix, and the characteristic pdf $f_S(s)$ or the so-called characterisation function $h_{2N}(\boldsymbol{z}\boldsymbol{\Sigma}^{-1}\boldsymbol{z}^T)$, where $\boldsymbol{\Sigma}$ is the covariance matrix of $\boldsymbol{Z}$. In most cases, it is far easier to derive the characterisation function than the characteristic pdf. In the case of the G distribution we found that the characterisation function is

$$\begin{aligned} h_{2N}(q) &= \frac{2^N A^{n+N-1} q^{n-1}}{K_\alpha(2B)\Gamma(n)} \sum_{k=1}^{N} \left[\binom{N-1}{k-1} \right. \\ &\times \frac{\Gamma(n)(-1)^{k+N}}{\Gamma(n-N+k)(Aq)^{N-k}} \left(1+\frac{Aq}{B}\right)^{\frac{\alpha-n-k+1}{2}} \\ &\times \left. K_{\alpha-n-k+1}\left(2B\sqrt{1+\frac{Aq}{B}}\right)\right] \end{aligned} \qquad (7)$$

where $q = \boldsymbol{z}\boldsymbol{\Sigma}^{-1}\boldsymbol{z}^T$ and

$$\begin{aligned} A &= \frac{K_{\alpha+1}(2\sqrt{\gamma\lambda})\Gamma(n+1)}{2K_\alpha(2\sqrt{\gamma\lambda})\Gamma(n)} \\ B &= \sqrt{\gamma\lambda}. \end{aligned}$$

The characterisation function $h_{2N}(q)$ has to fulfill the monotonicity requirement [1, 8]. A sufficient condition for this requirement is that

$$n \leq 1 \qquad (8)$$

Thus the multivariate pdf of a SIRV with a G-distributed first order amplitude is given by

$$f_{\boldsymbol{Z}}(\boldsymbol{z}) = (2\pi)^{-N}|\boldsymbol{\Sigma}|^{-1/2} h_{2N}(\boldsymbol{z}\boldsymbol{\Sigma}^{-1}\boldsymbol{z}^T) \qquad (9)$$

4. DETECTION OF SIGNALS IN G DISTRIBUTED CLUTTER

Herein we design optimal schemes for coherent detection of deterministic signals in G distributed clutter. The problem of detecting a deterministic signal $\boldsymbol{s} = Ae^{j\theta}\boldsymbol{v}$ in G-distributed clutter can be expressed in the following framework

$$\begin{aligned} \mathsf{H} &: \boldsymbol{r} = \boldsymbol{z} \\ \mathsf{K} &: \boldsymbol{r} = \boldsymbol{s} + \boldsymbol{z}, \end{aligned} \qquad (10)$$

where H denotes the null hypothesis, K the alternative hypothesis, and where $\boldsymbol{r} = [\boldsymbol{r}_I, \boldsymbol{r}_Q]$, $\boldsymbol{z} = [\boldsymbol{z}_I, \boldsymbol{z}_Q]$, and $\boldsymbol{s} = [\boldsymbol{s}_I, \boldsymbol{s}_Q]$, are real vectors with $2N$ entries representing the observations of the received signal, interference, and deterministic signal, respectively.

Provided that the covariance matrix $\boldsymbol{\Sigma}$ is known, one can use a whitening transformation without penalty [10], providing that the parameter $n \leq 1$. Whitening the received signal leads to the following framework

$$\begin{aligned} \mathsf{H} &: \boldsymbol{x} = \boldsymbol{n} \\ \mathsf{K} &: \boldsymbol{x} = \boldsymbol{u} + \boldsymbol{n}, \end{aligned} \quad (11)$$

where $\boldsymbol{x}$ is the whitened version of the received signal vector $\boldsymbol{r}$, and $\boldsymbol{n}$ and $\boldsymbol{u}$ represent the whitened versions of the interference vector $\boldsymbol{z}$ and deterministic signal vector $\boldsymbol{s}$, respectively.

First, we consider the case of a known signal for which the log-likelihood ratio test (LLRT) can be constructed. This stage of the detection process is essential as the results are a reference point for subsequent and more complex analyses into cases where some or all of the parameters of the signal need to be estimated.

The LLRT has the following form for known signals in G distributed clutter.

$$\begin{aligned} \Lambda(\boldsymbol{x}) &= \log\left[\frac{f(\boldsymbol{x} \mid \mathsf{K})}{f(\boldsymbol{x} \mid \mathsf{H})}\right] = \log\left[\frac{f_{\boldsymbol{N}}(\boldsymbol{x} - \boldsymbol{u})}{f_{\boldsymbol{N}}(\boldsymbol{x})}\right] \\ &= g(||\boldsymbol{x} - \boldsymbol{u}||) - g(||\boldsymbol{x}||) \underset{H}{\overset{K}{\gtrless}} T \end{aligned} \quad (12)$$

where T is an appropriate threshold. The function $g(\cdot)$ is given by

$$\begin{aligned} g(x) &= \log\Bigg[\sum_{k=1}^{N} \binom{N-1}{k-1} \frac{\Gamma(n)(-1)^{k+N}}{\Gamma(n-N+k)} \\ &\times \frac{||x||^{2(n-N+k-1)}}{A^{N-k}} \left(1 + \frac{A||x||^2}{B}\right)^{\frac{\alpha-n-k+1}{2}} \\ &\times K_{\alpha-n-k+1}\left(2B\sqrt{1 + \frac{A||x||^2}{B}}\right)\Bigg] \end{aligned} \quad (13)$$

In the case where both the phase and the amplitude of the signal are unknown we use the generalised log-likelihood ratio test (GLLRT) where the unknown parameters are replaced by their maximum likelihood estimates. The GLLRT can be expressed as follows,

$$\Lambda(\boldsymbol{x}) = \log\left[\frac{\max_{A,\theta} f_{\boldsymbol{N}}(\boldsymbol{x} - Ae^{j\theta}\boldsymbol{p})}{f_{\boldsymbol{N}}(\boldsymbol{x})}\right] \underset{H}{\overset{K}{\gtrless}} T. \quad (14)$$

The maximum likelihood estimators for the amplitude A and phase θ are given by

$$\hat{A} = \frac{|\langle \boldsymbol{x}, \boldsymbol{p} \rangle|}{||\boldsymbol{p}||^2},$$

and

$$\hat{\theta} = \phi$$

respectively, where ϕ is the phase of the inner product $\langle \boldsymbol{x}, \boldsymbol{p} \rangle$ and $\boldsymbol{p}$ is the whitened version of $\boldsymbol{v}$ [7].

The substitution of these equations into (14) for the unknown amplitude and phase case, results in the following

$$\Lambda(\boldsymbol{x}) = g\left(\sqrt{||\boldsymbol{x}||^2 - \frac{|\langle \boldsymbol{x}, \boldsymbol{p} \rangle|^2}{||\boldsymbol{p}||^2}}\right) - g(||\boldsymbol{x}||) \underset{H}{\overset{K}{\gtrless}} T \quad (15)$$

4.1. Performance Analysis

The performance of the detectors, which were developed in the previous section, needs to be evaluated using computer simulations since the pdfs of $\Lambda(\boldsymbol{x})$ in the expressions (12) and (14) under the hypothesis and the alternative cannot be expressed in closed form. We assumed that the parameters of the G distribution were known and they were arbitrarily assigned values within the parameter space defined under the requirement of monotonicity criterion. This situation occurs when the clutter statistics have already been estimated prior to the detection process.

Examples of the performance of the detectors of known signals and unknown signals are provided in the receiver operating characteristics (ROC) of Figure 1 and Figure 2 respectively. In this case the values of the parameters of the distribution were assigned the values of $\alpha = -1$, $\gamma = 0.405$, $\lambda = 1$ and $n = 1$. The number of integrated pulses, N, was set to 4. The ROCs were obtained for 10,000 realisations of the clutter process using Monte Carlo simulations.

The ROC plots indicate that even for a very low number of integrated pulses the unknown signal detector can provide good performance when compared to the reference ROC which was generated using known signals.

The development of ROCs for the case where both the signal and the clutter parameters are completely unknown is a topic of research which is currently under investigation.

5. CONCLUSION

With the aid of the theory of Spherically Invariant Random Vectors, it is possible to conclude that the G distribution can certainly be used in the coherent detection of radar signals in non-Gaussian clutter, provided that the associated monotonicity requirement is met. Optimal detection structures for both the reference, known signal case and the unknown phase and amplitude signal case have been presented. In addition to this a simplification for the estimation of the parameters of the distribution has also been presented. These developments may lead to more widespread use of the G distribution in radar practice.

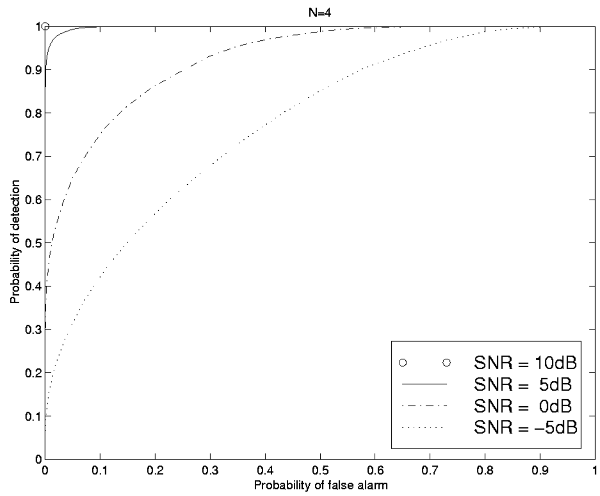

Figure 1: The receiver operating characteristics for a known signal embedded in G-distributed clutter for $N = 4$.

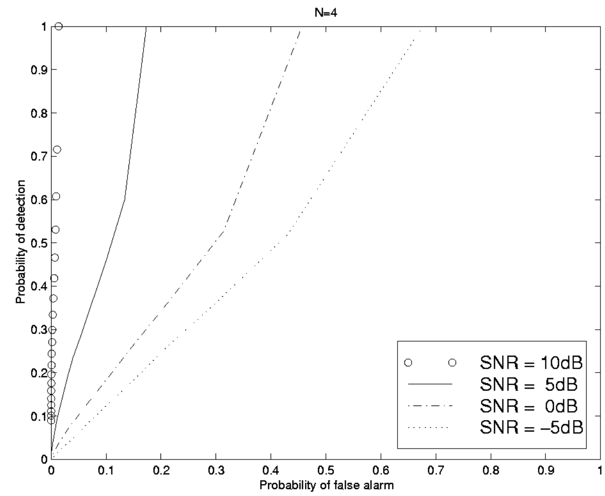

Figure 2: The receiver operating characteristics for a signal with unknown amplitude and phase embedded in G-distributed clutter for $N = 4$.

6. ACKNOWLEDGEMENT

The authors wish to thank Dr Abdelhak Zoubir for his encouragement, constructive comments and support during the project.

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