NEW REALIZATION METHOD FOR LINEAR PERIODIC TIME-VARYING FILTERS

Alban DUVERDIER and Bernard LACAZE***

* CNES, 18 av. Belin, BPI 2012, 31401 Toulouse Cedex 4, France
** ENSEEIHT/SIC, 2, rue Camichel BP7122, 31071 Toulouse Cedex 7, France
tel: +33 (0)5 61 28 31 79 / fax: +33 (0)5 61 28 26 13
email: Alban.Duverdier@cnes.fr

ABSTRACT

For channel modelisation, modulation and analogue scrambling, the modern telecommunications use often linear periodic time-varying filters. The authors recall the definition of these filters. In particular, it is shown that a stationary process subjected to a linear periodic filter becomes cyclostationary. In this paper, we show that any linear periodic filter can be realized by means of periodic clock changes. An original implementation method is then introduced. An example illustrates the periodic clock change implementation and presents the advantages of the new implementation technique in comparison to the classical one.

1. INTRODUCTION

In telecommunications, signals submitted to a Linear Periodically Time-Varying (LPTV) filter [1] are often encountered. Thus, this filter can correspond to a transmission channel modelisation [2], a modulation [3] or an analogue scrambling system [4]. In this paper, we show that linear periodic filters can be obtained by a new implementation method using periodic clock changes [5].

In the first section, we recall some definitions. We see that a stationary process subjected to a linear periodic filter becomes cyclostationary [6]. Next, we point out that linear periodic filters can be implemented originally by means of periodic clock changes. Finally, a periodic clock change is implemented to study the bounds computation and to present the important saving of time obtained.

2. DEFINITIONS

2.1. Stationary process

We let $Z = \{Z(t),\ t \in \mathbf{R}\}$ be a random stationary process, zero mean and mean square continuous. $\Theta_Z(\omega)$ is the Cramér-Loève spectral representation [7] of $Z(t)$ and the two are related by:

$$Z(t) = \int_{-\infty}^{+\infty} e^{i\omega t} d\Theta_Z(\omega) \tag{1}$$

We note $R_Z(\tau)$ its autocorrelation function and $S_Z(\omega)$ its spectrum defined by:

$$R_Z(\tau) = E\left[Z(t)Z^*(t-\tau)\right] = \int_{-\infty}^{+\infty} e^{i\omega\tau} dS_Z(\omega) \tag{2}$$

2.2. Linear periodic time-varying filter

Let $\tilde{h}$ be a continuous-time linear filter. Its response $X(t)$ to an input $Z(t)$ may in general be written as:

$$X(t) = \int_{-\infty}^{+\infty} h(t,s)Z(s)ds \tag{3}$$

where $h(t,s)$ is the filter impulse response. We will study the case where $\tilde{h}$ is an LPTV system [1], i.e. where there exists a period $T = 2\pi/\omega_0$ such that:

$$h(t+T, s+T) = h(t,s) \tag{4}$$

The time-varying frequency response of the LPTV filter $\tilde{h}$ is defined by:

$$H_t(\omega) = \int_{-\infty}^{+\infty} h(t, t-\tau)e^{-i\omega\tau} d\tau \tag{5}$$

It is worth noting that $H_t(\omega)$ is periodic in t. We then define its Fourier development, assumed to be sufficiently regular, by:

$$H_t(\omega) = \sum_{k=-\infty}^{+\infty} \psi_k(\omega) e^{ik\omega_0 t} \tag{6}$$

where the Fourier coefficients are expressed as:

$$\psi_k(\omega) = \frac{1}{T}\int_0^T H_t(\omega) e^{-ik\omega_0 t} dt \tag{7}$$

2.3. Periodic clock changes

The response of a stationary process $Z(t)$ subjected to a Periodic Clock Change (PCC) was defined in [5] by:

$$X(t) = g(t)Z\left[t - f(t)\right] \quad (8)$$

where $f(t)$ is a real measurable function and $g(t)$ is a real integrable function, $f(t)$ and $g(t)$ being $T = 2\pi/\omega_0$ periodic. If we denote by $\delta(t)$ the Dirac function, then, in the sense of distributions, the PCC corresponds to an LPTV filter whose impulse response is:

$$h(t,s) = g(t)\delta(t - f(t) - s) \quad (9)$$

A PCC is also an LPTV filter of frequency response:

$$H_t(\omega) = g(t)e^{-i\omega f(t)} \quad (10)$$

3. RESPONSE OF A STATIONARY PROCESS THROUGH A LINEAR PERIODIC FILTER

3.1. Continuous-time series representation of $X(t)$

$X(t)$ is the filtering of $Z(t)$ by the LPTV filter $\tilde{h}$ of impulse response $h(t,s)$ and frequency response $H_t(\omega)$. Using (1) and (5), the expression of X given by equation (3) becomes:

$$X(t) = \int_{-\infty}^{+\infty} H_t(\omega)e^{i\omega t}d\Theta_Z(\omega) \quad (11)$$

The Fourier representation of $H_t(\omega)$ (6) allows us to define a continuous-series representation of $X(t)$ such that:

$$X(t) = \sum_{k=-\infty}^{+\infty} e^{ik\omega_0 t}G_k(t) \quad (12)$$

with:

$$G_k(t) = \int_{-\infty}^{+\infty} e^{i\omega t}\psi_k(\omega)d\Theta_Z(\omega) \quad (13)$$

$G_k(t)$ is the response to $Z(t)$ through the linear time-invariant filter whose frequency response is $\psi_k(\omega)$, k'th coefficient of the Fourier development of $\tilde{h}$ frequency response.

3.2. Stochastic parameters of $X(t)$

Since $Z(t)$ is zero mean, then all $G_k(t)$ are zero mean. Therefore the above decomposition shows that $X(t)$ is a zero mean process. From the Wiener-Lee relations, we can easily obtain the autocorrelation function of $X(t)$ and its two-dimensional spectral density:

$$R_X(t,\tau) = \sum_{m,l=-\infty}^{+\infty} e^{im\omega_0 t} \int_{-\infty}^{+\infty} e^{i(\omega + l\omega_0)\tau}\psi_{m+l}(\omega)\psi_l^*(\omega)dS_Z(\omega) \quad (14)$$

$$dS_X(t,\omega) = \sum_{m,l=-\infty}^{+\infty} e^{im\omega_0 t}\psi_{m+l}(\omega - l\omega_0)\psi_l^*(\omega - l\omega_0)dS_Z(\omega - l\omega_0) \quad (15)$$

They are both periodic in t. $X(t)$ is thus cyclostationary in the wide sense [6].

4. REALIZATION OF LINEAR PERIODIC FILTERS BY MEANS OF PERIODIC CLOCK CHANGES

4.1. Classical method

Let $X(t)$ be the response of the stationary process $Z(t)$ through the linear periodic filter $\tilde{h}$. We have previously seen that $X(t)$ admits the continuous series representation (12). For $Z(t)$ bandlimited on $[-l\omega_0; l\omega_0[$, it allows to obtain $X(t)$ on $[-(N-l)\omega_0; (N-l)\omega_0[$ for $N > l$ by the classical implementation [6]:

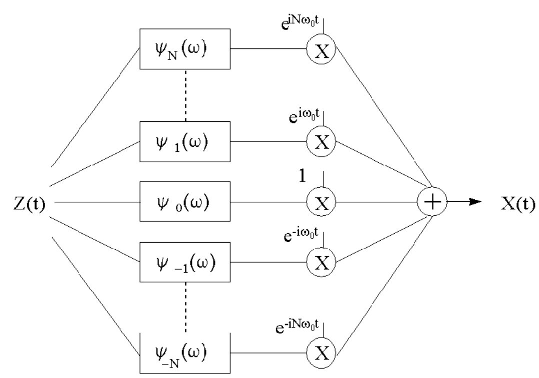

Figure 1: Classical implementation of an LPTV filter

This filter generation method is easy to realize. However, it has also to be judged on the two following criterions: the correspondence with a physical model and the implementation cost. On the one hand, a filter is physically described by its impulse response. Therefore, the $\{\psi_k(\omega)\}_{k\in\mathbf{Z}}$ functions and the above generation method do not directly represent the nature of the filter. On the other hand, the implementation cost depends on two parameters: the bandwith of the input signal and the bandwith for the observation window of the output signal. In a large band case, a lot of operations is then required. Thus, the above generation method presents important drawbacks.

4.2. Use of periodic clock changes

Let $\tilde{h}_0$ be a linear time-invariant filter of frequency response $H_0(\omega)$. The algorithm given in [8] based on the Prony's technique applied in the frequency domain allows the following approximation:

$$H_0(\omega) = \sum_{k=-N}^{N} g_k e^{-jf_k\omega} \quad (16)$$

where f_k and g_k are real coefficients. This result shows that the response $X(t)$ to the stationary process $Z(t)$ through the linear periodic filter $\tilde{h}$ can be approximated by:

$$X(t) = \sum_{k=-\infty}^{+\infty} g_k(t) Z(t - f_k(t)) \quad (17)$$

where the $\{f_k(t)\}_{k\in\mathbf{Z}}$ are real periodic measurable functions and the $\{g_k(t)\}_{k\in\mathbf{Z}}$ are real periodic integrable functions. In (8), we have given the definition of a particular class of linear periodic filters, the periodic clock changes. (17) shows that linear periodic filters can be approximated by a sum of periodic clock changes. The following implementation can then be proposed:

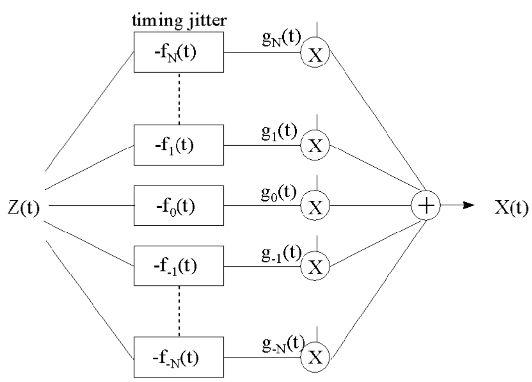

Figure 2: Implementation of an LPTV filter by means of PCC

Firstly, one interest of such an approximation is that it corresponds to the usual model of timing jitter. It has been particularly used in communication. Secondly, the implementation cost of this generation method only depends on complexity of the filter. This new method corrects all the disadvantages of the classical one: it corresponds to a physical model and its implementation cost just depends on the filter. Therefore, the generation of periodic clock changes presents one difficulty: bounds computation. Indeed, the initial signal is always defined on a finite window. After time jitter, some points go out of the windows and cannot be computed. The solution of this problem is to realize a block computation, the block being periodised. It is illustrated by the following example.

4.3. Example

We study now the practical implementation of a periodic clock change. Let $Z(t)$ be a stationary process of spectral support included in $[-\omega_0/2, \omega_0/2[$. $X(t)$, response of $Z(t)$ through a periodic clock change $\tilde{h}$, is observed on the spectral support $[-(2N+1)\omega_0/2, (2N+1)\omega_0/2[$. $\tilde{h}$ is given by:

$$\begin{cases} f(t) = -\alpha \sin(\omega_0 t) \\ g(t) = 1 \end{cases} \quad (18)$$

From (7) and (10), we deduce that in this case the $\{\psi_k(\omega)\}_{k\in\mathbf{Z}}$ are equal to:

$$\psi_k(\omega) = \int_{-\infty}^{+\infty} e^{i(\alpha\omega\sin(\omega_0 t) - k\omega_0 t)} dt = J_k(\alpha\omega) \quad (19)$$

where $J_k(\alpha\omega)$ is the k'th order Bessel function. We can then implement $\tilde{h}$ by the classical method or by means of a periodic clock change. As we are in a real case, we work on a finite temporal window of length W. We chose W multiple of T to respect the spectral expression of X. Figure 3 gives a realization of $Z(t)$ for $W = 10000$ and $T = 100$.

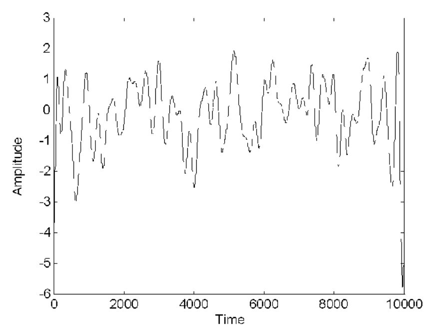

Figure 3: Input signal $Z(t)$

Figure 4 presents the realization of $X(t)$ obtained by the classical method for $\alpha = 1000$ and $N = 8$.
We compare this result to a generation done by means of periodic clock changes.Figure 5 shows that bounds error appears.
To avoid this error, a block computation can be introduced. The block of length W is submitted to a periodisation. It permits to define any point even after timing jitter, but conserves the spectral expression of X. The response obtained by means of periodic clock changes is then the same as this given by the classical method. Figure 6 presents the number of operations for the classical method implementation (——) and for the implementation by means of periodic clock changes (– – –) in function of the observed bandwith.
Our method gives the same results as the classical one and permits an important saving of time for a large observed bandwith.

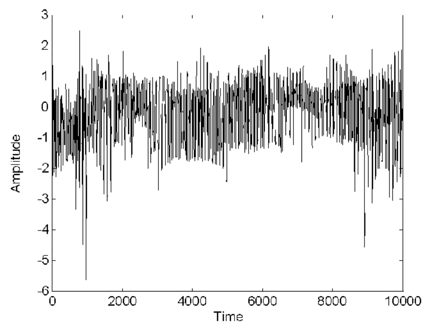

Figure 4: Observed signal $X(t)$ with classical method implementation

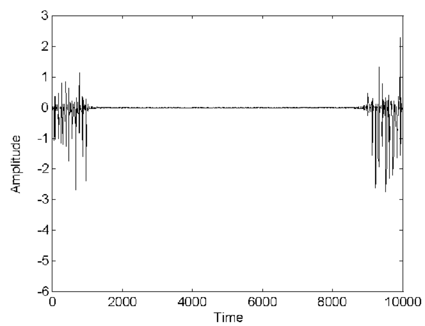

Figure 5: Error due to real time implementation by means of periodic clock changes

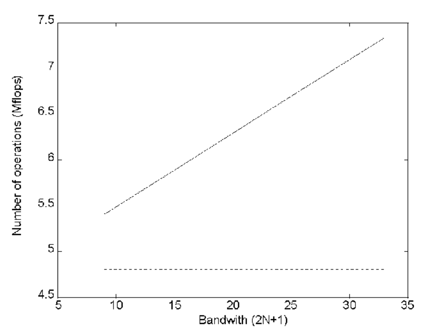

Figure 6: Number of operations needed for implementation

5. CONCLUSION

In this paper, we have presented a new method of linear periodic filter implementation. It is done by means of periodic clock changes. It is particularly easy to realize and permits to save a lot of time. The only difficulty of periodic clock changes use, bounds computation, has been solved. This result is verified on an example. It opens then the way to real-time implementation with complicated filters and high protected systems for channel modelisation, modulation and scrambling.

6. REFERENCES

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