A NEW ESTIMATION TECHNIQUE FOR NEAR-SHORE BATHYMETRIC MEASUREMENTS

S. Radhakrishnan Pillai

Zetron, Inc.
Advanced Development
P.O. Box 97004
Redmond, WA 98073
radha@matrix.poly.edu.

Hyun Seok Oh

Department of Electrical Engg.
Five MetroTech Center
Polytechnic University
Brooklyn, NY 11201
hyun@arma.poly.edu.

ABSTRACT

A new technique for depth estimation in airborne laser bathymetry is proposed. The technique involves the transmission of a nonlinear frequency-modulated signal, the detection of the signal reflected by the ocean, and its processing using an appropriate matched filter. On the basis of this technique, a receiver can be constructed that offers improved resolution between the signals reflected by the surface and bottom of the ocean which translates into improved accuracy of depth measurement.

1. INTRODUCTION

Acoustic echo sounding has dominated the field of bathymetry in the past. This technology has improved through the years with the introduction of more accurate and reliable equipment. Sonar systems based on acoustic echo sounding methods require surface vessels to carry them and thus the speed of acquisition of bathymetric data is limited by the speed of the vessel. Moreover, the survey of coastal waters is difficult if not impossible because hydrographic survey ships cannot operate safely in shallow waters [1].

The accuracy and reliability of depth estimation can be improved through the use of a variety of signal processing techniques [4]. In this paper, improve depth estimation is achieved through the use of a new technique that involves the transmission of a nonlinear frequency-modulation (FM) signal, the detection of the signal reflected by the ocean, and its processing using a matched filter. The proposed technique can be used to construct a receiver that improves the resolution between the surface and bottom reflections which results in improved accuracy of depth estimation.

2. RESOLUTION ENHANCEMENT

Let us assume that the target region is irradiated by a transmitter that sends a time-dependent signal $s(t)$. The leading edge of $s(t)$ reaches the first target (ocean surface) and after reflection it returns (or is echoed back) to the receiver T_1 seconds later. The remaining portion of the signal travels towards the second target (ocean bottom) and is also reflected back. The receiver detects the sum of the echoes from all the reflectors from the target scene. The received signal can be represented as

$$r(t) = \sum_{i=1}^{m} a_i s(t - T_i) + n(t), \quad 0 < t < T \tag{1}$$

where T_i is the two-way round trip delay to the ith target and $n(t)$ represents the noise present. In bathymetry, the signal-to-noise ratio (SNR) can be very poor and the problem is to estimate the unknown delays $T_1, T_2, \ldots, T_m$ from the received waveform $r(t)$.

When the transmitted signal $s(t)$ is a rectangular pulse of duration τ_0, then its Fourier transform is a sinc function with its first zero crossing at $\omega_0 = 2\pi/\tau_0$. Hence the bandwidth of $s(t)$, namely, ω_0, determines the range resolution. Clearly, when $s(t)$ is a rectangular pulse, one cannot achieve long duration in the time domain and wide bandwidth in the frequency domain since ω_0 is inversely proportional to the pulse duration τ_0.

Evidently, the shape and type of the signal transmitted is important in this application, and we propose to use a special type of FM signal known as *chirp* signal. When processed by an appropriate matched filter at the receiver, a chirp signal has a remarkable energy localization property. The phase angle change in a chirp signal $s_1(t)$ is a quadratic function of t, i.e.,

$$s_1(t) = \cos(\omega_0 t + bt^2), \quad 0 < t < T. \tag{2}$$

If we take the first derivative of the phase with respect to time, we get the instantaneous frequency as $\omega_0 + 2bt$. Thus in a chirp signal, the instantaneous frequency varies linearly with time.

Through this approach, an increase in τ_0 results in a better signal-to-noise ratio at the output of the receiver. To see this, consider the baseband signal $a(t)$ and let

$$b(t) = a(t)e^{-j\mu t^2}, \quad 0 < t < T \tag{3}$$

be the received signal. The duration of $b(t)$ is the same as that of $a(t)$ but its bandwidth is higher than that of $a(t)$. Note that $e^{-j\mu t^2}$ has a maximum instantaneous bandwidth of $2\mu T$, and hence the bandwidth of $b(t)$ is greater than $2\mu T$. Also since

$$|b(t)| = |a(t)|,$$

the energy in $b(t)$ and $a(t)$ are the same.

Consider the receiver which comprises a matched filter. The output of the matched filter, $g(t)$, can be obtained by using the convolution as

$$\begin{aligned} g(t) &= b(t) * h(t) = \int_{-\infty}^{+\infty} b(\tau)h(t-\tau)d\tau \\ &= e^{\jmath\mu t^2} \int_{-\infty}^{+\infty} a(\tau)e^{-\jmath 2\mu t\tau}d\tau = A(2\mu t)e^{\jmath\mu t^2} \quad (4) \end{aligned}$$

where $A(\omega)$ represents the Fourier transform of $a(t)$. Thus, the duration of the output pulse is given by

$$\text{width of } g(t) = \frac{\text{width of } A(\omega)}{2\mu}. \quad (5)$$

If the bandwidth of $A(\omega)$ is approximately equal to $1/T$, then we obtain the classical result

$$\text{width of } g(t) = \frac{1}{2\mu T}.$$

Thus by choosing μ large enough, the output pulse can be made as narrow as desired irrespective of the value of T. In effect, pulse compression by a factor γ given by

$$\gamma = \frac{\text{input pulse width}}{\text{output pulse width}} = \frac{T}{1/2\mu T} = 2\mu T^2 = \Delta\omega \cdot T \quad (6)$$

is achieved where $\Delta\omega \triangleq 2\mu T$ represents the instantaneous maximum bandwidth of the chirp signal $e^{-\jmath\mu t^2}$ at time $t = T$. Note that the compression factor can be increased by increasing the value of μ.

3. NONLINEAR FM TECHNIQUE

Interestingly, since the bandwidth of $A(\omega)$ in (5) plays a crucial role in the attainable pulse compression factor γ in (6), it is possible to reformulate (2) using a nonlinear FM technique. Towards this end, with $a(t)$ representing the same baseband signal as before, let the received signal be

$$c(t) \triangleq a(t)e^{-\jmath\mu t^3}, \quad 0 < t < T. \quad (7)$$

As before, the duration and energy of $c(t)$ are the same as those of $a(t)$ but its maximum instantaneous bandwidth is greater than $3\mu T^2$. If the matched filter has an impulse response

$$h_1(t) = e^{-\jmath\mu t^3}, \quad (8)$$

then the output of the matched filter, $g_1(t)$, is obtained as

$$\begin{aligned} g_1(t) &= c(t) * h_1(t) = \int_{-\infty}^{+\infty} c(\tau)h_1(t-\tau)d\tau \\ &= e^{-\jmath\mu t^3/4} \int_{-\infty}^{+\infty} a(\tau)e^{-\jmath 3\mu(\tau-\frac{t}{2})^2 t}d\tau. \quad (9) \end{aligned}$$

If we define

$$A_\mu(\omega) \triangleq \int_{-\infty}^{+\infty} a(t)e^{-\jmath\mu(t-\frac{\omega}{2})^2 t}dt \quad (10)$$

we notice that the bandwidth of $A_\mu(\omega)$ is different from that of $A(\omega)$. Using (10), (9) simplifies to

$$g_1(t) = A_{3\mu}(t)e^{-\jmath\mu t^3/4}. \quad (11)$$

Thus the output pulse width equals the width of $A_{3\mu}(t)$, and by selecting $a(t)$ and μ appropriately, it is possible to make the width of the output pulse as narrow as desired. As a result, by using a nonlinear FM signal, a higher pulse compression ratio can be realized, which would lead to improved energy localization.

3.1. Super-Chirp Signal

The FM signal

$$s_2(t) = \cos(\omega_0 t + bt^2 + ct^3), \quad 0 < t < T \quad (12)$$

where b and c are appropriate constants, is a desirable choice in practice, and the quadratic variation of the instantaneous frequency (the first derivative of the phase with respect to time) helps to strongly reject all other signals (and noise) at the output of the matched filter. We refer to $s_2(t)$ as a *super-chirp* signal.

With $b(t)$ representing the received signal, the desired receiver takes the matched filter. The output of the matched filter, $g(t)$, is given by

$$g(t) = b(t) * s_2(T-t). \quad (13)$$

The output $g(t)$ peaks at the unknown delays $T_1, T_2, \ldots, T_m$ and due to the energy compactification property in $s_2(t)$, these peaks are quite dominant. Thus an increase in the pulse duration τ_0 can simultaneously increase the overall power and bandwidth of the transmitted pulse. Thus an increased SNR will be achieved at the output of the matched filter which will result in improved resolution between the surface and bottom returns and, in consequence, to improved depth estimation.

3.2. Pulse Compression

In this context, we will develop an alternative method for signal pulse compression on the basis of the principles described. Unlike the linear FM chirp signal, the nonlinear FM super-chirp signal has a higher-order phase term which is proportional to t^3, in addition to a phase term which is proportional to t^2.

Let the nonlinear FM signal $b(t)$ be

$$b(t) = a(t)e^{-\jmath\mu t^2(1-\alpha t)}, \quad 0 \le t \le T. \quad (14)$$

In such a case, the impulse response $h(t)$ of the matched filter is of the form

$$h(t) = e^{\jmath\mu t^2(1+\alpha t)} \quad (15)$$

and its output $g(t)$ is given by the convolution of $b(t)$ and $h(t)$, i.e.,

$$g(t) = b(t) * h(t) = \int_{-\infty}^{+\infty} b(\tau)h(t-\tau)\,d\tau. \quad (16)$$

From (14) and (15), $b(t)$ and $h(t)$ are the standard chirp signal and the corresponding impulse response of the matched filter, respectively, if $\alpha = 0$. However, if $\alpha \neq 0$, then the modulation technique becomes nonlinear. In such a case, (16) can be further simplified to

$$g(t) = \int_{-\infty}^{+\infty} b(\tau)h(t-\tau)\,d\tau = e^{j\mu t^2(1+\alpha t)} A_{\mu,\alpha}(t) \quad (17)$$

where

$$A_{\mu,\alpha}(t) \triangleq \int_{-\infty}^{+\infty} a(\tau)e^{-j[2\mu t\tau + 3\alpha\mu t\tau(t-\tau)]} d\tau. \quad (18)$$

The integral in equation (18) cannot be evaluated in closed form. However, if $a(\tau)$ is sufficiently smooth or of a constant value, then a simple approximate solution can be found. Let μ represent the linear FM index and α the nonlinear modulation index. Clearly, if $\alpha = 0$, then

$$g(t) = e^{j\mu t^2} \int_{-\infty}^{+\infty} a(\tau)e^{-j2\mu t\tau}\,d\tau = e^{j\mu t^2} A(2\mu t) \quad (19)$$

where $A(\omega)$ represents the Fourier transform of $a(t)$. It can be shown that the limit in (18) tends to approximate a delta function and, in the present context, the special form of the delta function

$$\lim_{\gamma\to 0} \frac{1}{\gamma\sqrt{j\pi}} e^{j\tau^2/\gamma^2} = \delta(\tau) \quad (20)$$

turns out to be useful. This representation of $\delta(\tau)$ leads to the relation

$$\int_{-\infty}^{+\infty} \psi(\tau)e^{j(\tau-\tau_0)^2/\gamma^2}\,d\tau \approx \gamma\sqrt{j\pi}\psi(\tau_0) \quad (21)$$

as $\gamma \to 0$. With the help of equations (19) and (20), as $\alpha \to \infty$, equation (18) can be simplified further as

$$\begin{aligned} A_{\mu,\alpha}(t) &= e^{-j\frac{\mu t(3\alpha t+2)^2}{12\alpha}} \int_{-\infty}^{+\infty} a(\tau)e^{j3\alpha\mu t(\tau - \frac{3\alpha t+2}{6\alpha})^2}\,d\tau \\ &= e^{-j\frac{\mu t(3\alpha t+2)^2}{12\alpha}} Q(t) \quad (22) \end{aligned}$$

where

$$Q(t) \triangleq \int_{-\infty}^{+\infty} a(\tau)e^{j3\alpha\mu t(\tau - \frac{3\alpha t+2}{6\alpha})^2}\,d\tau. \quad (23)$$

To make use of the above approximation, let

$$\gamma(t) \triangleq \frac{1}{\sqrt{3\alpha\mu t}} \quad (24)$$

and

$$\tau_0(t) = \frac{3\alpha t+2}{6\alpha} = \left(\frac{t}{2} + \frac{1}{3\alpha}\right). \quad (25)$$

Note that $\tau_0(t) \to t/2$ as $\alpha \to \infty$. Using equations (24) – (25), $Q(t)$ in (23) simplifies to

$$Q(t) = \int_{-\infty}^{+\infty} a(\tau)e^{(\tau-\tau_0)^2/\gamma^2}\,d\tau \quad (26)$$

and as $\gamma \to 0$ we obtain the result

$$Q_0(t) = \lim_{\gamma\to 0} Q(t) = \sqrt{j\pi}\,\gamma(t)a[\tau_0(t)]. \quad (27)$$

Therefore, the matched filter output for a nonlinear FM signal $g(t)$ can be written as

$$\begin{aligned} g(t) &= \lim_{\alpha\to\infty} e^{j\mu t^2(1+\alpha t)} e^{-j\mu t(3\alpha t+2)^2/12\alpha} Q_0(t) \\ &= \lim_{\alpha\to\infty} \sqrt{j\pi} e^{j\mu t^2(1+\alpha t)} e^{-j\mu t(3\alpha t+2)^2/12\alpha} \left\{\gamma(t)a[\tau_0(t)]\right\}. \end{aligned} \quad (28)$$

Equation (28) shows that the envelop of the matched filter output $g(t)$ is given by $\gamma(t)a[\tau_0(t)]$ if $\alpha\mu t \to \infty$. Since $\gamma(t)$ rapidly approaches zero in this case, excellent output pulse compression can be realized.

3.3. Time-Bandwidth Relation

For a chirp signal, if the linear modulation index μ increases, then the matched filter linearly compresses the output signal $g(t)$ by a factor of $2\mu T$. However, the bandwidth of $G(\omega)$ also increases linearly by a factor of $2\mu T$. Therefore, the time-bandwidth relation of the output signal $g(t)$ can be changed linearly by a factor of the linear modulation index μ.

Now let us examine the time-bandwidth relation of the matched filter output for the case of a nonlinear FM signal. From (28), the bandwidth of the matched filter output is predominantly determined by $\gamma(t)$. From (24), the width of $\gamma(t)$ depends on α, μ, and t, and it decreases as α increases.

Note that the maximum instantaneous bandwidth (BW) of a chirp signal $e^{-j\mu t^2}$ at $t = T$ is given by

$$\mathrm{BW}_{chirp} = \frac{d}{dt}\mu t^2\Big|_{t=T} = 2\mu T. \quad (29)$$

For a nonlinear FM signal, the maximum instantaneous bandwidth at $t = T$ is given by

$$\begin{aligned} \mathrm{BW} &= \frac{d}{dt}\,\mu t^2(1+\alpha t)\big|_{t=T} = 2\mu T(1 + \frac{3\alpha T}{2}) \\ &= 2\mu T(1+\eta) = BW_{chirp}(1+\eta) \end{aligned} \quad (30)$$

where

$$\eta \triangleq \frac{3\alpha T}{2}. \quad (31)$$

Here, η represents the new bandwidth increment factor compared to that of the standard chirp signal.

If $a(t)$ is a rectangular pulse in the interval $-T/2 \leq t \leq T/2$, then

$$a(t) = \begin{cases} 1, & -T/2 \leq t \leq T/2, \\ 0, & \text{otherwise.} \end{cases} \quad (32)$$

The input signal $b(t)$ is given by

$$b(t) = a(t)\cos[\mu t^2(1-\alpha t)] \quad (33)$$

and the impulse response of the matched filter $h(t)$ assumes the form

$$h(t) = \cos[\mu t^2(1+\alpha t)]. \quad (34)$$

The output of the matched filter is given by the convolution $b(t) * h(t)$, as before. Fig. 1 shows the matched filter output $g(t)$ for $\mu = 50$, $T = 1$, and various values of α. If $\alpha = 0$, from (19), $g(t)$ is the output of the standard chirp signal (see Fig. 1(a)). As α increases, $g(t)$ depends only on the function $\gamma(t)$ (see Fig. 1(d)). If $\alpha > 0$, then $g(t)$ becomes more compressed. And if α becomes sufficiently large, then the envelope of $g(t)$ follows the function $\gamma(t)$.

3.4. Signal Separation

It would be interesting to address the issue of signal separation at this point. If the return signals are very close to each other, the matched filter may not separate the signals. Recall that for a rectangular input envelope, the width of the matched-filter output for a chirp signal is $T_0 = 1/(2\mu T)$ and if two signal components are separated by $\Delta < T_0$ then, for a given μ, the standard chirp will not resolve these components. However, there is no problem if the proposed nonlinear FM signal technique is used owing to the improved resolution of $\gamma(t)$. This is demonstrated in Fig. 2 where $g(t)$ is plotted for various values of α for the case of two signal returns, one at $T_1 = 0.0$ and the other at $T_2 = 0.05$ s. If $\alpha = 0$, which corresponds to the case of a standard chirp signal, the matched filter does not separate the two signals. However, as α is increased, the two signals begin to separate for values of α greater than 1 or 2 and are well separated for values of α above 5 (see Fig. 2(c) and (d)).

To conclude, we have developed a new technique for pulse compression based on a nonlinear frequency modulation. The technique entails a nonlinear modulation index α, which can be used to control the duration of the output pulse. The technique can be used to resolve several signal returns whose output pulse width is less than the width of the chirp pulse by simply selecting the appropriate value of α.

By utilizing an FM pulse, we can achieve the frequency-spread characteristics of a short pulse within the envelope of a long duration signal. Thus by using a quadratic FM pulse with the frequency span set by the resolution required and with the duration set by the energy required for the range of depths to be measured, we can enhance the resolution between the surface and bottom returns of the blue-green laser signal thereby increasing the accuracy of depth measurement.

Other applications of the proposed technique might include detection and estimation of fish stocks and the measurement of sea-water turbidity and coastal pollution.

4. CONCLUSIONS

A new technique for the estimation of time delay for ocean depth measurement has been introduced. The technique involves the transmission of a chirp-like signal and the processing of the reflection by an appropriate matched filter. The proposed scheme improves the resolution between the blue-green surface and bottom returns thereby enhancing the accuracy of the depth measurement. Results obtained so far show that the technique works very well for return signals that are heavily corrupted with noise or noise-like clutter that can originate from any number of sources.

5. REFERENCES

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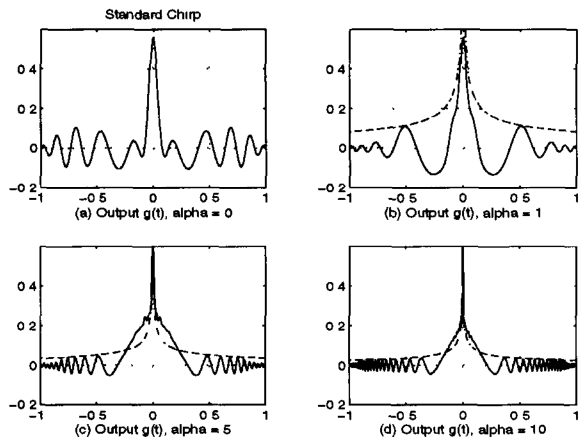

Figure 1: Comparison of matched filter output $g(t)$ (solid) and $\gamma(t)$ (dashed) for the various values of α ($\alpha = 0, 1, 5, 10$).

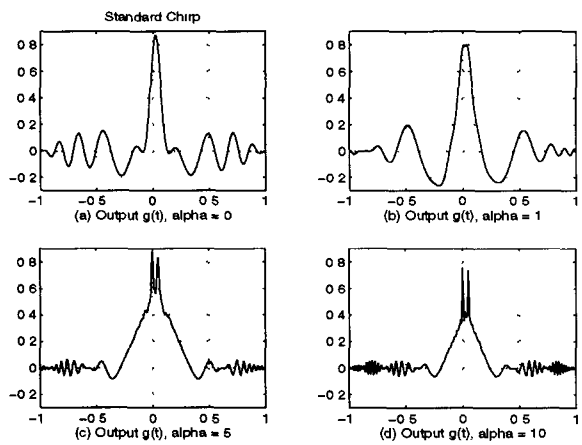

Figure 2: Comparison of the matched filter output $g(t)$ for the two signal returns ($T_1 = 0.0$, $T_2 = 0.05$) for the various values of α ($\alpha = 0, 1, 5, 10$).