

TEXTURE ANALYSIS OF AN IMAGE BY USING A ROTATION-INVARIANT MODEL

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ABSTRACT

Texture analysis is an important problem in image processing because it conditions the quality of image segmentation and interpretation. We propose in this communication a texture model which is invariant by rotation and whose parameters allow to characterize at the same time the type of texture and its tonal primitive. The originality of the model proposed lies in the use of the Wold decomposition to modelize the 1D normalized autocovariance. This function is computed from the 2D normalized autocovariance of a texture. Finally, parameters of the model are estimated by using a genetic algorithm. Experimental results on textures from the Brodatz album and synthetic textures show a modeling error lower than 0.06.

1. INTRODUCTION

Natural image processing put in obviousness the necessity to build reliable models for image analysis taking into account some textured regions for a posterior interpretation. Nevertheless, the definition of texture remains unclear in the literature [7], [9]. The texture is usually defined following either two approaches:

- descriptive approach : a texture is described through several characteristic properties :
 - “A texture is a region of an image, for which a window with minimal dimensions can be defined, such that the visual perception of an observation inside the window, is the same for all possible translation of the window inside the region” [9],
 - the notion of texture is only valid for a set of resolutions [8].
- constructive approach : according to Haralick [4], a texture can be decomposed following two dimensions. The first concerns the description of the tonal primitives that compose it and the second defines the interactions between these tonal properties. He distinguishes three types of texture :
 - structural or macroscopic textures: these textures have a regular spatial structure generated by the repetition of a primitive,
 - random or microscopic textures: the texture is considered as the realization of a 2D stochastic process where it is difficult to define an elementary primitive,

- hybrid textures : the interactions between the tonal primitives is random.

We opt for this last definition. However, the notion of a texture's tonal primitive remains vague. We propose here to precise the definition of the texture tonal primitive as :

“The tonal primitive of a texture is the smallest spatial structure in a texture whose shape is governed by the same probability density within a texture”.

These two approaches have led to the elaboration of the two main techniques of texture analysis. The first consists in analyzing a texture using attributes of the grey level co-occurrence or the run-length matrix [4]. This characterization of textures does not always give a complete description of the texture and necessitates a great number of parameters. The second approach modelizes the texture as the realization of a 2D random field [3], [2]. Hypothesis of utilization of these models limit their applications to certain types of textures.

To avoid these drawbacks, we propose in this paper to analyze a texture from its normalized autocovariance function. The Wold decomposition of the autocovariance function allows to modelize efficiently the different textures. Moreover, parameters of the model can be used to identify textures by considering their tonal primitive (microscopic, macroscopic) and their deterministic or stochastic characteristic. This model of homogeneous texture is also invariant by rotation. Parameters of the model are estimated by using a genetic algorithm.

2. DEVELOPPED METHOD

In this section, we propose a model of the 1D normalized autocovariance function by using the Wold decomposition. We compute this function from the 2D normalized autocovariance of the texture. Finally, we present the method used to estimate the parameters of the model.

2.1. Wold decomposition of the 1D normalized autocovariance

The Wold decomposition of a homogeneous random field is the three superposition of mutually orthogonal components : a purely-indeterministic field, a generalized evanescent field and a harmonic field [8]. We propose to modelize the 1D normalized autocorrelation by using the Wold decomposition. This function is often used to evaluate spatial dependencies between some pairs of pixels. In

the case of a centered image, this function is defined as the autocovariance

A 1D harmonic field I_1 can be modeled by a sum of p harmonics :

$$I_1(r) = \sum_p \gamma_p \sin(2\pi f_p r + \phi_p)$$

where γ_p is the p th harmonic component of the field I_1 , f_p the associated frequency and ϕ_p the phase.

The autocovariance of a harmonic field I_1 is given by :

$$FAC_{I_1}(r) = \gamma_0^2 + 2 \sum_{p>0} \gamma_p^2 \cos(2\pi f_p r + \phi_p)$$

Similarly, the autocovariance of a purely-indeterministic field I_2 is a function that can be modeled as follows [3] :

$$FAC_{I_2}(r) = e^{-\alpha r}$$

A 1D generalized evanescent field I_3 can be modeled as :

$$I_3(r) = s(r) \cdot \sum_p \gamma_p \sin(f_p r + \phi_p)$$

where s is a 1D random process.

The associated autocovariance function can be defined as :

$$FAC_{I_3}(r) = e^{-\alpha_1 r} + e^{-\alpha_2 r} \sum_p \gamma_p \cos(2\pi f_p r + \phi_p)$$

The autocovariance of a generalized evanescent field is characterized by a fast decrease for small displacement distances and by oscillations of lessened amplitude.

Thus, the 1D autocovariance function of a random field I can be modeled as following :

$$FAC_I(r) = e^{-\alpha r} + \gamma \cdot e^{-\beta r} \cos(2\pi f r + \phi) + \delta + \epsilon(r) \quad (1)$$

The term $\epsilon(r)$ corresponds to the modeling error of the normalized autocovariance function.

In the case of a purely-indeterministic field, the second term of this expression has to be insignificant (that is realized from a value of δ very small and a high value of β or γ small). In the harmonic case, the coefficient β has to be close to 0. Finally, the evanescent case is characterized by an average value of $\beta > 0$ for modelize amortized oscillations present in the normalized autocovariance function.

The size of the texture tonal primitive can be estimated from the parameters of the model. In the harmonic case, the size of the texture tonal primitive is approached by the period $1/f$. In others cases, M the tonal primitive size is approached by considering the distance for which $F(M)$ becomes insignificant.

2.2. Computation of the 1D normalized autocovariance of a texture

We present a method to compute the 1D normalized autocovariance of a texture from its 2D normalized autocovariance function. The resulting function is called F and is computed as :

$$F(r) = \frac{1}{\pi r} \sum_{(i,j) \in \hat{C}_r} F\tilde{A}C(i, j) \quad \forall r > 0$$

where $\hat{C}_r = \{(i, j) / i > 0, \sqrt{i^2 + j^2} = r\}$ is the set of points on the semi-circle of radius r . Let be $m = \sqrt{i^2 + j^2}$, $F\tilde{A}C$ is defined as :

$$F\tilde{A}C(i, j) = \begin{cases} FAC(i, j) & \text{if } m \in \mathbb{N} \\ \frac{\sum_{k=1}^v d(s^{(k)}, m) I(s^{(k)})}{\sum_{k=1}^v d(s^{(k)}, m)} & \text{otherwise} \end{cases}$$

The first term is the 2D normalized autocovariance function defined as :

$$FAC(i, j) = \frac{1/N_{\Delta} \sum_{s=(k,l) \in R_{\Delta}} I(s) I(k+i, l+j)}{1/N \sum_{k,l} I^2(k, l)}$$

where N_{Δ} and R_{Δ} are respectively the number of points and the region on which the product $I(s)I(k+i, l+j)$ is computed. N is the total number of pixels, I is the luminance function (its mean is equal to zero), i and j are the horizontal and vertical displacement.

The second term of this expression corresponds to the interpolation of the point m on the circle $\hat{C}_r$ and the term $d(s^{(i)}, m)$ corresponds to the Euclidean distance between two sites of the image to analyze (see Figure 1). The value v defines the considered number of neighbors in the interpolation phase.

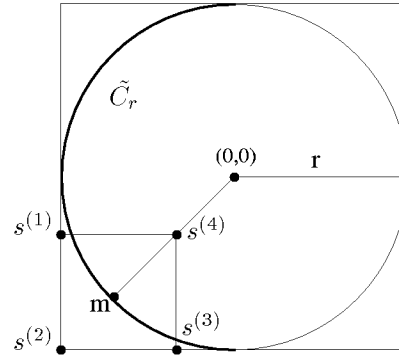


Figure 1: Interpolation of a point on $\hat{C}_r$

Let $F(r)$ be the average correlation of a pixel with others distant of r in all directions, this function is defined as an 1D autocorrelation function. Indeed, it is easy to show that this function is invariant by translation (the F function is computed from the 2D autocovariance which has this property). In addition, the $F(r)$ function allows a rotation invariant analysis of the considered texture. This property is obtained by taking into account correlations

in all directions.

The F function allows us to estimate the parameters of the model of equation (1).

2.3. Model parameters estimation

The F function previously computed is analyzed as an 1D autocorrelation function. Parameters of the model are estimated by minimizing the following function :

$$\sum_{i=1}^{R_q} (F(r_i) - \hat{F}(r_i))^2$$

where R_q is the maximal radius value of analysis and $\hat{F}$ the model of the function F .

So as to determine parameters minimizing this functional, we used a genetic algorithm [6] (GAOT : Genetic Algorithm for Optimization Toolbox). This approach is particularly adapted to our problem since the function to minimize is not convex (because of the presence of a cosine in the model). Other approaches such as conjugated gradient or simulated annealing would give in most cases only local extrema [5].

Genetic algorithms determine solutions of a function by simulating the evolution of a population until only the most adapted individuals survive. The choice of an important initial population guarantees the convergence to a global minimum. The survivors are individuals obtained by crossing over, mutation or selection of individuals of the previous generation.

3. EXPERIMENTAL RESULTS

We have tested this model on a representative texture database from the Brodatz album [1] and synthetic textures composed of 64 images. We have computed the normalized autocovariance function for each texture of this database as well as the associated function $r \mapsto F(r)$.

We give as illustration the result of the modeling (see Figure 4) for some textures (see Figure 5). Our experimentations show the stability of the model parameters for similar textures. The modeling error for all textures in the database is lower than 0.06 (see Table 1).

Nevertheless, the model that we propose allows to consider only one harmonic component in the considered texture. When this hypothesis is not verified, the period estimation is wrong.

The Figure 3 shows two examples of invariance by rotation of the proposed model applied to two textures for four orientations ($\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$). The error committed due to orientation ($\sum_{i=1}^{R_q} (F^0(r_i) - F^\theta(r_i))$, $\theta = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$) in these two cases is lower than 0.05.

4. CONCLUSION AND PERSPECTIVES

We have proposed a rotation invariant texture model. It has been obtained from a Wold-like model of the 1D normalized autocovariance. The Parameters of the model have been computed by using a

texture	α	γ	β	$T = \frac{1}{f}$	ϕ	δ	ϵ
noise	3.21	0.02	0.25	7.73	48.71	0.	0.
sand	0.33	0.22	0.18	32.34	39.36	0.	0.
water	0.42	0.13	0.10	44.53	42.07	0.02	0.
bubbles	0.22	0.16	0.25	13.34	33.30	0.01	0.01
chess	0.44	0.30	0.02	22.33	36.48	0.	0.06
canvas	0.82	0.17	0.09	8.59	36.36	0.	0.02

Table 1: Parameters of the model for different textures

genetic algorithm.

Experimental results show that the 1D normalized autocovariance allows to analyze a texture. Moreover, the proposed model provides a better discrimination of textures by considering their types (deterministic, stochastic) and their tonal primitive (microscopic, macroscopic). These informations will facilitate subsequent treatments particularly in image segmentation and interpretation.

Perspectives of this study concern the computation of the number of harmonic components of the texture. This model can also be used in texture segmentation or in image retrieval.

5. REFERENCES

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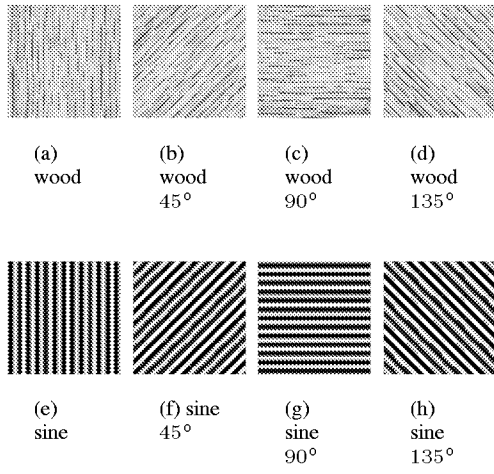


Figure 2: Two textures with 4 different orientations

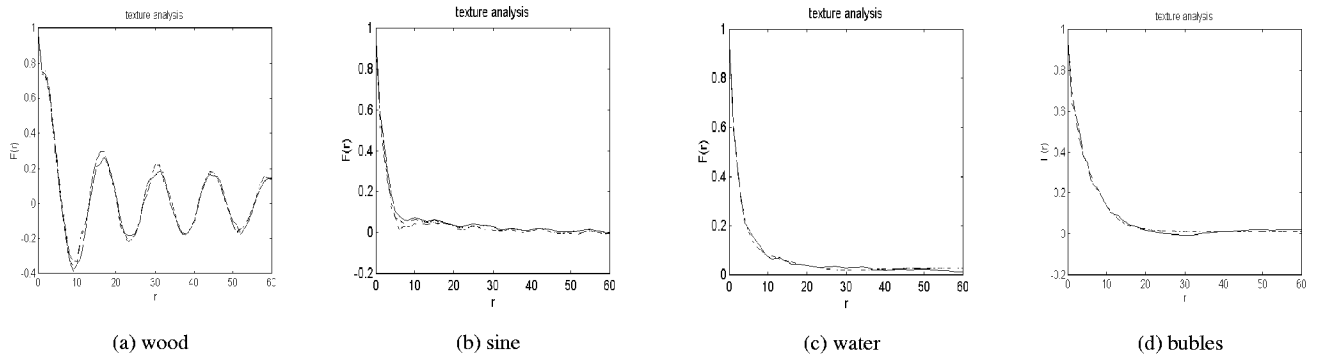
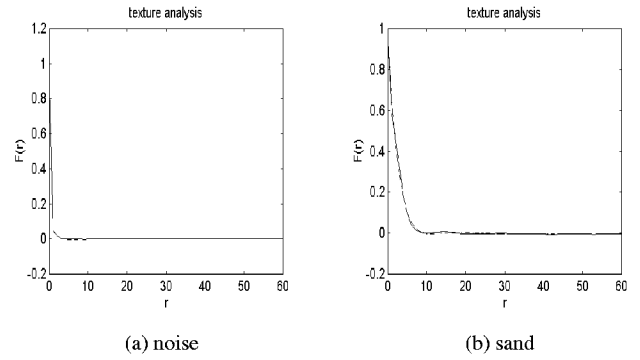


Figure 3: Modeling results of previous textures for four orientations

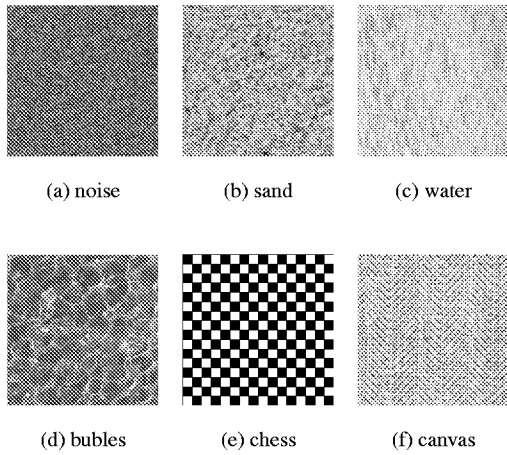


Figure 4: Texture samples from the database

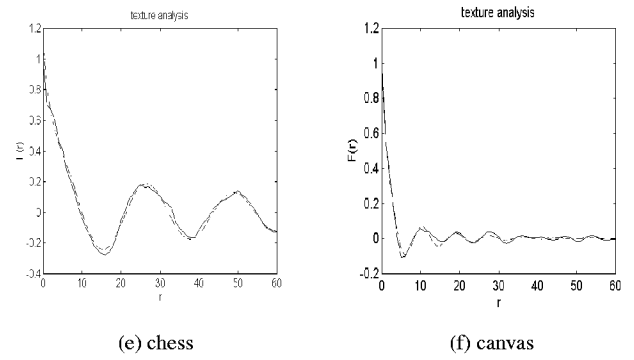


Figure 5: texture modeling results ($\text{—} : F(r), \dots : \tilde{F}(r)$)