APPLICATION OF BLIND SECOND ORDER STATISTICS MIMO IDENTIFICATION METHODS TO THE BLIND CDMA FORWARD LINK CHANNEL ESTIMATION.

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ABSTRACT

Blind channel estimation for periodic sequence DS-CDMA systems can be cast into the framework of "structured" blind estimation of multi-input / multi-output (MIMO) FIR systems, where the structure is imposed by the user's signatures. A possible approach to tackle this problem consists in looking for a structured solution to one of the so-called "blind" MIMO-FIR system identification techniques proposed recently. This is the approach undertaken, among others, by Wang and Poor [16], who proposed an adaptation of the subspace method originally developed by Moulines *et al*. [8] for single-input multiple outputs (SIMO) FIR systems, and later extended by Abed-Meraim *et al* [1] to MIMO-FIR systems. In this contribution, we follow this approach to consider the particular blind forward channel estimation problem, and improve quite significantly the results presented in [16].

1. INTRODUCTION

The suppression of the multiuser interference in DS-CDMA systems operating over dispersive channel is known to be a difficult problem. The problem is particularly severe for DS-CDMA systems using short periodic spreading sequences (e.g. the European UMTS system), because the channel delay spread is typically longer than the duration of one information symbols, mixing multiuser and inter-symbol interferences. In such a context, classical techniques (such as the RAKE receiver) can no longer be used and there is a need to develop more sophisticated receivers coupling multiuser channel identification/equalization and multiuser detection techniques. Considerable research effort has been devoted to these topics recently. In particular, a number of works have been dedicated to the special case where the channels are to be identified/equalized without any training sequence (the so-called *blind* context) : see e.g. [2], [7], [12], [14], [15].

After chip-matched filtering and sampling the received signal at the chip rate, this problem turns out to be equivalent to the blind identification of a multi-inputs (the number of users) / multi-outputs (the spreading factor) FIR transfer function (see [16], among others). In the following, this transfer function is denoted $\mathbf{H}(z)$. The multi-inputs / multi-outputs (MIMO) FIR blind identification problem, which is itself connected to the blind source separation problem, has been studied extensively recently. It was shown that, under appropriate conditions, the unknown transfer function can be identified from the sole knowledge of the covariance of the received signals up to a square polynomial matrix multiplicative factor [1]. When no additional information is available, it is possible to identify the multiplicative factor, but the algorithm to do this task requires to know exactely the order of the channels, and is thus not attractive for practical implementations [3].

In the DS-CDMA case, a much more efficient procedure can be derived. because the transfer function $\mathbf{H}(z)$ to be identified is highly structured (assuming that spreading sequences of the users are known). A natural approach to solve the blind identification problem in that context consists in using one of the existing second order blind identification algorithm, and to look for a relevant structured solution.

In this paper, we follow this approach in order to consider the blind forward link channel estimation problem. We first remark that the use of the MIMO generalization of the subspace method of [8], suggested in [16], yields to consistent estimates only under restrictive assumptions. However, under certain reasonnable conditions on the channel and on the code sequences of the users, we show that the MIMO transfer function to be identified can be estimated up to a constant invertible matrix by using the so-called linear prediction approach introduced by Slock ([11]) and refined in [1] and [4]. We finally establish that taking into account the particular structure of $\mathbf{H}(z)$ allows to identify directly the remaining invertible matrix. In contrast with [16], we show that no extra assumption on the channel is needed.

This paper addresses identifiability issues rather than concrete estimation algorithms. However, estimation algorithms can be derived from our results in a straightforward manner.

This paper is structured as follows. In section 2, we present the forward link channel estimation problem of CDMA systems and express $\mathbf{H}(z)$ in terms of the channel and of the code sequences of the various users. In section 3, we give some conditions under which it is possible to use the linear prediction approach. If these conditions are met, $\mathbf{H}(z)$ can be identified up to a constant invertible matrix. We finally show in section 4 that this matrix can be identified by taking into account the particular structure of $\mathbf{H}(z)$.

General notations.

Polynomials (scalar, vector, matrix valued) $\mathbf{F}(z)$ considered in this paper are functions of the backward shift operator z^{-1}, i.e. $\mathbf{F}(z) = \sum_{k=0}^{M} F(k)z^{-k}$. We use boldface for scalar, vector or matrix polynomials.

Let $\mathbf{f}(z) = \sum_{k=0}^{M} f(k)z^{-k}$ be a (scalar valued) degree M FIR filter. Then, for each integer m, we denote by $\mathcal{C}_m(\mathbf{f})$ the $(M+m+1) \times (m+1)$ matrix defined by

$$\mathcal{C}_m(\mathbf{f}) = \begin{pmatrix} f(0) & 0 & \dots & 0 \\ \vdots & & \ddots & 0 \\ \vdots & & & f(0) \\ f(M) & & & \vdots \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \dots & 0 & f(M) \end{pmatrix} \quad (1)$$

2. PRESENTATION OF THE MODEL.

We consider a multi-user communication system based a direct sequence spread spectrum. We denote by N the spreading factor, and assume that the spreading sequence of each user is periodic with period N; the spreading sequence of user k is denoted $c_k \triangleq (c_k(0), \dots, c_k(N-1))^T$. It is assumed that

$$C = [c_1, \cdots, c_K] \quad (2)$$

is full-rank and that $c_k(N-1) \neq 0$, for $1 \leq k \leq K$. We assume that a certain base station transmits simultaneously at rate $1/T_s$ K symbol sequences $(b_k(n))_{n\in\mathbb{Z}}$ for $k = 1, K$ to K different mobile receivers ($K < N$). We denote by $y_c(t)$ the composite continuous-time signal received by the mobile of interest. As it is usual in the context of the forward link, we assume that the receivers at the mobile units have synchronized with the base station transmitters. The signal $y_c(t)$ is first filtered by a chip-matched filter and then sampled at the chip rate. The resulting discrete-time signal is denoted $y(n)$.

We denote by $\mathbf{c}_k(z) = \sum_{j=0}^{N-1} c_k(j)z^{-k}$ the degree $N-1$ FIR filter associated to the spreading sequence of the user k and by $\tilde{b}_k(n)$ the zero-padded sequence

$$\tilde{b}_k(nN) = b_k(n)$$
$$\tilde{b}_k(nN+j) = 0 \text{ for } j \neq 0 \text{ modulo } N$$

Then, the received signal $y(n)$ may be written as (see e.g. [13])

$$y(n) = \sum_{k=1}^{K} [\mathbf{c}_k(z)\mathbf{g}(z)]\tilde{b}_k(n) + v(n) \quad (3)$$

where $v(n)$ is an additive noise supposed to be white with unknown variance σ^2. $\mathbf{g}(z) = \sum_{j\geq 0} g(k)z^{-k}$ represents an **unknown** discrete time FIR filter resulting from the dispersive channel between the base station and the mobile of interest, and from the transmit and receive filters. We assume that $\deg(\mathbf{g}(z)) = l$, and define L by $(L-1)N < l \leq LN$. We also denote $r \triangleq l - (L-1)N$.

Since the receivers are synchronized, we take $g(0) \neq 0$. In the following, we denote $\mathbf{h}_k(z) = \sum_{j=0}^{l+N-1} h_k(j)z^{-j} = \mathbf{c}_k(z)\mathbf{g}(z)$ the composite signature of the kth user. Put $Y(n) = (y(nN), \dots, y(nN+N-1))^T$ and $V(n) = (v(nN), \dots, v(nN+N-1))^T$. The polyphase decomposition of $\mathbf{h}_k(z)$ writes

$$\mathbf{h}_k(z) = \mathbf{H}_{k,0}(z^N) + \mathbf{H}_{k,1}(z^N)z^{-1} + \dots + \mathbf{H}_{k,N-1}(z^N)z^{-(N-1)}$$

It is easily seen that (see [13] for more details)

$$Y(n) = \sum_{k=1}^{K} [\mathbf{H}_k(z)]b_k(n) + V(n) = [\mathbf{H}(z)]B(n) + V(n) \quad (4)$$

where $B(n) = (b_1(n), \dots, b_K(n))^T$, $\mathbf{H}_k(z) = (\mathbf{H}_{k,0}(z), \dots, \mathbf{H}_{k,N-1}(z))^T$ and $\mathbf{H}(z) = (\mathbf{H}_1(z), \dots, \mathbf{H}_K(z))$. Note that $\deg(\mathbf{H}_k(z)) = L$ for each k, and denote $\mathbf{H}_k(z) = \sum_{j=0}^{L} H_k(j)z^{-j}$, and H_k the $(L+1)N$–dimensional vector $H_k = (H_k(0)^T, \dots, H_k(L)^T)^T$. It follows from the definition of the polyphase decomposition of $\mathbf{h}_k(z), 1 \leq k \leq K$, that

$$H_k = (h_k^T, \underbrace{0, \dots, 0}_{N-1-r})^T \quad (5)$$

Note in particular that for each k, the last $(N-1-r)$ coefficients of $H_k(L)$ are zeros. In this paper, we assume that the code sequences of the K users are a priori known by the receiver of the mobile of interest. Therefore, $\mathbf{H}(z)$ is structured, in the sense that: $\mathbf{h}_k(z) = \mathbf{g}(z)\mathbf{c}_k(z)$. Note that this last relation is equivalent to the identity

$$h_k = \begin{pmatrix} g(0) & 0 & \dots & 0 \\ \vdots & & \ddots & 0 \\ \vdots & & & g(0) \\ g(l) & & & \vdots \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \dots & 0 & g(l) \end{pmatrix} \begin{pmatrix} c_k(0) \\ c_k(1) \\ \vdots \\ c_k(N-1) \end{pmatrix} = \mathcal{C}_{N-1}(\mathbf{g})c_k \quad (6)$$

In the sequel, we show that $\mathbf{g}(z)$ can be identified from the covariance coefficients of $Y(n)$. The main emphasis will be on identifiablity properties: practical algorithms (using sample estimate of the covariance coefficients) can be derived from these results in a straightforward manner. It is assumed in the sequel that L is known. The knowledge of L is not mandatory, but simplifies the statements of the results. In any case, this assumption is not restrictive because it often occurs that $L = 1$ in CDMA systems.

3. THE PROPERTIES OF $\mathbf{H}(Z)$.

As the $N \times K$ transfer function $\mathbf{H}(z)$ depends on $\mathbf{g}(z)$ in a very particular way, Wang and Poor [16] proposed the following two-steps procedure :

- Use the MIMO generalization of the subspace method to identify $\mathbf{H}(z)$ up to an invertible constant matrix.
- Use the particular structure of $\mathbf{H}(z)$ to raise the indetermination.

According to results of [1], the first step will yield a proper result if and only if the three following conditions are fulfilled: **(i)** $\deg(\mathbf{H}_1(z)) = \cdots \deg(\mathbf{H}_K(z)) = L$, **(ii)** matrix $H(L)$ is full column rank ($\mathbf{H}(z)$ is column-reduced), and **(iii)** $\mathbf{H}(z)$ is irreducible, i.e.

$$\text{Rank}(\mathbf{H}(z)) = K \text{ for each } z \neq 0 \text{ (including } z = \infty) \quad (7)$$

In the present context, **(i)** is always satisfied. On the contrary, $\mathbf{H}(z)$ does not satisfy **(ii)** in general. Recall that the last $(N - 1 - r)$ rows of $H(L) = (H_1(L), \ldots, H_K(L))$ are zero. Hence, if $(N - 1 - r) > K$, the coefficient of $\mathbf{H}(z)$ of degree L, $H(L)$, is not full-rank. In this case, it can be shown (see [6]) that the subspace method leaves a more complicated indetermination on $\mathbf{H}(z)$. Hence, the method outlined in [16] needs some adaptations.

We now investigate under which conditions the transfer function $\mathbf{H}(z)$ is irreducible. Note that an irreducibility condition of $\mathbf{H}(z)$ was given in [13] and [14] in the more general uplink context. Here, we specialize the condition of [13] to the forward link, the specificity of which allows to derive easily interpretable criteria. Assume that the K spreading sequences are the first K columns of a $N \times N$ orthogonal matrix, say O, i.e.

$$O = (c_1, \ldots, c_K, c_{K+1}, \ldots, c_N).$$

Proposition 1 *Assume that the FIR filter $\mathbf{g}(z)$ has no more than $(N - K - 1)$ zeros located on a circle. Assume in addition that, for any $(N - K - 1)$-tuplets of integers, $(k_1, \ldots, k_{N-K-1})$, the $(N - K) \times (N - K - 1)$ matrix defined by*

$$\begin{pmatrix} c_{K+1}(ze^{2i\pi k_1/N}) & \ldots & c_{K+1}(ze^{2i\pi k_{N-K-1}/N}) \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ c_N(ze^{2i\pi k_1/N}) & \ldots & c_N(ze^{2i\pi k_{N-K-1}/N}) \end{pmatrix} \quad (8)$$

is full rank column for each $z \neq 0, \infty$. Then, $\mathbf{H}(z)$ is irreducible.

Proof. see [6].

This result shows that if the the matrix O is appropriately chosen, then for a large class of channels, the matrix $\mathbf{H}(z)$ is irreducible. One should note that if $\deg(\mathbf{g}(z)) = l < N - K$, then $\mathbf{g}(z)$ satisfies the condition of Proposition 1.

Remark. A similar result has been obtained earlier by Scaglione *et al* ([9], [10]) for Transmitter Induced Cyclostationnary (TIC) systems. This similarity follows from the multirate model of the received signal (3). If instead of representing different symbol sequences emitted to various users, the sequences $(b_1(n), \ldots, b_K(n))$ come from a single sequence to be transmitted at rate T_s/K, then Eq.(3) corresponds to a single user TIC system of redundancy factor N/K. This observation indicates that our results apply not only to the blind identification of multiuser systems in the forward link context, but also to the blind equalization of TIC systems.

4. IDENTIFICATION OF THE CHANNEL VIA THE LINEAR PREDICTION APPROACH.

As mentionned above, the matrix $H(L)$ is in general not full column rank. Therefore, the subspace method does not allow to identify $\mathbf{H}(z)$ up to an invertible $K \times K$ matrix. However, if $\mathbf{H}(z)$ is irreducible (e.g. if the conditions of Proposition (1) are met), the so-called linear prediction approach make possible the identification of $\mathbf{H}(z)$ up to a constant unitary matrix ([11], [1], [4]). We refer the reader to the paper [1] (see section IV-C) for more details on this approach.

Therefore, it is in principle possible to identify (in practice to estimate consistently) $\mathbf{H}(z)$ up to a constant invertible matrix. In this section, we show that the taking into account of the particular structure of $\mathbf{H}(z)$ allows to raise the above mentionned indeterminacies without requiring any extra hypothesis on the transfer function $\mathbf{g}(z)$ of the channel. This is in contrast with [16] where a purely technical extra condition on $\mathbf{g}(z)$ is formulated. In particular, it is useful to mention that we do not need any a priori information on the degree of $\mathbf{g}(z)$, except that it is compatible with the degree L of $\mathbf{H}(z)$, which is assumed known. In other words, we assume that it is known that $(L-1)N + 1 \leq \deg(\mathbf{g}(z) \leq LN$.

We assume that one has identified an order L FIR $N \times K$ transfer function $\mathbf{H}'(z)$ equal to $\mathbf{H}(z)R$ where R is an unknown constant unitary matrix. Let us put $H = (H(0)^T, \ldots, H(L)^T)^T$ and $H' = (H'(0)^T, \ldots, H'(L)^T)^T$. Then, $H' = HR$, so that the knowledge of $\mathbf{H}'(z)$ is equivalent to the knowledge of the space $\text{Range}(H)$ generated by the columns of the matrix H. $\text{Range}(H)$ can therefore be identified (resp. consistently estimated) from the exact (resp. estimated) second order statistics of the observation. We now show that the knowledge of $\text{Range}(H)$ allows to retrieve $\mathbf{g}(z)$.

Proposition 2 *Let Π be the orthogonal projection matrix on the space $(\text{Range}(H))^\perp$ and let $\mathbf{f}(z) = \sum_{k=0}^{LN} f(k)z^{-k}$ be a scalar polynomial of degree less than or equal to LN. Assume that the polynomials $(\mathbf{c}_1(z), \ldots, \mathbf{c}_K(z))$ have no common zero. Then,*

$$\Pi \mathcal{C}_{N-1}(\mathbf{f})C = 0 \iff \mathbf{f}(z) = \alpha \mathbf{g}(z), \quad (9)$$

where α is a constant.

Proof. Let $\mathbf{f}(z)$ be a solution of (9). Then, (9) implies that the range of $\mathcal{C}_{N-1}(\mathbf{f})C$ is included in $\text{Range}(H)$. But, it is easily seen that the K columns of $\mathcal{C}_{N-1}(\mathbf{f})C$ coincide with the K vectors associated to the coefficients of the K polynomials $(\mathbf{f}(z)\mathbf{c}_k(z))$ for $k = 1, K$. In a quite similar way, the columns of H coincide with the vectors constructed from the polynomials $(\mathbf{g}(z)\mathbf{c}_k(z))_{k=1,K}$. Using this polynomial interpretation of $\mathcal{C}_{N-1}(\mathbf{f})C$ and H, it follows that (9) holds iff the linear space $\text{span}(\mathbf{f}(z)\mathbf{c}_k(z)), k = 1, K) = \{\sum_{k=1}^{K} \alpha_k \mathbf{c}_k(z)\mathbf{f}(z), \alpha_k \in \mathbb{C}\}$ is included in $\text{span}(\mathbf{g}(z)\mathbf{c}_k(z), k = 1, K)$. Therefore, for each k, the polynomial $\mathbf{f}(z)\mathbf{c}_k(z)$ can be written as

$$\mathbf{f}(z)\mathbf{c}_k(z) = \sum_{j=1}^{K} \alpha_{k,j} \mathbf{g}(z)\mathbf{c}_j(z) \quad (10)$$

We first remark that the degree of the righthandside of (10) is less than or equal to $N + l - 1$. As $\deg(\mathbf{f}(z)\mathbf{c}_k(z)) = \deg(\mathbf{f}(z)) + \deg(\mathbf{c}_k(z)) = \deg(\mathbf{f}(z)) + N - 1$, we get that $\deg(\mathbf{f}(z)) \leq l$. Let $z_1, \ldots, z_l$ be the zeros the of $\mathbf{g}(z)$. (10) implies that for each k and j, $\mathbf{f}(z_j)\mathbf{c}_k(z_j) = 0$. For each j, there is at least an index k for which $\mathbf{c}_k(z_j) \neq 0$. Therefore, $\mathbf{f}(z_j) = 0$ for each $j = 1, l$, i.e. $\mathbf{f}(z) = \mathbf{g}(z)\mathbf{r}(z)$ for some scalar polynomial $\mathbf{r}(z)$. But, as $\deg(\mathbf{f}(z)) \leq l$, $\mathbf{r}(z)$ must be reduced to a scalar constant. This concludes the proof.

Let us comment on this result. We first remark that $\mathbf{g}(z)$ can be identified by solving a linear equation. This shows that the practical implementation of this approach is quite easy. Next, we note here that the assumed degree of $\mathbf{f}(z)$ is LN, i.e. the greatest possible degree compatible with the a priori knowledge of L (we recall that L is assumed to be known). However, this possible degree overdetermination has no consequence on the consistency of the method because the unique solution of (9) is the degree l transfer function $\mathbf{g}(z)$. Therefore, the proposed method does not suffer from an overdetermination of the degree of the channel.

5. CONCLUSION.

In this paper, we have studied the blind CDMA forward link channel estimation problem. Our approach, similar to that of [16], consists in looking for a structured solution to an existing blind second order MIMO method based on the second order statistics of the observation. We have shown that the use of the subspace method seems not appropriate. In this paper, we have proposed to use the linear prediction approach which allows to identify $\mathbf{H}(z)$ up to a constant invertible matrix if $\mathbf{H}(z)$ is irreducible. We have given reasonnable sufficient irreducibility conditions of $\mathbf{H}(z)$ in terms of the channel and of the codes of the various users. Finally, we have shown that the indeterminacies leaved by the linear prediction approach can be raised using the structure of $\mathbf{H}(z)$. In particular, the channel can be identified, even if no a priori information on the channel order is available. Our contribution mainly concerns identifiability issues rather than concrete estimation algorithms. However, estimation algorithms can be derived in a straightforward manner from our results.

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