# SYSTEM IDENTIFICATION USING ORTHOGONAL FUNCTIONS AND APPLICATION TO ACOUSTIC ECHO CANCELLATION

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## ABSTRACT

In this paper, a new Laguerre domain adaptive filter algorithm, which will be refered to as the Laguerre domain adaptive filter II (LDAF II) has been proposed. The performance of the adaptive filtering algorithms simulated for acoustic echo cancellation application. The performances of the algorithm are specified using various quantities. All the results of the work, for different performance quantities, are presented with several graphics and they are compared with Legendre functions based adaptive filter (LFB ADF) [1], and LMS adaptive filter (LMS ADF).

## **1. INTRODUCTION**

The concept of adaptive filtering constitutes an important part of statistical signal processing. Whenever there is a requirement to process signals that result from unknown statistics of an environment, the use of an adaptive filter offers an attractive solution to the problem. Thus, in communications, adaptive filters are successfully applied in such diverse fields as echo cancellation, channel equalization, linear prediction, and spectral estimation.

In the time domain approach for finite impulse response (FIR), the filter taps are estimated by adaptive algorithms such as the least mean square (LMS) [2], the gradient lattice [3], the least squares lattice [4] or the recursive least squares (RLS) [2].

The time domain LMS adaptive filter algorithm has been used in a variety of applications because of its robustness and its well-understood behaviour. Convergence time and stability of the algorithm depend on the ratio between the largest and the smallest eigenvalues associated with the correlation matrix of the input sequence. Slow convergence rate can be expected when this ratio is large. In practical applications the input sequence is usually speech whose correlation matrix has highly disparate eigenvalues.

Thus, transform domain adaptive filtering was introduced to accelerate this rate and also reduce the complexity of the tapped delay line ADFs[5]. Several Frequency domain adaptive filters are proposed. Recently, a Laguerre domain adaptive filter is proposed by Mandyam where only the input is transformed to the Laguerre domain [6]. These Laguerre domain components are weighed and summed to produce the filter output. This output is then subtracted from the time-domain desired response. However in the proposed Laguerre domain adaptive filter II structure [7] in this paper, the desired response is also transformed into Laguerre domain. Then LMS algorithm is applied to each bin of the filter independently. This way reduces the error between the transformed values of the desired response and the received speech signal. The performance of this filter is simulated for acoustic echo cancellation application. Echo arises in hands-free telephony due to impedance mismatch in the hybrid (network echo) and due to acoustic feedback from the loudspeaker to the microphone (acoustic echo) [8].

Organization of the paper is as follows: Section II introduces the background theory for the Discrete Laguerre transform and general transform domain adaptive filtering. Section III describes the proposed LADF II. Simulation results and the concluding remarks are summarized in Section IV.

## 2. BACKGROUND METERIAL

### 2.1 Discrete Laguerre Transform

Discrete unitary transforms are useful in signal processing applications, such as transform domain adaptive filtering in the areas of speech processing and adaptive line enhancers. A wellknown method for generating these unitary transforms from a set of orthonormal polynomials is Gauss-Jacobi procedure [6]. Using this procedure, Mandyam and Ahmed [6] derived discrete Laguerre transform (DLT) from the orthonormal set of Laguerre functions.

The n-th order Laguerre functions (starting from 0) can be defined as [6]

$$l_n(p,x) = (-1)^n \sqrt{(2p)} e^{-px} L_n(2px)$$
(1)

where

$$L_{n}(x) = \frac{e_{x} d_{n}}{n! dx_{n}} (x^{n} e^{-x}), \qquad (2)$$

and p is nonzero constant  $(\frac{d}{dx}$  is the derivation operator). Following recursive representation of the n-th Laguerre function can be obtained from (1) and (2) for n>1[6]

$$l_{n}(x,p) = \frac{1}{n} l_{n-1}(x,p) [2px-2n+1] - \frac{n-1}{n} (n-1)^{2} l_{n-2}(x,p).$$
(3)

The DLT of a discrete N-point signal u[n] can be written as

$$Z[f] = \sum_{k=0}^{N-1} a_k l_f(x_k, p) u[k] \quad f = 0, 1...N-1$$
(4)

Here,  $\{x_n\}$  is the set of the discretization points derived from the roots of  $l_N(x,p)$  and  $a_k$  coefficients are given by [6]

$$a_{k} = \frac{-b_{n}}{b_{n+1}} \frac{1}{l_{n}'(x_{k})l_{n+1}(x_{k})}$$
(5)

#### 2.2 Transform Domain Adaptive filtering

A block diagram of the transform domain adaptive filter is shown in Figure (1). The input vector  $\mathbf{x}_n$  is first transformed into another vector  $\mathbf{z}_n$ .

$$\mathbf{z_n} = \left[ z_n(0) \, z_n(1) \dots \, z_n(\mathbf{M-1}) \right]^{\mathrm{T}} \tag{6}$$

using an orthogonal transformation

$$\mathbf{z}_{\mathbf{n}} = \mathbf{W} \mathbf{x}_{\mathbf{n}} \tag{7}$$

where W is a unitary matrix of rank M. Now, the vector  $z_n$  is multiplied by the transform domain weight vector

$$\Omega_{\mathbf{n}} = [\Omega_0(\mathbf{n}) \,\Omega_1(\mathbf{n}) \dots \,\Omega_{\mathbf{M}-1}(\mathbf{n})] \tag{8}$$

to form the adaptive output. The output and the corresponding error signal are

$$\mathbf{y}(\mathbf{n}) = \mathbf{z_n}^{\mathrm{T}} \mathbf{\Omega_n} \tag{9}$$

and

$$e(n) = d(n) - y(n)$$
 (10)

respectively. The weight update equation is

$$\Omega_{n+1}(i) = \Omega_n(i) + 2\mu_e(n)z_n(i)$$
(11)

where

$$\mu_{i} = \frac{\mu}{E[z_{n}^{2}(i)]}, \quad i = 0, 1, \dots M-1$$
 (12)

is the adaptive step size for the i-th transform component and the  $\mu$  is a positive constant that governs the rate of convergence. As it is shown in [5], new tap weights are equal to the optimum (Wiener) solution  $\boldsymbol{\omega}_{0}$  multiplied by the transform matrix  $\mathbf{W}$ .

$$\mathbf{\Omega}^{\star} = \mathbf{W} \mathbf{R}^{-1}_{\mathbf{x}\mathbf{x}} \mathbf{p}_{\mathbf{x}\mathbf{d}}.$$
 (13)

where  $\mathbf{p}_{xd}$  is the cross-correlation vector between the tap input vector  $\mathbf{x}_n$  and the desired response d(n), and  $\mathbf{R}_{xx}$  is the correlation matrix of the tap input vector  $\mathbf{x}_n$ .

Therefore, for properly chosen orthogonal transform **W**, some reduction in the eigenvalue spread can be expected. As a consequence of this, the transform domain adaptive algorithm can be expected to have better convergence properties than the corresponding time domain adaptive algorithm. As an application, in the next section, proposed Laguerre domain adaptive filter II implementation is considered.

## 3. Laguerre Domain Adaptive Filter II

In the proposed adaptive filter structure, adaptation of the filter is done in the Laguerre domain. The difference between the proposed adaptive filter and the adaptive filter mentioned above [6] is that in the proposed adaptive filter, the desired value is also transformed to the Laguerre domain. The advantages of this step is to reduce the error between the transformed values of the desired response and the received speech signal.

A block diagram of the Laguerre domain adaptive filter II is shown in Figure (2). The input vector and the desired vector are first transformed into Laguerre domain vectors  $z_n$  and  $\eta_n$ 

 $\mathbf{z}_{n} = [\mathbf{z}_{n}(0) \mathbf{z}_{n}(1) \dots \mathbf{z}_{n}(M-1)]^{T}$ 

and

$$\boldsymbol{\eta}_{\mathbf{n}} = [\boldsymbol{\eta}_{\mathbf{n}}(0) \, \boldsymbol{\eta}_{\mathbf{n}}(1) \dots \boldsymbol{\eta}_{\mathbf{n}}(\mathbf{M}-1)]^{\mathrm{I}}$$
(15)

using DLT,

$$\mathbf{z_n} = \mathbf{L}\mathbf{x_n}, \tag{16}$$
$$\boldsymbol{\eta_n} = \mathbf{L}\mathbf{d_n} \tag{17}$$

(14)

where L is the unitary Laguerre transform matrix and  $d_n$  and  $x_n$  are desired response and input vectors respectively. Now, the each  $z_n(i)$  is multiplied by the corresponding transform domain weights as shown in vector form in Eq. (18)

$$\boldsymbol{\Omega}_{\mathbf{n}} = \left[\Omega_{0}(n) \, \Omega_{1}(n) \, \dots \, \Omega_{M-1}(n)\right]^{\mathrm{T}}$$
(18)

to form the adaptive outputs. The outputs and the error signals are

$$y_n(i) = \Omega_n(i)z_n(i) \tag{19}$$

and

$$\mathbf{e}_{\mathbf{n}}(\mathbf{i}) = \mathbf{\eta}_{\mathbf{n}}(\mathbf{i}) - \mathbf{y}_{\mathbf{n}}(\mathbf{i}) \tag{20}$$

respectively. The weight update equation is

$$\Omega_{(n+1)}(i) = \Omega_n(i) + 2\mu_i e_n(1) z_n(i), i = 0, 1, \dots, M-1$$
(21)

where

$$\mu_{n+1}(i) = \frac{\alpha}{q_n(i)}$$
(22)

$$q_n(1) = (1-\beta) q_{n-1}(i) + \beta E[z_{n(i)}^2], i = 0, 1, ..., M-1$$
 (23)

 $\mu(i)$  is the adaptive step size for the i-th transform component. From the last section, if the LMS algorithm is applied independently to i-th bin, different  $\mu_I$  can be chosen for each bin. Since in practical applications this is not useful, an estimate of  $\mu$ , by normalizing a constant convergence factor  $\alpha$  by an estimate of the energy at the i-th transform component, is used instead. This normalization is similar to the process used in the lattice adaptive filter [2].

The filter structure implemented in this paper as an acoustic echo canceller is given in the Figure (3). The input of the both unknown echo source and the LDAF II is the same signal from the far end speaker.(x(n)) The output of the unknown system (the desired signal d(n)) and additive noise from the environment are fed into the LDAF II. Then the output of the LDAF II is residual echo, i.e. error e(n) in the above structure.

#### **IV. RESULTS AND CONCLUSION**

In this work, An acoustic echo canceller for detection of an echoed signal in a car environment is considered. Th echo cancellation is performeed as a test case due to evaluate the effectiveness of proposed algorithm. The echo path in a car environment was simulated using a finite impulse response filter. Real speech signal was used to test the echo canceller. Echo Return Loss Enhancement (ERLE) is used as the performance index of the algorithms. ERLE is defined as the ratio of the energy in the residual echo e(n) to the energy in the original echo d(n):

ERLE = 
$$\frac{E\{e^{2}(n)\}}{E\{d^{2}(n)\}}$$
 (24)

In this work the LDAF II, LMS, and Legendre and Laguerre function based adaptive filtering algorithms are implemented and compared based on their convergence rate, steady state ERLE, and complexity. All the filters have 256 taps. LMS, while simple to implement, has poor convergence rate when the eigenvalue spread of the signal is high. As it can be seen in Figure (4) the proposed structure, LDAF II has fastest convergence, but with significant increase in complexity. This can be shown in Figure (5) Legendre ADF structure provides faster convergence and better steady state ERLE than LMS, Figure (6). The complexity of the LDAF II is caused by taking the DLT of the both input and the desired response. As mentioned in the [6], increase in the filter order requires the finding the roots of the large order discrete laguerre functions which is very complex The complexity of the filter structure may be overcome by using a very large-scale integrated (VLSI) custom chip. Further research to reduce the complexity of the LDAF II algorithm would be worthwhile due to the desirability

of its fast convergence at higher filter orders than presently implementable on DSPs.

#### 5. REFERENCES

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Figure 1 Block diagram of the Transform domain ADF

and





Figure 4 ERLE of LMS ADF

Figure 2 Block diagram of the Laguerre domain ADF



Figure 3 Block diagram of the LDAF II in echo cancellation



Figure 5 ERLE of LDAF II



Figure 6. ERLE of Legendre ADF