SPECTRAL LINE RLS ADAPTIVE FILTERING ALGORITHM

Dinko Begusic, Darel Linebarger, Eric M. Dowling, Balaji Raghothaman*

Dept. of Electrical Eng. EC33,
University of Texas at Dallas,
Richardson, TX 75083-0688, USA.
begusic@fesb.hr

ABSTRACT

A family of adaptive filtering algorithms for processing signals which have energy concentrated in a relatively small number of component subspaces in the spectral domain is introduced. The approach is based on transform domain signal decomposition and linear least squares filtering of the selected subset of transform domain signal components. The derivation is based on the linear least squares adaptive filtering framework introduced in our previous work [1]. Fast convergence and computational efficiency are the main characteristics of the resulting algorithms. The method is applied to the problem of adaptive line enhancement comb filtering and DFT is used as a transform method. It is also shown that the resulting adaptive structure is capable of handling the case of non-coinciding frequencies. The performance of the algorithm is evaluated through a series of simulation experiments.

1. INTRODUCTION

In many signal processing applications the signal characteristics vary in the spectral domain. Using a spectral decomposition of a signal and matching the characteristics of an adaptive signal processing system to the characteristics of the signal in spectral subspaces makes it possible to optimize the usage of the computational power as well as increase the overall performance and efficiency of the system. The areas of transform domain, subband and subspace signal processing have recently been in the focus of the research interest [2, 3, 4]. Different techniques have been developed to increase the energy compaction. Spectral decomposition is usually combined with either simple and robust LMS or more sophisticated but usually higher performance RLS methods to obtain efficient adaptive filtering algorithms [5].

In this work we follow the approach of combining the energy compaction property of spectral decomposition with high performance of RLS methods to obtain an adaptive algorithm suitable for processing a class of signals which have energy concentrated in a relatively small number of component subspaces in the spectral domain. We refer to our approach as the spectral line recursive least squares adaptive filtering algorithm (SL RLS). We derive it by using the linear least squares adaptive filtering framework introduced in [1]. The original framework combines spectral decomposition with linear least squares adaptive filtering techniques. Spectral decomposition may be implemented by different types of transformations. Transforms which may be used include data dependent transforms (e.g. Singular Value Decomposition -SVD or Karhunen-Loeve Transform - KLT) and data independent transforms (e.g. Discrete Fourier Transform - DFT or Discrete Cosine Transform - DCT). By projecting the data vectors into a set of component spectral subspaces we can extract components that belong to a specified component subspaces or composite subspace. This approach gives us control of the selection of the portion of data vectors used in the RLS adaptive filter coefficients' adaptation process in both the original data domain and the spectral domain. The linear RLS adaptive filtering problem can be implemented in overdetermined, exactly determined or underdetermined form, depending on the rank of the problem which can be chosen by appropriate time-domain window boundary selection. Algorithms can be implemented in either a sample-by-sample or block versions.

This paper is organized as follows. In section 2 we define the generalized form of the algorithm. We also focus on a computationally efficient low rank - low projection order form of the algorithm and use DFT as an example of transformation. In the third section we describe the application of the proposed algorithm to an adaptive line enhancer/comb filter. In the fourth section we present typical examples of simulation results.

2. DEFINITION AND DERIVATION OF THE SPECTRAL LINE RLS ALGORITHM

Consider the linear least squares adaptive filtering problem. Let $\mathbf{X}(k)$ be an $N \times M$ data matrix

$$\mathbf{X}(k) = [\mathbf{x}_1(k) \quad \mathbf{x}_2(k) \quad \ldots \quad \mathbf{x}_M(k)] \tag{1}$$

where $\mathbf{x}_i(k), i = 1, 2, .., M$ are $N \times 1$ input data vectors at time k, and $\mathbf{y}(k)$ is the corresponding $M \times 1$ reference signal sample vector

$$\mathbf{y}^T(k) = [y(k) \quad y(k-1) \quad \ldots \quad y(k-M+1)]. \tag{2}$$

The $M \times 1$ adaptive filtering residual error vector at time k can be defined as

$$\mathbf{e}(k) = \mathbf{y}(k) - \mathbf{X}^H(k)\mathbf{h}(k) \tag{3}$$

where $\mathbf{h}(k)$ is an $N \times 1$ adaptive filter coefficient vector. The adaptive coefficient vector increment $\Delta\mathbf{h}(k+1)$ which is optimal in the linear least squares sense satisfies the following equation [1]:

$$\mathbf{X}^H(k)\Delta\mathbf{h}(k+1) = \mathbf{e}(k). \tag{4}$$

For $1 < M < N$ the system (4) represents an underdetermined LS problem and the solution leads to the affine projection algorithm

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(APA) [1]. For $M > N$ the system (4) is overdetermined and the solution leads to sliding window covariance (SWC) RLS algorithm [1]. Instead of solving the LS problem directly in this form we can modify it by introducing the product of a unitary matrix $\mathbf{Q}$ and its hermitian transpose $\mathbf{Q}^H$ in (4). By doing so we obtain a transform domain decomposition based linear least squares adaptive filtering framework in two forms [1]:

$$(\mathbf{Q}\mathbf{Q}^H)(\mathbf{X}^H(k)\Delta\mathbf{h}(k+1) = \mathbf{e}(k) \tag{5}$$

where the size of the transform matrix is M, and

$$\mathbf{X}^H(\mathbf{Q}\mathbf{Q}^H)\Delta\mathbf{h}(k+1) = \mathbf{e}(k) \tag{6}$$

where the size of the transform matrix is N.

The first form was analyzed previously [1, 2]. In this work we focus on the second form. If $\mathbf{Q}$ is a full rank unitary transform matrix the problem reduces to the ordinary linear least squares adaptive filtering problem. If the column vectors of the matrix $\mathbf{Q}$ are orthogonal we can decompose the product $\mathbf{Q}\mathbf{Q}^H$ into a sum of projection operators as follows:

$$\mathbf{Q}\mathbf{Q}^H = \sum_{i=1}^{N} \mathbf{q}_i\mathbf{q}_i^H \tag{7}$$

where $\mathbf{q}_i, i = 1, \cdots, N$ are the column vectors of the matrix $\mathbf{Q}$. Now if we want to project the quantities involved in the adaptation process into a subspace defined by a selected set of column vectors of the matrix $\mathbf{Q}$ we can define the projection operator into a selected rank - L subspace by reducing the column space of the matrix $\mathbf{Q}$ to a selected set of L column vectors:

$$\mathbf{Q}_L\mathbf{Q}_L^H = \sum_{i=1}^{L} \mathbf{q}_i\mathbf{q}_i^H \tag{8}$$

where $\mathbf{Q}_L$ is an $N \times L$ matrix of L column vectors of the matrix $\mathbf{Q}$. Now we can rewrite the equation (6) in the following form:

$$(\mathbf{X}^H\mathbf{Q}_L)(\mathbf{Q}_L^H \quad \Delta\mathbf{h}(k+1)) = \mathbf{e}(k) \tag{9}$$

and compute the minimum norm least squares solution for the $L \times 1$ transformed coefficient vector increment given by

$$\Delta\tilde{\mathbf{h}}(k+1) = \mathbf{Q}_L^H\Delta\mathbf{h}(k+1) \tag{10}$$

Generally the solution will have the following form:

$$\Delta\tilde{\mathbf{h}}(k+1) = (\mathbf{Q}_L^H\mathbf{X}^H)^+\mathbf{e}(k) \tag{11}$$

where $+$ denotes pseudoinverse operation.

By introducing the symbol $\tilde{\mathbf{X}}$ for the $L \times M$ transformed data matrix

$$\tilde{\mathbf{X}}(k) = \mathbf{Q}_L^H\mathbf{X}(k) \tag{12}$$

we can rewrite the expression for the coefficient vector increment in a more compact form. For $M > L$ the LS problem is overdetermined and the solution has the following form:

$$\Delta\tilde{\mathbf{h}}(k+1) = (\tilde{\mathbf{X}}\tilde{\mathbf{X}}^H)^{-1}\tilde{\mathbf{X}}\mathbf{e}(k) \tag{13}$$

If $M < L$ the problem is underdetermined and the solution has the following form:

$$\Delta\tilde{\mathbf{h}}(k+1) = \tilde{\mathbf{X}}(\tilde{\mathbf{X}}^H\tilde{\mathbf{X}})^{-1}\mathbf{e}(k) \tag{14}$$

The adaptive coefficient vector update equation is given by [1]:

$$\tilde{\mathbf{h}}(k+1) = \tilde{\mathbf{h}}(k-K+1) + \mu\Delta\tilde{\mathbf{h}}(k+1) \tag{15}$$

where μ is adaptation step size and K is the adaptation period.

The residual vector can also be expressed in terms of the transformed quantities $\tilde{\mathbf{X}}(k)$ and $\tilde{\mathbf{h}}(k)$:

$$\mathbf{e}(k) = \mathbf{y}(k) - \tilde{\mathbf{X}}^H(k)\tilde{\mathbf{h}}(k) \tag{16}$$

By using the equations (12), (16), (13) or (14), and (15) we obtain the complete general form of the SL RLS algorithm which is summarized in Table I. From these equations we can see that the filtering operation is performed in the original data domain (which is usually the time domain), but the data vectors involved in the coefficient adaptation process, as well as the coefficient vector, are decomposed in the spectral domain. That operation can be seen as spectral analysis, subspace decomposition or subband decomposition depending on the nature of the transform which is used. The selection of the data vectors used is defined by the form and the shape of the window in the spectral domain. The choice of the window can be fixed, dynamic or adaptive (data, state and/or time dependent).

1.	$\tilde{\mathbf{X}}(k) = \mathbf{Q}_L^H\mathbf{X}(k)$
2.	$\mathbf{e}(k) = \mathbf{y}(k) - \tilde{\mathbf{X}}^H(k)\tilde{\mathbf{h}}(k)$
3.a.	$\Delta\tilde{\mathbf{h}}(k+1) = (\tilde{\mathbf{X}}\tilde{\mathbf{X}}^H)^{-1}\tilde{\mathbf{X}}\mathbf{e}(k)$, $M > L$
3.b.	$\Delta\tilde{\mathbf{h}}(k+1) = \tilde{\mathbf{X}}(\tilde{\mathbf{X}}^H\tilde{\mathbf{X}})^{-1}\mathbf{e}(k)$, $M < L$
4.	$\tilde{\mathbf{h}}(k+1) = \tilde{\mathbf{h}}(k-K+1) + \mu\Delta\tilde{\mathbf{h}}(k+1)$

Table 1: Summary of the SL RLS Algorithm

We can now estimate the computational complexity of the general form of SL RLS algorithm by considering the number of complex multiplications associated with each equation assuming the adaptation period equals K. Transformation of the input data vector associated with equation (12) requires KLN multiplications. The residual computation defined by equation (16) requires ML multiplications. Coefficient vector increment calculation defined in (13) requires $(2L^2 + L)M$ multiplications and the $L \times L$ matrix inversion, while the corresponding calculation defined in (14) requires $(2M^2 + M)L$ multiplications and the $M \times M$ matrix inversion. This gives a total of $KLN + 2M(L^2 + L)$ multiplications and $L \times L$ matrix inversion for each K sampling periods in the case of the overdetermined system, and a total of $KLN + 2L(M^2 + M)$ multiplications and the $M \times M$ matrix inversion for each K sampling periods in the case of the underdetermined system. For particular transformation matrices the actual computational complexity can be significantly reduced by eliminating the redundancy associated with the input data vectors and the transform matrix column vectors. For example, if the DFT is used as a transformation method, the Goertzel algorithm can be used for each spectral line and the number of multiplications required for computation of equation (12) can be reduced to KL . Also in the case of the real sinusoids the sample correlation matrix will have a centrohermitian structure and the computational reduction of 75% for its inversion is possible [6].

3. ADAPTIVE LINE ENHANCEMENT USING SL RLS

The adaptive line enhancer (ALE), which is also called the spectral line enhancer, is a device that may be used either for detection of a periodic or narrowband signal in a noncoherent background, or for elimination of a narrowband interference from a broadband signal of interest [5, 7]. In this work we focus on the use of ALE for adaptive comb filtering aimed for extracting multiple sinusoids from a broadband signal. The conventional ALE may be implemented using either an adaptive FIR or IIR filter. Recently, new approaches are reported based on using the subspace filter [4] and subband (multirate) technique [3]. Using the property of signal energy compaction in subspace or subband signal decomposition the subspaces or subbands containing only noise (or broadband signal) can be ignored in the process of adaptive narrowband signal enhancement or extraction. Hence, the adaptation process is applied on the reduced rank subspace or reduced frequency range and the computational power of the algorithm is used more efficiently. We follow the approach of reducing the computational load by focusing our efforts on the frequency range occupied by the signal of interest. Since in the case of the adaptive comb filter it is only a discrete set of spectral lines that form the signal of interest for adaptation process we use the DFT decomposition of the signal and reduce it to a discrete set of spectral lines.

The frequency domain ALE is analyzed in [8] and it is shown that the optimal coefficient set has a matrix form with nonzero off-diagonal elements. Using the N-point DFT (FFT) based ALE yields limited performance if adaptation is performed independently for all spectral lines because it corresponds to adaptation using only the diagonal elements of the optimal coefficient matrix and ignoring the off-diagonal terms. Thus the suboptimal scheme was proposed which takes into account the diagonal and two closest off-diagonals of the weight matrix. A similar problem caused by aliasing is noticed in the case of subband filtering based approach [9], and the solution is proposed in a form of additional auxiliary subband that employs cross-terms. In the DFT the leakage effect caused by aliasing is more severe than in the case of the subband approach because of the low attenuation of the sidelobes. The case of coinciding frequencies (the frequency of interest is transformed exactly to one spectral line by using N-point DFT) is practically improbable [8], so we have to base the solution on the assumption of noncoinciding frequencies which involves the leakage effect.

Consider the case where the input signal $x(k)$ is in the form of a sum of p complex sinusoids:

$$x(k) = \sum_{i=1}^{p} A_i e^{j(\omega_i k + \Phi_i)} + \nu(k). \quad (17)$$

Note that the case of real sinusoids is a special case of (17). Fourier transform of this signal consists of p spectral lines (one for each complex sinusoid) and the transform of the noise term:

$$\tilde{x}(k) = \sum_{i=1}^{p} A_i e^{j\Phi_i} \delta(\omega - \omega_i) + \tilde{\nu}(k). \quad (18)$$

In the case of coinciding frequencies, filtering of the signal vector $x(k)$ by a set of appropriate column vectors of N-point DFT matrix would separate p spectral lines combined in $\tilde{x}(k)$ and the signal could be represented by p uncorrelated spectral domain samples. However, in the case of non-coinciding frequencies, due to the leakage effect, filtering of the input signal vector with any of the DFT column vectors yields a linear combination of all the terms present in the Fourier transform:

$$\tilde{x}_{jl}(k) = \mathbf{x}_{jl}(k)\mathbf{q}_l = \sum_{i=1}^{p} A_i e^{j\Phi_i} q_l(\omega_j) + \tilde{\nu}(\omega) q_l(\omega) \quad (19)$$

where $q_l(\omega_j) = \mathbf{q}_l(\omega = \omega_j)$. So, the input data matrix consists of correlated spectral domain data samples. The data can be decorrelated using the LS method and selecting the data correlation matrix rank of the order equal to p. Hence, the SL RLS algorithm with $L = p$ is adequate for this type of problem. It is also possible to use a modified Goertzel algorithm which makes it possible to calculate an N-point DFT at any value of the frequency (not only at integer multiples of the sampling frequency divided by N) [10].

4. SIMULATION RESULTS

In this section we present some typical examples of the simulation results showing the performance of the ALE comb filter based on the SL RLS algorithm. We consider the problem of removing multiple real sinusoidal signals from a composite input signal which consists of a sum of the broad-band and sinusoidal signal components:

$$x(k) = \sum_{i=1}^{p} A_i \sin(\omega_i k + \Phi_i) + \nu(k) \quad (20)$$

We use the example with $p = 3$ sinusoidal components with amplitudes $A_1 = 1, A_2 = 0.35, A_3 = 0.2$, phases $\Phi_1 = \Phi_2 = \Phi_3 = 0$, and frequencies $\omega_1 = 8.1(2\pi/N)$, $\omega_1 = 11.2(2\pi/N)$, $\omega_1 = 16.3(2\pi/N)$ where $N = 64$ represents the size of the DFT, and $\nu(n)$ is the uniformly distributed white noise with zero mean and variance 0.00001.

In Fig.1 the averaged convergence curves are shown for the three algorithms: SL RLS, APA and NLMS. For APA and the SL RLS algorithm the value of the stepsize parameter μ is selected to be equal to 0.1, while for NLMS algorithm it is set to 1. The adaptation is performed on a sample-by-sample basis. For APA the projection order $M = 6$. For SL RLS the spectral domain projection order $L = 6$ and the time domain projection order is $M = 6$. Simulation results show that the SL RLS algorithm converges much faster than the NLMS algorithm and that its performance is very similar to the performance of the full band APA algorithm. Selecting stepsize close to the value of 1 for APA and SL RLS algorithm increases the convergence rate of the algorithms but at the expense of increasing the magnitude of the residual. In Fig.2 the impact of the adaptation stepsize μ on the performance of the SL RLS algorithm is shown. Averaged convergence curves for the three values of the adaptation stepsize parameter 0.1, 0.04, 0.03 are shown. There is a tradeoff between the convergence speed and the misadjustment associated with the choice of this parameter but its impact is significant only for values of the stepsize close to 1. In Fig.3 the influence of the selection of the time domain projection order on the performance of the SL RLS algorithm is shown. Averaged convergence curves for the three values 6, 4, and 3 of the projection order parameter are shown. Simulations show strong impact of the time domain projection order on the convergence speed of the algorithm.

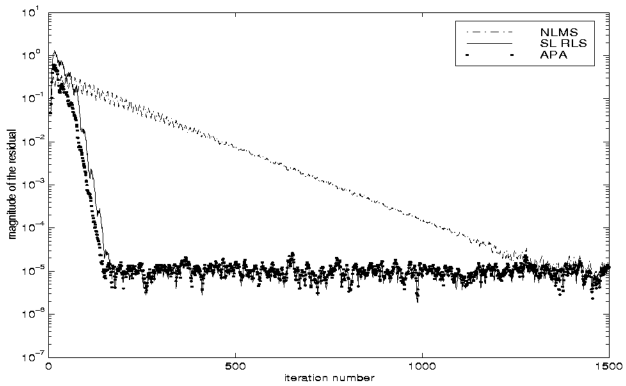

Figure 1: The convergence curves for ALE comb filter using three algorithms: NLMS, APA, and SL RLS.

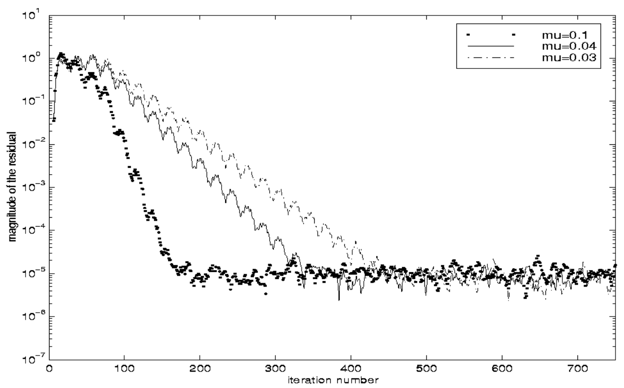

Figure 2: Convergence curves for ALE comb filter using SL RLS algorithm with different stepsizes: $\mu = 0.1$, $\mu = 0.04$, and $\mu = 0.03$.

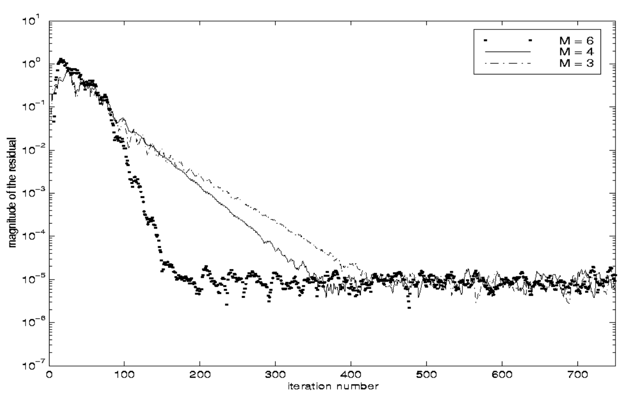

Figure 3: Convergence curves for ALE comb filter using SL RLS algorithm with different projection orders: $M = 6$, $M = 4$, and $M = 3$.

5. CONCLUSION

In this paper we have presented a new family of adaptive filtering algorithms based on the transform domain signal decomposition and linear least squares filtering of the selected subset of the transform domain signal components. We showed that the RLS method can efficiently solve the problem of crosscorelation between the non-ideally separated spectral components while transform domain decomposition makes it possible to reduce the spectral dimension of the input signal. This approach is particularly suitable for types of signals and applications where transform domain decomposition results in high energy compaction. The main characteristics of the resulting algorithms are fast convergence and computational efficiency. The performance of the method is shown in an example of ALE comb filter where DFT is used as transformation method. The focus of our further work will include the other types of transforms such as DCT or Wavelet transform.

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