PERFORMANCE OF ORDERED STATISTICS DECODING FOR ROBUST VIDEO TRANSMISSION ON THE WSSUS CHANNEL

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ABSTRACT

This paper investigates the performance of ordered statistics decoding of linear block codes with binary differential phase-shift-keying (2DPSK) transmission on the wide-sense-stationary uncorrelated-scattering (WSSUS) Rayleigh fading channel. For typical mobile speed 60 mph, tropospheric scatter radio communication at carrier frequency 900 MHz and very low bit rate video communication at transmission speed 32 kbit/s, the channel is modeled as a frequency non-selective, slow fading environment without inter-symbol interference (ISI). At bit error rate (BER) 10^{-5}, 34.5 dB and 38 dB gains compared to uncoded 2DPSK are obtained for the decoding of the (24, 12, 8) extended Golay code and the (128, 64, 22) extended BCH code with sufficient degree of interleaving.

1. INTRODUCTION

With the release of the ITU-T H.263 video coding standard [5], video telephony at bit rate less than 32 kbit/s has become realizable nowadays. However, as the coded video bitstreams are highly compressed, they tend to be more susceptible to errors induced from the transmission process than other type of data. Many error control schemes have been proposed with regard to robust video transmission. In general, they can be classified into two categories: (1) techniques to reduce bit error rate (BER) during transmission, e.g., forward error correction (FEC) [3]; and (2) techniques to increase peak signal-to-noise (PSNR) as well as subjective video quality after the corrupted video data are received [6].

In this study, we are interested in the FEC schemes to protect coded video data transmitted through radio communication channel. In particular, the error performance of ordered statistics decoding [3] on the wide-sense-stationary uncorrelated-scattering (WSSUS) multipath channel [1] which exhibits uncorrelated dispersiveness in time delay and Doppler shifts is investigated

This paper is organized as follows. Section 2 describes the system design, in which ordered statistics decoding of block codes, block interleaving and binary differential phase-shift-keying (2DPSK) transmission are integrated to overcome the bursty error channel. The log-likelihood ratio of received symbols due to 2DPSK and the WSSUS channel is derived. In Section 3, the error performance of ordered statistics decoding on the WSSUS channel is analyzed for sufficient degree of interleaving. Simulation results are presented in Section 4. A summary and concluding remarks are given in Section 5.

2. SYSTEM DESIGN

2.1 System Structure

The WSSUS channel characterized by multipath and fading exhibits bursty error features. To effectively combat the noisy channel, we propose the system which integrates ordered statistics decoding of linear block codes with a block interleaver [2, chap. 8, pp. 469]. The coded data is interleaved in such a way that the bursty channel is transformed to a channel having independent errors. In addition, a simple and effective non-coherent detection scheme known as 2DPSK [2, chap. 5, pp. 274-278] is adopted in our design. The block diagram of the proposed system is shown in Figure 1.

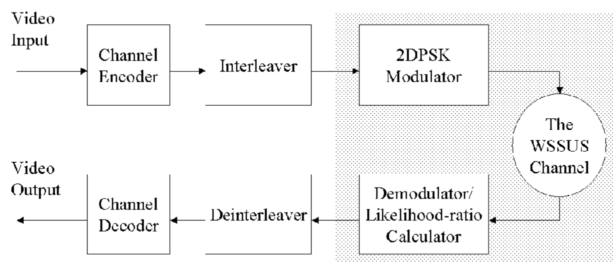

Figure 1. System block diagram. The discrete-time representation associated with the shaded area is illustrated in Figure 3.

2.2 Channel Model

The WSSUS channel model is determined by a two-dimensional scattering function in terms of the echo delay due to multipath effects and the Doppler frequency due to the mobile movement. The Monte Carlo based approximation [1] assumes that the time-varying channel is composed of M independent echoes. Each echo corresponds to a phase ϕ_n, a delay τ_n and a Doppler frequency f_{D_n}, where ϕ_n, τ_n and f_{D_n} are continuous, mutually independent random variables (RVs), $1 \leq n \leq M$. The pdfs of τ_n and f_{D_n} are shown to be proportional to the delay power spectrum and the Doppler power spectrum, respectively [1]. We use exponential distribution to model the delay power profile and the Clarke's spectrum [4, chap. 4, pp. 177-181] to model the Doppler power spectrum, whereas the phase ϕ_n is uniformly distributed from 0 to 2π.

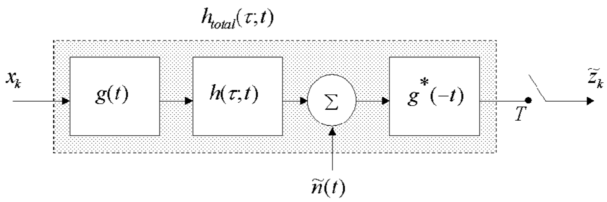

Figure 2. The baseband model for communication over the WSSUS fading channel.

Then the instantaneous channel impulse response at time t to an impulse input at time t - τ is approximated as

$$h(\tau,t) = \lim_{M\to\infty} \frac{1}{\sqrt{M}} \sum_{n=1}^{M} e^{j(\phi_n + 2\pi f_{Dn} t)} \cdot \delta(\tau - \tau_n) \cdot \qquad (1)$$

To derive the discrete-time channel representation, we denote the components of the overall baseband model as in Figure 2, where $\{x_k\}$ is the data sequence, $g(t)$ and $g^*(-t)$ are the time-invariant impulse responses of the transmitter and receiver, $\tilde{n}(t)$ is a complex AWGN process, T is the symbol duration and $\{\tilde{z}_k\}$ is the sampled output. The time-varying overall transmitter-receiver plus channel impulse response is then given as

$$h_{total}(\tau,t) = \lim_{M\to\infty} \frac{1}{\sqrt{M}} \sum_{n=1}^{M} e^{j(\phi_n + 2\pi f_{Dn} t)} \cdot g_{total}(\tau - \tau_n), \qquad (2)$$

where $g_{total}(t) = g(t) \otimes g^*(-t)$.

Consider an H.263 video communication application on the WSSUS channel with the data rate 32 kbit/s (including both compressed video and channel coding redundancy), the mobile speed 60 mph and carrier frequency 900 MHz. Consequently,

- symbol duration T = 31.25 μs;
- maximum Doppler frequency $f_{D_{\max}}$ = 80 Hz;
- multipath duration σ_T ≈ 10^{-6} s [2, pp. 772];
- transmission bandwidth B_s ≈ 32 kHz;
- coherence time T_c ≈ 12.5 ms;
- coherence bandwidth B_c ≈ 1 MHz.

Because $B_s << B_c$, $T_c >> T$ and $T >> \sigma_T$, the channel is modeled as a frequency non-selective, slow fading environment without inter-symbol interference (ISI).

With the channel characteristics, the channel output at instant t = kT given x_k is transmitted is

$$\tilde{z}_k = \tilde{h}_k x_k + \tilde{n}_k, \qquad (3.1)$$

with

$$\tilde{h}_k = h_{total}(0,kT) = \lim_{M\to\infty} \frac{1}{\sqrt{M}} \sum_{n=1}^{M} e^{j(\phi_n + 2\pi f_{Dn} kT)} \cdot g_{total}(-\tau_n), \qquad (3.2)$$

where $x_k = (-1)^{d_k} = (-1)^{b_k \oplus d_{k-1}} \in \{+1,-1\}$ is an interleaved code bit, $b_k, d_k \in GF(2)$. And $\tilde{h}_k$ is a complex, zero-mean Gaussian process with Rayleigh distributed amplitude A and uniformly distributed phase θ . Also, $\tilde{n}_k = n_{k,r} + jn_{k,i}$ is a sample of the noise process $\tilde{n}(t) \otimes g^*(-t)$, modeled as a zero-mean, complex Gaussian RV with independent, identical components of variance $N_0/2$. The discrete-time representation is shown in Figure 3.

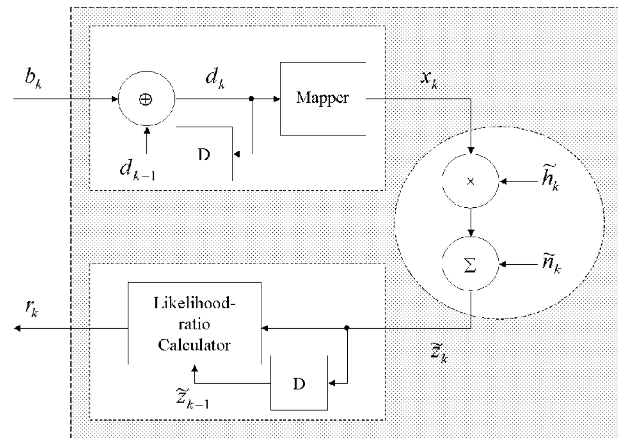

Figure 3. Discrete-time representation of the 2DPSK modulator, the WSSUS channel and the receiver.

2.3 Likelihood Ratio

The value of received symbol is proportional to the log-likelihood ratio for BPSK transmission on AWGN channels [3]. Hence, the hard-decision reliability of the received symbol is its absolute value. As a result, ordered statistics decoder operates on a block of received sequence directly for such systems. In our case, the likelihood ratio has to be re-derived to cope with different channel characteristics and transmission strategy. Because successive tap gains, $\tilde{h}_{k-1}$, $\tilde{h}_k$, $\tilde{h}_{k+1}$, etc., are highly correlated due to slow fading, we assume that the channel tap gain $\tilde{h}_k = A \cdot e^{j\theta}$ at time t = kT equals to the previous tap gain $\tilde{h}_{k-1}$. That is, $\tilde{z}_{k-1} = A \cdot e^{j\theta} \cdot x_{k-1} + \tilde{n}_{k-1}$, and $\tilde{z}_k = A \cdot e^{j\theta} \cdot x_k + \tilde{n}_k$. It is derived that

$$\begin{cases} \tilde{n}_k - \tilde{n}_{k-1} = \tilde{z}_k - \tilde{z}_{k-1} & \text{for} \quad x_k = x_{k-1} \\ \tilde{n}_k + \tilde{n}_{k-1} = \tilde{z}_k + \tilde{z}_{k-1} & \text{for} \quad x_k \neq x_{k-1} \end{cases} . \qquad (4)$$

Figure 4 illustrates the detection mechanism for the case $x_k = x_{k-1} = +1$. Let $\tilde{Z}_{k-1}$ and $\tilde{Z}_k$ denote the received random symbols at time index k-1 and k, and define event $E = \{\tilde{z}_k < \tilde{Z}_k < \tilde{z}_k + d\tilde{z}, \tilde{z}_{k-1} < \tilde{Z}_{k-1} < \tilde{z}_{k-1} + d\tilde{z}\}$, where $d\tilde{z}$ is a very small deviation in the complex plane. Thus, the log-likelihood ratio of the differentially encoded bit is given by

$$L(\tilde{z}_k, \tilde{z}_{k-1}) = \ln \frac{P(E \mid X_k = X_{k-1})}{P(E \mid X_k \neq X_{k-1})}, \qquad (5.1)$$

$$= \ln \frac{p_{\tilde{N}_k - \tilde{N}_{k-1}}(\tilde{z}_k - \tilde{z}_{k-1})}{p_{\tilde{N}_k + \tilde{N}_{k-1}}(\tilde{z}_k + \tilde{z}_{k-1})}, \qquad (5.2)$$

$$\propto \mathrm{Re}(\ \tilde{z}_k^* \tilde{z}_{k-1}\ ), \qquad (5.3)$$

where X_{k-1} and X_k are the transmitted random symbols and $p_{\tilde{N}_k \pm \tilde{N}_{k-1}}(\cdot)$ denotes the pdfs of the noise RVs $\tilde{N}_k \pm \tilde{N}_{k-1}$. Consequently, the associated reliability measure is defined as $|r_k|$, where

$$r_k = \mathrm{Re}(\ \tilde{z}_k^* \tilde{z}_{k-1}\ ) . \qquad (6)$$

With the new reliability measure due to 2DPSK modulation, the ordered statistics decoding is employed.

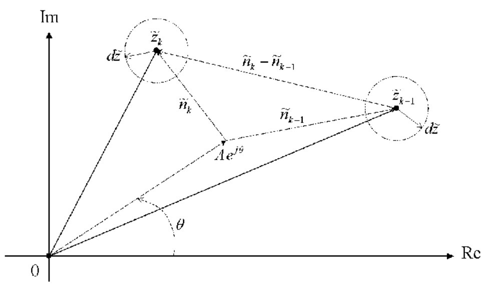

Figure 4. Graph description of the 2DPSK detection on the slowly fading channel, where $x_k = x_{k-1} = +1$.

2.4 Ordered Statistics Decoding

Ordered statistics decoding reduces search space for maximum likelihood decoding (MLD) performance by taking advantage of the reliability information from the received symbols. To illustrate the idea, consider the binary transmission system described in Section 2.2 where a binary (N, K, d_H) linear block code C is used for error control over the discrete-time channel. For each block of K bits from the input video sequence, a codeword $\overline{c}$ = (c_1, c_2,…, c_N) in C is generated at the channel encoder output, where c_i is an element of $GF(2)$, $1 \le i \le N$. The $\{b_k\}$ sequence is formed by interleaving successive codewords. With 2DPSK modulation, the sequence is differentially encoded, mapped into the bipolar sequence $\{x_k\}$ and sent through the channel. At the output of the likelihood-ratio calculator, the corresponding real number sequence $\{r_k\}$ is received. By deinterleaving $\{r_k\}$, the block of received symbols $\overline{r}$, associated with the transmitted codeword $\overline{c}$, is obtained, where $\overline{r}$ = (r_1, r_2,…, r_N). The components of $\overline{r}$ are independent for sufficient degree of interleaving. The hard-decision of each symbol r_i is based on the sign of r_i, whereas the associated reliability is determined by $|r_i|$.

For each $\overline{r}$, the ordered statistics decoder performs two permutations (λ_1, λ_2), followed by two decoding steps, i.e., order-0 and order-l decoding, where l is an integer, $l > 0$. λ_1 reorders the components of each $\overline{r}$ based on their reliabilities, while λ_2 reorders them again to find the K most reliable independent (MRI) positions [4]. Thereafter, Order-0 decoding constructs a codeword corresponding to the hard decision of the MRI positions. This codeword is expected to have as few information bits in error as possible. Furthermore, order-l decoding improves the result obtained from order-0 decoding progressively until the asymptotically optimum error performance is achieved. For codes and channel signal-to-noise ratios (SNRs) of practical interests, the optimum codeword candidate $\overline{c}^*$,

$$\overline{c}^* = \arg \max_{\overline{c}} \sum_{i=1}^{N} (-1)^{c_i} r_i \ , \tag{7}$$

will most likely be found at a small value of l [4].

3. PERFORMANCE

In the following discussion, the initial reference symbol x_0 is set as +1. Without loss of generality, we assume that all zero codewords are sent through the discrete-time channel. For an arbitrarily code bit c_i = 0, the corresponding bipolar symbols $x_k = x_{k-1} = +1$ are transmitted. At the receiving end, the likelihood ratio r_k is obtained as

$$r_k = A^2 + A\left[(n_{k-1,r} + n_{k,r})\cos\theta + (n_{k-1,r} + n_{k,r})\sin\theta\right] + \left[n_{k-1,r}n_{k,r} + n_{k-1,i}n_{k,i}\right]. \tag{8}$$

For SNRs of practical interests, $n_{k-1,r}n_{k,r} + n_{k-1,i}n_{k,i}$ is relatively small compared with other terms. Hence, we approximate the distribution of the received likelihood ratio at high SNRs as

$$p_R(r_k) \cong \frac{1}{\sigma^2\sqrt{2\pi N_0}} \int_0^{\infty} \exp\left\{-\frac{(r_k/a - a)^2}{2N_0} - \frac{a^2}{2\sigma^2}\right\} da \ , \tag{9}$$

where σ^2 is the variance of the zero-mean Gaussian RV whose envelope has the corresponding Rayleigh distribution. Assuming sufficient degree of interleaving, the components of $\overline{r}$ corresponding to the transmitted all-zero codeword are independent. Denote u_i as the symbol representing the likelihood ratio at the i-th position after permutation λ_1, i.e. $\overline{u} = \lambda_1(\overline{r})$ such that $|u_1| > |u_2| > \cdots > |u_N|$. Note that the components of $\overline{u}$ are exactly the *same* as those of $\overline{r}$, only the *ordering* is different. At high SNRs, the marginal pdf of the ordered symbol is approximated as

$$p_{U_i}(u_i) \cong \frac{N!}{(i-1)!(N-i)!}\left(1 - \int_{-|u_i|}^{|u_i|} p_R(r)dr\right)^{i-1}\left(\int_{-|u_i|}^{|u_i|} p_R(r)dr\right)^{N-i} p_R(u_i) \ . \tag{10}$$

Thus, the probability that the hard decision of the i-th symbol u_i is in error is given by

$$Pe(i) = P(U_i < 0) = \int_{-\infty}^{0} p_{U_i}(u_i)du_i \ . \tag{11}$$

The probabilities that the hard decisions of two or more ordered symbols of sequence $\overline{u}$ are in error can be evaluated in the same way.

Define P_{CM} as the optimum bit error probability of soft-decision decoding algorithms based on maximizing correlation metric. Let P_{bl} be the average bit error rate when there are more than l hard-decision errors in the first K MRI positions of $\overline{r}$. It follows from union bound that the bit error rate $P_b(l)$ of order-l decoding is

$$P_b(l) \le P_{CM} + P_{bl} \ . \tag{12}$$

Although the channel characteristics are different, the derivations in [3] for evaluating P_{bl} remain valid. To evaluate P_{CM}, denote the all zero codeword $\overline{c}_1 = \overline{0}$. Thus, the probability of deciding an arbitrary codeword $\overline{c}_2$ as the estimated result is

$$P_2 = P(\hat{c} = \overline{c}_2 \mid \overline{c}_1 \text{ is transmitted}) \ , \tag{13.1}$$

$$= P\left(\sum_{i=1}^{N}(-1)^{c_{1,i}} R_i < \sum_{i=1}^{N}(-1)^{c_{2,i}} R_i\right), \tag{13.2}$$

$$= P\left(\sum_{i=1}^{w_2} R_i < 0\right), \tag{13.4}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{0}\int_{-\infty}^{\infty} e^{-j\omega z}\left(\Phi_R(\omega)\right)^{w_2} d\omega dz \ , \tag{13.5}$$

where $\Phi_R(\omega) = E[e^{j\omega R}]$ and w_2 is the Hamming weight of $\bar{c}_2$. The block error probability $P_{CM,block}$ is bounded as

$$P_{CM,block} \le \sum_{m=2}^{2^K} P_m \approx \frac{n_d}{2\pi} \int_{-\infty}^{0} \int_{-\infty}^{\infty} e^{-j\omega z} \left(\Phi_R(\omega)\right)^{d_H} d\omega dz , \qquad (14)$$

where n_d is the number of minimum-weight codewords in C. The BER is approximated as $P_{CM} \cong \frac{d_H}{N} P_{CM,block}$ [7].

4. SIMULATIONS

In our simulations, the number of echoes M is 50 and the rolloff factor of the raised cosine pulse shaping filter is chosen as 0.5. Figures 4 and 5 depict the error performance of the (24, 12, 8) extended Golay code and the (128, 64, 22) extended BCH code with interleaving degree v = 200 and 50, respectively. For each code, simulation results for various orders of decoding are plotted in terms of BER vs. SNR. The SNR is defined as $E_b < A^2 > / N_0$, where E_b is the energy per information bit, $< A^2 >$ is the time average of the Rayleigh amplitude. For the Golay code, Figure 4 shows that order-1 decoding already achieves the practically optimum performance, no significant improvement can be obtained with higher order of decoding. At BER 10^{-5}, order-1 decoding of the Golay code with v = 200 has 34.5 dB coding gain compared to uncoded 2DPSK. For the BCH code, Figure 5 shows that order-3 decoding is practically optimum. Order-3 decoding of the BCH code with v = 50 achieves 38 dB coding gain over uncoded 2DPSK at BER 10^{-5}. For the decoding of the Golay code with v = 200, each received r_k is separated from previous symbol r_{k-1} by 200×24/32k = 0.15 sec, which is much larger than the coherence time (≈ 12.5 ms) of the channel. For the decoding of the BCH code with v = 50, the separation is 50×128/32k = 0.2 sec. Hence for both cases, the successive received symbols within a block are affected differently by the channel. Simulation results show that no further performance improvement can be obtained in both cases by increasing v.

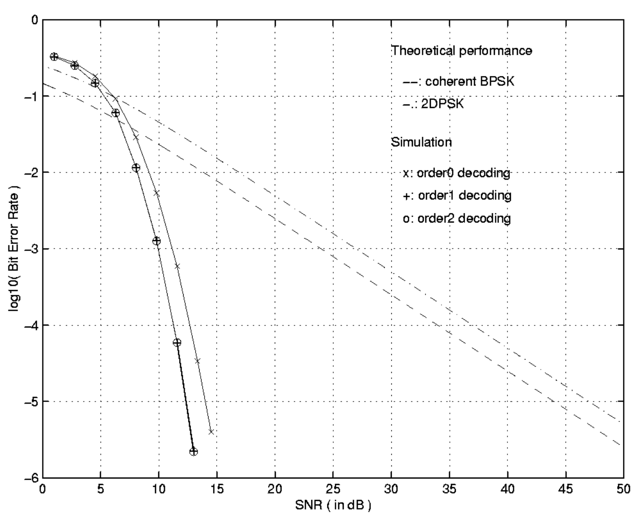

Figure 4. Performance of the (24,12,8) extended Golay code with interleaving degree 200.

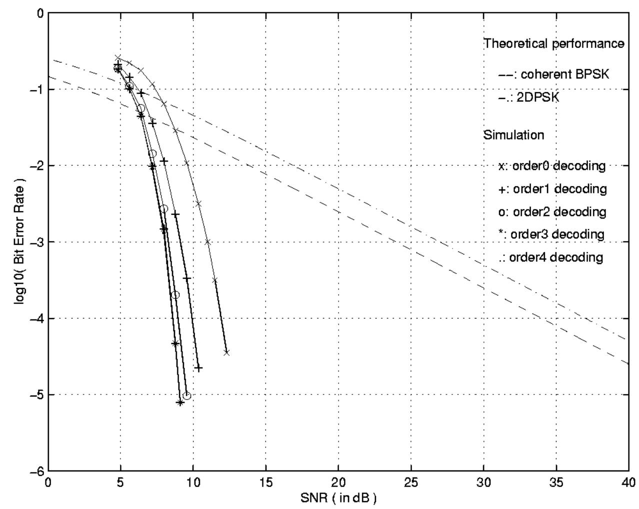

Figure 5. Performance of the (128,64,22) extended BCH code with interleaving degree 50.

5. CONCLUSION

In this study, ordered statistics decoding [3] is applied to the WSSUS multipath channel to protect video sequence transmitted at 32 kbit/s. Simulations are conducted over the channel with 2DPSK transmission for ordered statistics decoding of the (24, 12, 8) extended Golay code and the (128, 64, 22) extended BCH code. The log-likelihood ratio due to 2DPSK and the WSSUS channel is derived and its statistics after ordering is used to analyze the system performance. Results demonstrate that the purpose of error protection over the WSSUS channel can be effectively achieved by ordered statistics decoding of channel codes. This is important for delay constrained applications for which ARQ is not feasible due to intolerable round trip delay.

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