SPATIAL AVERAGING OF TIME-FREQUENCY DISTRIBUTIONS

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ABSTRACT

This paper presents a novel approach based on time-frequency distributions (TFDs) for separating signals received by a multiple antenna array. This approach provides a significant improvement in performance over the recently introduced spatial time-frequency distributions, specifically for signals with close time-frequency signatures. In this approach, spatial averaging of the time-frequency distributions of the sensor data is performed to eliminate the interactions of the sources signals in the time-frequency domain, and as such restore the realness property and the diagonal structure of the source TFDs, which are necessary for source separation. It is shown that the proposed approach yields improved performance over both cases of no spatial averaging and averaging using time-frequency smoothing kernels.

1. INTRODUCTION

In this paper, we introduce a new technique for source separation based on time-frequency distribution methods. The sources have different time-frequency signatures and instantaneously mixed at the array sensors. The number of sensors is assumed to be equal to or greater than twice the number of sources. The time-frequency distributions (TFDs) of the data across the array are computed and used to construct spatial time-frequency distribution matrices (STFDs). By forcing the hermition Toeplitz structure of the STFDs and perform spatial symmetric averaging over two parts of the array, we achieve significant improvement of source separation over the case where no spatial averaging is performed.

Recently, time-frequency distributions have been applied to direction finding and blind source separation problems in array processing. The spatial time-frequency distributions are introduced in [1] and represented by a spatial matrix whose elements are the time-frequency distributions of the data across the multi-sensor array. The successful application of STFDs to separating sources with identical spectra, but different time-frequency signatures, is shown in [2]. In this application, STFD matrices computed at different t-f points are incorporated into a joint-diagnalization technique based on generalized Jacobi transform to estimate the mixing, or array manifold, matrix. This matrix is then used to estimate the sources' signals up to a multiplicative complex scalar and the order of the sources. The general theory of solving blind source seperation problems using spatial arbitrary joint variable distributions, including those of time and frequency, is given in [3]. In [4], the two arbitrary variables are chosen as the time-lag and frequency-lag, and the source separation was performed using spatial ambiguity functions. The use of STFDs as an eigenstructure-based approach for direction finding is given in [5], where the Time-Frequency MUSIC technique is proposed to estimate the signal and noise subspaces.

The importance of joint-diagonalization (JD) in the STFD context is that the diagonal structure, the distinct eigenvalues, and the full rank properties of the signal TFD matrix, necessary for source separation, can be easily violated when operating with a single t-f point. The cross time-frequency distributions of the source signals yield non-zero complex values at the off-diagonal elements, rendering the estimation of the mixing matrix difficult, or even impossible. Also, the noise contribution to all matrix elements at low SNR cannot be ignored. As the interactions of the source signals vary over the time-frequency plane, the incorporation of several STFD matrices at different t-f points into JD enhances diagonalization and leads to a successful separation of signal arrivals. It is noted that the primary motivation of using smoothing kernels and resorting to other variables than time and frequency, specifically the ambiguity-domain variables, is to allow the selection of joint-variable points where the interactions of the source signals are insignificant.

The fundamental role of the proposed technique of symmetric spatial averaging of STFDs is the effective elimination of the signals' intermodulations. It effectively restores the diagonal structure and realness property of the signal TFD matrix. Symmetric spatial averaging is a simple, well-known technique in conventional array processing [6]. It uses additional array sensors to reduce cross-correlation in coherent and correlated signal environments, and thereby permits proper angle-of-arrival estimations and source separations. It is shown that adopting this technique in the underlying TFD-based source separation JD problem gives robustness to t-f point selections and leads to improved performance over other TFD-based techniques, specifically for sources whose time-frequency signatures are not very distinct.

2. SPATIAL TIME-FREQUENCY DISTRIBUTIONS

The data vector for N-element array is given by

$$\mathbf{x}(t) = \mathbf{y}(t)+\mathbf{n}(t) = \mathbf{A}\mathbf{s}(t)+\mathbf{n}(t). \tag{1}$$

In vector forms, $\mathbf{x}(t)=[x_0(t), \ldots, x_{N-1}(t)]^T$ is a noisy instantaneous linear mixture of the source signals $\mathbf{s}(t)=[s_1(t), \ldots, s_n(t)]^T$ and $\mathbf{n}(t)$ is the additive noise. The mixing matrix $\mathbf{A}$ is the transfer function between the sources and the array sensors.

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The discrete-time form of Cohen's class of TFD for signal $x(t)$ is given by [7]

$$D_{xx}(t,f)=\sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\phi(m,l)x(t+m+l)x^*(t+m-l)e^{-j4\pi fl} \quad (2)$$

where t and f represent the time index and the frequency index, respectively. The kernel $\phi(m, l)$ characterizes the TFD and is a function of both the time and lag variables. The cross-TFD of two signals $x_i(t)$ and $x_j(t)$ is defined by

$$D_{x_ix_j}(t,f)=\sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\phi(m,l)x_i(t+m+l)x_j^*(t+m-l)e^{-j4\pi fl} \quad (3)$$

The spatial time-frequency distribution (STFD) incorporates both equations (2) and (3), and is defined in [2] by,

$$\mathbf{D}_{\mathbf{xx}}(t,f)=\sum_{l=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}\mathbf{\Phi}(m,l)\otimes\mathbf{x}(t+m+l)\mathbf{x}^H(t+m-l)e^{-j4\pi fl} \quad (4)$$

where $\left[\mathbf{D}_{\mathbf{xx}}(t,f)\right]_{i,j}=D_{x_ix_j}(t,f)$, for i, j=0, …, N-1, $\otimes$ denotes the Hadamard product, and $\left[\mathbf{\Phi}(m,l)\right]_{i,j}=\phi_{i,j}(m,l)$ is the time-frequency kernel associated with the pair of the sensor data $x_i(t)$ and $x_j(t)$. Under the linear data model of Eq. (1) and assuming noise-free environment, the STFD matrix takes the following simple structure

$$\mathbf{D}_{\mathbf{xx}}(t,f)=\mathbf{A}\mathbf{D}_{\mathbf{ss}}(t,f)\mathbf{A}^H \quad (5)$$

where $\mathbf{D}_{\mathbf{ss}}(t,f)$ is the signal TFD matrix whose entries are the auto- and cross-TFDs of the sources. Eq. (5) is similar to that commonly used in conventional blind source separation and direction-of-arrival (DOA) estimation problems [8,9], relating the signal correlation matrix to the data spatial correlation matrix. If $\mathbf{D}_{\mathbf{ss}}(t,f)$ is a full-rank matrix, the two subspaces spanned by the principle eigenvectors of $\mathbf{D}_{\mathbf{xx}}(t,f)$ and the columns of $\mathbf{A}$ become identical. In this case, directional finding techniques based on eigenstructures can be applied. If $\mathbf{D}_{\mathbf{ss}}(t,f)$ is diagonal, i.e., the signal cross-TFDs at the time-frequency point (t,f) are zeros, the mixture matrix and the signal waveform can be recovered using blind source separation methods [1,2]. In these methods, in order to avoid potential problems associated with using a single STFD, STFDs at different (t,f) points are incorporated into a joint-diagonalization scheme. Although JD of the STFDs is effective in most cases, signals with close time-frequency signatures are still difficult to separate. As shown below, spatial averaging can be used to facilitate signal separation.

3. SPATIAL AVERAGING TIME-FREQUENCY DISTRIBUTIONS

Symmetric spatial averaging method was proposed by Pillai [6] to restore the full-rank property of the signal covariance matrix in the presence of coherent signals. In this section, we extend the spatial averaging method to TFD analysis, and propose the signal separation method by joint diagonalization (JD) based on spatial averaging TFDs.

Without loss of generality, we consider M=2, i.e., only two sources, $s_1(t)$ and $s_2(t)$. The result can be easily extended to multiple sources. By ignoring the effect of noise, the received signal at i-th array sensor is represented as

$$x_i(t)=x_i^{(1)}(t)+x_i^{(2)}(t)=s_1(t)e^{-jd_i\omega_1}+s_2(t)e^{-jd_i\omega_2} \quad (6)$$

where $\omega_k=2\pi\sin\phi_k/\lambda$ (k =1,2) is the spatial radian frequency, λ is the RF wavelength, and d_i is the distance between 0-th and i-th array sensors. We assume the array is equi-spaced linear array. The cross-TFD of $x_i(t)$ and $x_j(t)$ is

$$\begin{aligned}
&D_{x_ix_j}(t,f)\\
&=D_{x_i^{(1)}x_j^{(1)}}(t,f)+D_{x_i^{(2)}x_j^{(1)}}(t,f)+D_{x_i^{(2)}x_j^{(2)}}(t,f)+D_{x_i^{(1)}x_j^{(2)}}(t,f)\\
&=\left[D_{s_1s_1}(t,f)+D_{s_2s_1}(t,f)e^{-jd_i(\omega_2-\omega_1)}\right]e^{-j(d_i-d_j)\omega_1}\\
&\quad+\left[D_{s_2s_2}(t,f)+D_{s_1s_2}(t,f)e^{jd_i(\omega_2-\omega_1)}\right]e^{-j(d_i-d_j)\omega_2}
\end{aligned} \quad (7)$$

Since the cross-terms (second term in each bracket in (7)) are generally complex, it is clear that the TFD matrix $\mathbf{D}_{\mathbf{xx}}(t,f)$ will not provide proper phase information for recovering the DOA of the arrived signals when cross-terms are present. However, such phase information can be restored by using spatial averaging methods. The spatial averaging of TFD allows the signal separation even when the TFDs of multiple signals have very similar shapes and are highly overlapping.

Let the number of array sensors be $2N$-1with the array center is the zeroth sensor, as shown in Fig.1. The TFD of $x_0(t)$ and $x_i(t)$, i= 0, 1, 2, …, N-1, is

$$\begin{aligned}
&D_{x_0x_i}(t,f)\\
&=\left[D_{s_1s_1}(t,f)+D_{s_2s_1}(t,f)\right]e^{jd_i\omega_1}+\left[D_{s_2s_2}(t,f)+D_{s_1s_2}(t,f)\right]e^{jd_i\omega_2}
\end{aligned} \quad (8)$$

where we note d_0=0. Similarly, the TFD of $x_0(t)$ and $x_{-i}(t)$ is

$$\begin{aligned}
&D_{x_0x_{-i}}(t,f)\\
&=\left[D_{s_1s_1}(t,f)+D_{s_2s_1}(t,f)\right]e^{-jd_i\omega_1}+\left[D_{s_2s_2}(t,f)+D_{s_1s_2}(t,f)\right]e^{-jd_i\omega_2}
\end{aligned} \quad (9)$$

The spatial averaging of (8) and (9) is given by

$$\tilde{D}_{xx}^{(i)}(t,f)=\left\{D_{x_0x_i}(t,f)+D_{x_0x_{-i}}^*(t,f)\right\}/2=b_1e^{jd_i\omega_1}+b_2e^{jd_i\omega_2} \quad (10)$$

where

$$b_1=D_{s_1s_1}(t,f)+\mathrm{Re}\left(D_{s_2s_1}(t,f)\right)$$

$$b_2=D_{s_2s_2}(t,f)+\mathrm{Re}\left(D_{s_1s_2}(t,f)\right)$$

Since the terms in the brackets are all real, the TFD in (10) correctly represents the phase information caused by the propagation delay between array sensors, even when the cross-terms are complex. The matrix formed from the TFDs (10)

$$\tilde{\mathbf{D}}_{\mathbf{xx}}(t,f)=\begin{bmatrix}\tilde{D}_{xx}^{(0)}(t,f) & \tilde{D}_{xx}^{(1)}(t,f) & \cdots & \tilde{D}_{xx}^{(N-1)}(t,f)\\ \tilde{D}_{xx}^{(1)*}(t,f) & \tilde{D}_{xx}^{(0)}(t,f) & \cdots & \tilde{D}_{xx}^{(N-2)}(t,f)\\ \vdots & \vdots & \ddots & \vdots\\ \tilde{D}_{xx}^{(N-1)*}(t,f) & \tilde{D}_{xx}^{(N-2)*}(t,f) & \cdots & \tilde{D}_{xx}^{(0)}(t,f)\end{bmatrix} \quad (11)$$

is hermitian and Toeplitz. It is referred to as the spatial averaging TFD (SATFD) matrix. In the noise-free environment, the SATFD matrix can be expressed as

$$\tilde{\mathbf{D}}_{\mathbf{xx}}(t,f) = \mathbf{A}\tilde{\mathbf{D}}_{\mathbf{ss}}\mathbf{A}^H \quad (12)$$

where

$$\tilde{\mathbf{D}}_{\mathbf{ss}}(t,f) = \mathrm{diag}[b_1 \quad b_2] \quad (13)$$

are the equivalent TFD of the signal vectors. Note that $\tilde{\mathbf{D}}_{\mathbf{ss}}(t,f)$ no longer expresses the actual TFD. Clearly, (12) has the same format as (5), and $\tilde{\mathbf{D}}_{\mathbf{ss}}(t,f)$ here is diagonal even when the cross-terms of the TFD of the signals are present. Therefore, the spatial averaging method will ensure the validity of the TFD-based signal separation in the presence of cross-TFD.

4. SIMULATION RESULTS

Equi-spaced 5-element linear array is used for simulation with the interelement spacing 0.5λ. When spatial averaging method is used, two sub-arrays are formed, each with 3 elements. Two sources of chirp signals

$$s_1(t) = e^{-j\mu\frac{t^2}{2}}, \; s_2(t) = e^{-j\mu\frac{t^2}{2} - j\omega t} \quad (14)$$

are used, where μ and ω are chosen to be 0.008π and 0.02π, respectively. The DOAs of the two signals are assumed 30° and 60° from the broadside direction. No noise is considered here.

Fig.2(a) shows the Wigner-Ville distribution of each source signal, and Fig.2(b) shows the respective distributions after signal separation. It is clear that the array fails to separate $s_1(t)$ and $s_2(t)$.

In the TFD-based signal separation method, applied in Fig. 2, three points (t, f) are used for joint diagonalization at t = 32, 64, and 96. The frequency f is chosen so that the TFD at the first array sensor is the largest for a given t.

To show the effect of using a smoothing kernel, similar simulation is performed with the Choi-Williams kernel [10] with σ = 0.1. The result is shown in Fig.3. A rectangular window with 31 samples in both time and frequency scale is used. Since the two signals are closely spaced in the t-f domain, the cross-terms reduction furnished by the Choi-Williams kernel is limited, and again the array fails to separate the two signals.

Fig.4 shows the separated signals under the same conditions when the proposed spatial averaging method is applied. The signals are perfectly separated, except for their order.

5. CONCLUSIONS

Symmetric averaging of spatial time-frequency distributions has been introduced. The averaging improves the performance of source separation using joint-diagonalization techniques. It amounts to forming a spatial hermition Toeplitze matrix using the time-frequency distributions of the data across one half of the array. This matrix is then added to the spatial matrix corresponding to the other half of the array. The effect of this averaging is to remove interaction between the source signals in the time-frequency domain. Joint digonalization (JD) using a generalization of Jacobi transform is then applied to estimate the mixing matrix. By reducing the interaction of the source signals, the JD algorithm yields improved performance over the case when no averaging is performed. The paper presented an example of separating two chirps signals whose time-frequency signatures are slightly different. The proposed approach has successfully separated the two signatures, while other non-averaging methods fail.

6. REFERENCES

[1] A. Belouchrani and M. Amin, "Source separation based on the diagonalization of a combined set of spatial time-frequency distribution matrices," in *Proc. IEEE ICASSP'97*, Germany, April 1997.

[2] A. Belouchrani and M. Amin, "Blind source separation based on time-frequency signal representation," *IEEE Trans. Signal Processing*, Nov. 1998.

[3] A. Belouchrani and M. Amin, "Blind source separation using joint signal representations," in *Proc. SPIE Conf. on Advanced Algorithms and Architectures for Signal Processing*, San Diego, CA, Aug. 1997.

[4] M. Amin and A. Belouchrani, " Blind source separation using the spatial ambiguity functions," in *Proc. IEEE Int. Symp. on Time-Frequency and Time-Scale Analysis*, Pittsburgh, Pennsylvania, Oct. 1998.

[5] M. Amin and A. Belouchrani "Time-frequency MUSIC: an array signal processing method based on time-frequency signal representation," in *Proc. SPIE Conf. on Radar Processing, Technology and Applications*, San Diego, CA, July 1997.

[6] S. U. Pillai, *Array Signal Processing*, Springer-Verlog, 1989.

[7] L. Cohen, *Time-frequency Analysis*, Prentice Hall, 1995.

[8] L. Tong, Y. Inouye, and R-W. Liu, "Waveform-preserving blind estimation of multiple independent sources," *IEEE Trans. Signal Processing*, vol,41, no.7, pp.2461-2470, July 1993.

[9] A. Belouchrani, K. A. Meraim, H.-F. Cardoso, and E. Muylines, "A blind source separation techniques using second order statistics," *IEEE Trans. Signal Processing*, vol.45, no.2, pp.434-444, Feb. 1997.

[10] H. I. Choi and W. J. Williams, "Improved time-frequency representation of multicomponent signals using exponential kernels," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, no.6, pp.862-871, June 1989.

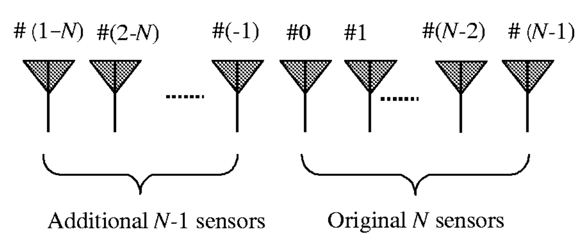
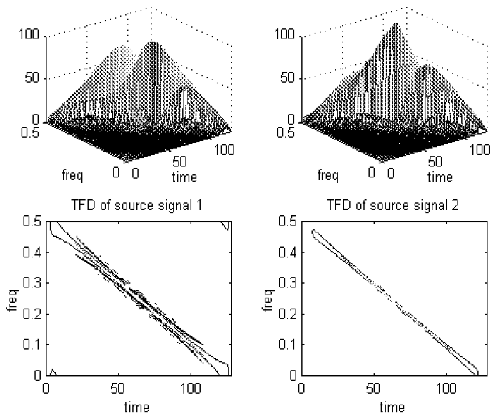
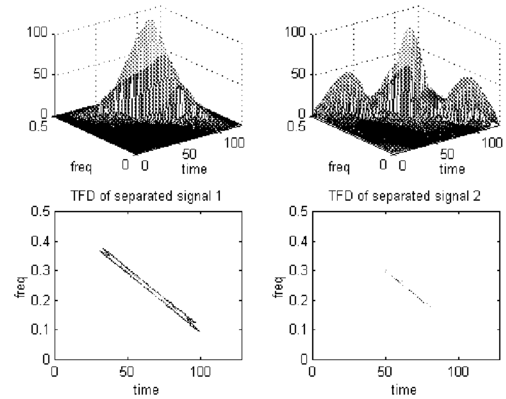


Fig.1 Array configuration for spatial averaging

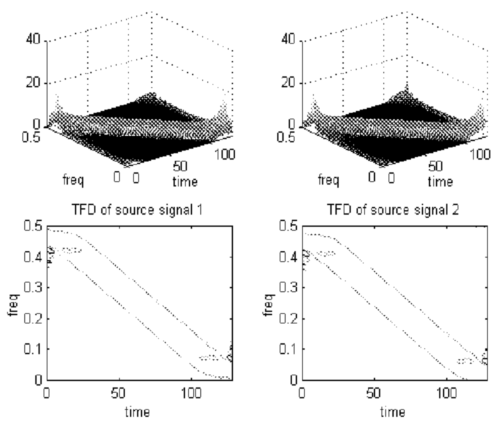


(a) TFD of the sources

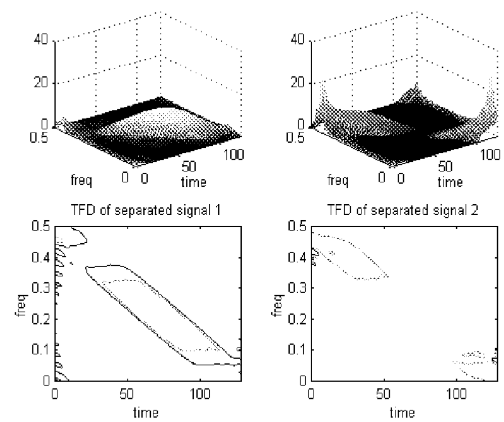


(b) TFD of the separated signals

Fig.2 TFD of the sources and the separated signals using Wigner-Ville distribution

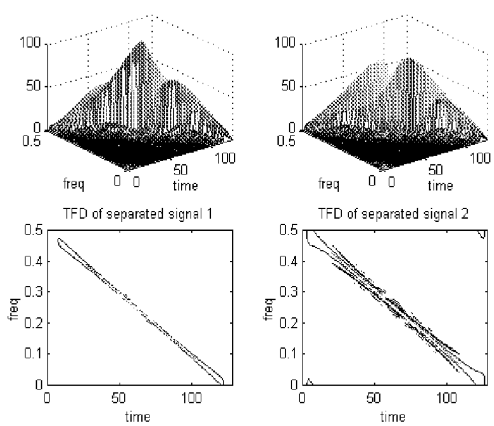


(a) TFD of the sources

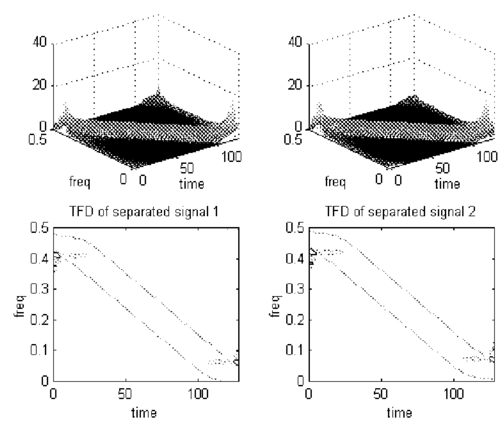


(b) TFD of the separated signals

Fig.3 TFD of the sources and the separated signals using Choi-Williams distribution



(a) Wigner-Ville distribution



(b) Choi-Williams distribution

Fig.4 Separated signals with spatial averaging