THE DETECTION OF RADAR PULSE SEQUENCES BY MEANS OF A CONTINUOUS WAVELET TRANSFORM

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ABSTRACT

A wavelet transform is introduced as a means of detecting the characteristic scale or period T of a radar pulse sequence in an incoming stream of pulses. A particular choice of mother wavelet contains a fixed number, M, of cycles of a complex exponential, which provides a resolution of $1/M$ times T. The transform operates on interleaved pulse sequences and, by thresholding, determines T for each component sequence. The detector is robust against missing pulses and timing jitter and is sensitive to simple, staggered and complex pulse sequences. The method is an improvement on other established approaches, such as the time-difference-of-arrival (TDOA) histogram and the periodogram.

1. INTRODUCTION

A surveillance system used to detect and identify radars will measure certain characteristics of the radar emissions to ascertain the nature of the source [1, 2, 3]. The majority of the radars that one encounters in practice emit energy in predefined sequences of pulses. The timing of these pulses follow simple or complex patterns described by measurable parameters, such as stagger or pulse repetition intervals.

It is useful, for the purpose of detecting and identifying radars in the environment, to treat these sequences as signals. The problem of determining the presence of a specific emitter in the environment is then a problem of detection. The presence or absence of an emitter is a function of the incoming stream of pulses.

A significant problem arises when the signals from two or more emitters overlap in time. The pulses arrive in natural time order and so become interleaved. This complicates the task of identifying individual sequences. It is sometimes difficult, or impossible, to determine which pulses belong to which emitters, or even how many emitters are represented in an interleaved pulse stream.

An established procedure for processing interleaved pulse sequences is based on time-difference-of-arrival (TDOA) histogramming [4, 5, 6]. The essence of this method is first to do a coarse search in PRI using TDOA histogramming, removing any sequences with the detected PRI's from the interleaved pulse stream, and then to repeat the process on the remaining data. Another approach is to search a periodogram for pulse repetition frequencies (PRF's) [7, 8, 9].

A number of complications impact the effectiveness of these two approaches. The traditional method, TDOA histogramming, (1) produces false detections at multiples of the true PRI's, (2) is sensitive to the bin width, (3) fails to detect complex pulse sequences and (4) is a computation-intensive procedure. Worst of all, interleaved pulse streams (5) produce a background of false pulse gaps in the histogram, which bury the true PRI's. The periodogram (1) produces false detections at multiples of the true PRF's, (2) fails to detect short sequences, (3) fails to detect complex or wobulated sequences, and (4) provides an inefficient search strategy. Another well-known technique, the fast-folding method [10], requires a histogram search for each folding period.

In this paper, we describe a method [11], based on a continuous wavelet transform, for detecting radar pulse sequences. It operates directly on interleaved pulse streams. The detector function contains two arguments: one corresponding to location in time and the other corresponding to a characteristic period or scale, T. A single adjustable parameter M fixes the resolution of the detector.

The new method overcomes the complications of the other approaches: (1) harmonics are suppressed by a factor of four or more, (2) cross interference effects between interleaved pulse sequences are minimized, (3) detection of wobulated and complex stagger sequences is improved, and (4) the procedure of searching for T is fast.

2. DETECTION METHOD

Our approach to the solution of this problem is first to represent the set of received TOA's, $\{t_j\}$, as a superposition of impulses, $s(t) = \sum_j \delta(t - t_j)$, and then to apply a continuous wavelet transformation to the signal based on the *mother wavelet*

$$\psi(t) = M^{-\frac{1}{2}} \chi(t/M) e^{2\pi i t}, \tag{1}$$

where $\chi(t)$ is a rectangular window of unit length and M is the minimum length pulse train we expect to receive or wish to detect. Wavelet $\psi(t)$ satisfies the admissibility condition for a CWT [12] when M is a positive integer. The transform is reminiscent of a Fourier transform, with a window of length MT. But notice the window length varies with T.

The wavelet transform of signal $s(t)$ gives

$$D(T, t) = \frac{T}{M} \left| \int dt' \, \psi^* \left(\frac{t' - t}{T} \right) s(t') \right|^2 . \tag{2}$$

We flag a detection whenever D exceeds a threshold a. The detection represents the presence of a pulse train at T. The normalization ensures that a is of order unity.
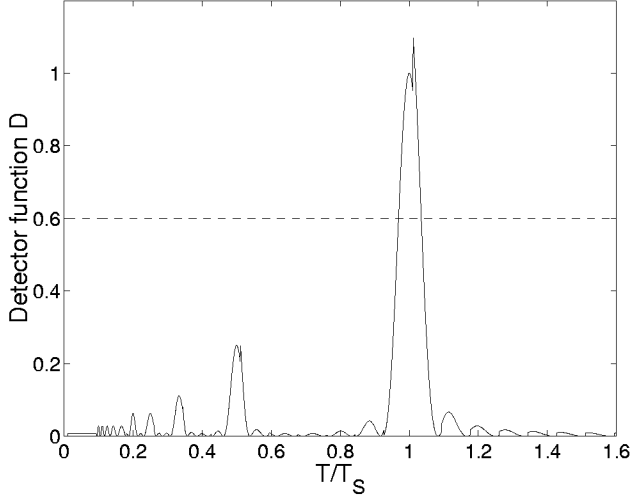
Figure 1: The detector function $D(T,t)$ for a simple pulse train with PRI T_S and $M = 12$. In this plot, t is set to zero.

Figure 2: The CWT detector function for the interleaved set of three pulse trains.

The detector is sensitive to radar pulse sequences of the following three types: simple, staggered and complex. The simple sequence contains a single PRI, T_S. The times of arrival in this sequence are

$$t_j = (j - 1)T_S + t_\phi, \quad j = 1, \ldots, N, \tag{3}$$

Here, t_ϕ is the reference time or *phase*. The staggered sequence has M pulse gaps, $T_1, \ldots T_M$, that repeat in M-count cycles: $T_{j+M} = T_j$. The times of arrival are

$$t_1 = t_\phi \tag{4}$$
$$t_j = t_{j-1} + T_j, \quad j = 2, \ldots, N.$$

The possibilities for complex sequences are unlimited. A wobulated sequence, for example, is produced by

$$T_j = A \cos(Bj + C) \tag{5}$$

where A, B and C are constants.

For the simple pulse train of Equation 3, the detector function works out simply to equal

$$D(T,t) = \frac{\sin^2[N(T,t)\pi T_S/T]}{M^2 \sin^2(\pi T_S/T)} \tag{6}$$

where $N(T,t)$ is the number of pulses in a window of length MT at t. As T and t vary, pulses drop in and out of the window, causing $D(T,t)$ to jump discontinuously. A plot of D for a single pulse train is shown in Figure 1. A nominal threshold is indicated by a dashed line. The threshold rejects all T except $|T - T_S| \lesssim T_S/M$. There is ample margin for fluctuations in the heights of the first and second peaks. This margin provides robustness against the effects of missing pulses, timing jitter and the interference of interleaved pulse sequences.

The second, third and fourth harmonics, at $T_S/2$, $T_S/3$ and $T_S/4$, are suppressed by factors of 4, 9 and 16, respectively.

Parameter M controls the resolution of the detector. The resolution is inversely proportional to M, $\delta T/T = \delta f/f \sim 1/M$.
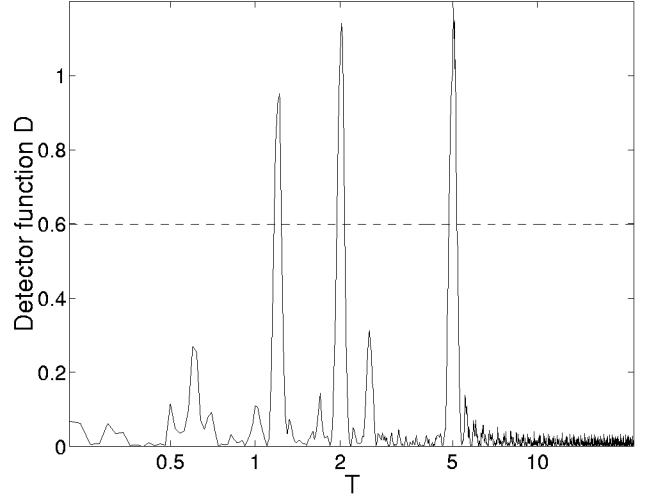
This means, given a fixed value for M, that pulse sequences at different length scales are treated equivalently, that is, $D(T,t)$ is scale invariant. A large range of PRI's occur in practice, on the order of from 10 us to 10 ms, which covers three orders of magnitude. A single value M applies to the full range.

A practical value for application to radar is $M = 12$. Larger values of M provide higher resolutions, but then $D(T,t)$ fails to detect short pulse sequences. A radar scan dwell may contain only ten or twenty pulses. At $M = 12$, the detector's resolution is about 10 per cent and it is sensitive to scans of eight or ten pulses. The wavelet detector does not estimate T as precisely as, say, a least-squares fit. It is intended for use in a first pass on an incoming pulse stream to determine the number of emitters, to locate the associated pulse sequences in time and to find approximate values for T. Once the sequences are located, individual pulses can be isolated and precise estimates of inter-pulse parameters can be found.

The wavelet detector is sufficiently robust to assign a characteristic quantity T to each simple, staggered or complex radar sequence in an interleaved pulse stream.

A useful property follows from Equation 2. An interleaved pulse stream, $s(t)$, is a sum of terms $s_1(t) + s_2(t) + s_3(t) + \cdots$ corresponding to the radar pulse sequences contributing to $s(t)$. The inner product $\int \psi^* s$ is a linear transform. If the magnitude-squared cross terms are small compared to 1, $D(T,t)$ behaves like the sum $D_1(T,t) + D_2(T,t) + D_3(T,t) + \cdots$ of detection functions of independent pulse sequences. In this case, the characteristic $T's$ can be deduced from $D(T,t)$ without de-interleaving the pulse stream. Often, though not always, the interference condition is satisfied when the characteristic T's differ by $\delta T \gtrsim T/M$.

3. SIMULATION RESULTS

An example with three interleaved pulse sequences is shown in Figure 2. In this example, all three sequences are of the simple type with PRI's of 1.2, 2.0 and 5.0. The detector function crosses
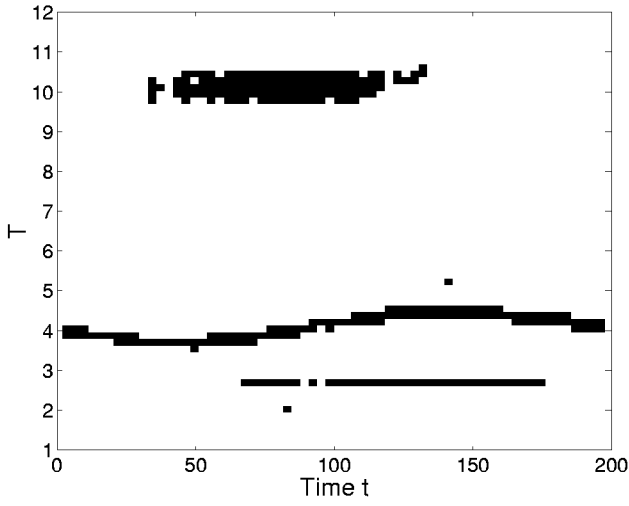
Figure 3: Detections $D(T, t) > 0.5$, marked ■, in a pulse stream of three interleaved pulse sequences: one simple, one staggered and one wobulated.
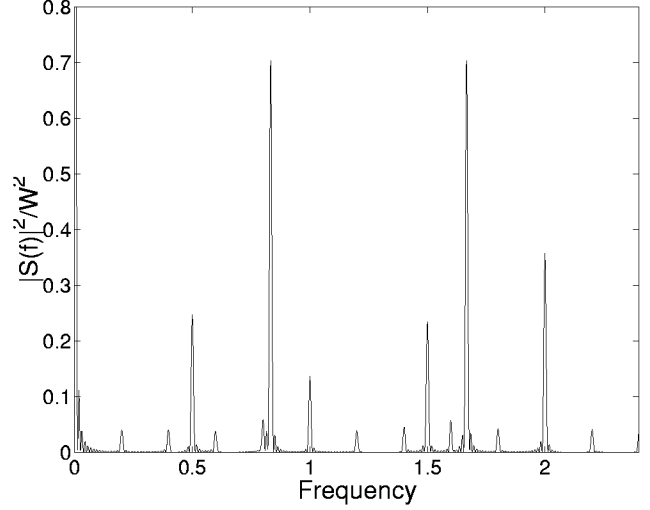

Figure 4: The periodogram of the interleaved set used in Figure 2. The window length is $W = 80$. The peaks at PRF's 0.833, 0.5 and 0.2 correspond to the PRI's 1.2, 2.0 and 5.0 respectively.

the threshold at just these three PRI's.

A more complex example is presented in Figure 3. This is a plot of detections over a period of time for a stream of three interleaved pulse sequences: one simple, one staggered and one wobulated. The simple sequence has a PRI T_S=10 and extends from t=20 to 140. The staggered sequence has four stagger intervals T_1=3.15, T_2=2.45, T_3=2.80 and T_4=2.10, with an average length of 2.63, and extends from t=62.1 to 186.0 The wobulated sequence has a mean PRI equal to 4.0 and the PRI varies by $\pm$10%. It contains 50 pulses per wobulation cycle. Detections corresponding to these three pulse sequences are easily recognized in the diagram. Notice the width of each band of detections increases with T. This is as expected. The scale-invariant $D(T, t)$ has constant relative resolution, $\delta T \sim MT$. We notice a small number of false and missing detections. These are caused by the interleaving cross terms that we discussed before.

Figures 4 and 5 show periodogram and TDOA histogram plots for the same interleaved set of three simple pulse trains as in Figure 2. The most apparent advantage of the wavelet detector is the one-to-one correspondence between the sharp peaks and the individual pulse sequences. There is one threshold crossing for each PRI. In the periodogram, Figure 4, the correspondence between peaks and pulse sequences is not as clear. Some of the peaks are harmonics of the true PRI's. It is not clear what threshold should be used. The histogram, in Figure 5, is worse. Not one PRI stands out in this example. The histogram is swamped by the background of false pulse gaps generated by the interleaving of the simple pulse sequences.

The TDOA histogram is sensitive to a parameter, the bin width. The windowed Fourier transform depends on a parameter W, the window length. The window W must be sufficiently long to contain several pulses of the longest PRI of interest. W, then, is too long to detect short sequences of small PRI's. A fixed window width W gives optimum results in only a narrow range of PRI's. The wavelet transform, on the hand, depends only on a dimensionless parameter M, which controls the relative resolution $\delta T/T$. A single value of M optimizes the window lengths MT over the
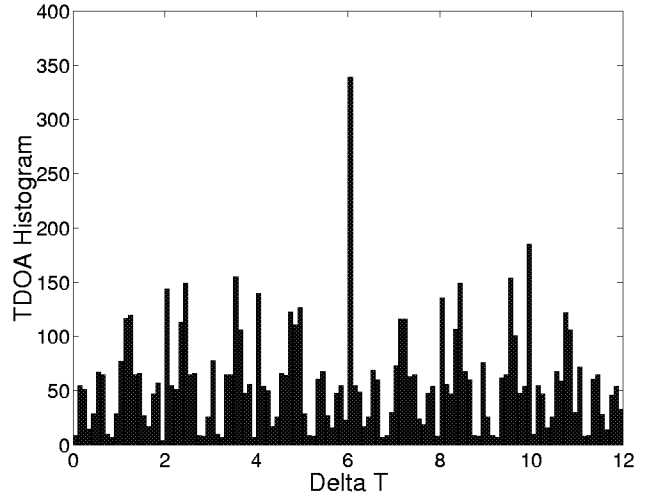

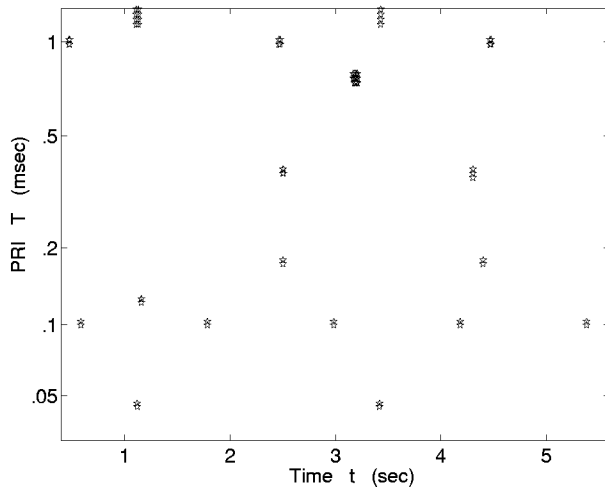Figure 5: A Time Differences of Arrival (TDOA) histogram for the interleaved set of Figure 2.

Figure 6: Example of the operation of the detector for a simulation of seven scanning radars. Detections are marked where $D(T,t) > 0.6$.

whole applicable range of T.

In Figure 3, the wavelet detector gives clear indication of a wobulated pulse sequence. The periodogram and the histogram would not. In the periodogram, with any W greater than a fraction of the wobulation period, the energy will smear over a band with numerous undulations and peaks. The histogram would present a band of raised bins rather than a single, distinct peak.

It is easy to demonstrate robustness of the wavelet detector against missing pulses and jitter. A few missing pulses in an interval of length MT will lower the peak in Figure 1. To drop below a threshold of .6, forty per cent of the pulses must be lost. Time jitter equal to a few percent of T will make no practical difference to the magnitude of $D(T,t)$ at the peak.

The computational requirements of our method are also considered and are shown to be modest: approximately one multiplication per pulse per sample T. A further advantage is that the computations can be carried out in parallel in T and recursively in time.

Figure 6 is a simulation of a practical application for radar PRI detection with seven simulated scanning emitters. An off-the shelf DSP processor can execute the algorithm, unparallelized, at 4,000 to 10,000 pulses per second.

All PRI values and scan times are correctly reported in the example of Figure 6. Two groups of detections, appearing, at $t = 1.1$ and 3.4 seconds where $T > 1$ ms, are spurious. The spurious detections are produced by the short bursts of PRI less than 0.05 ms in the much larger window MT for $T > 1$ ms. Spurious detections like these are avoided by restricting the range of T or by blanking small pulse intervals when searching the larger PRI's.

4. CONCLUSION

A continuous wavelet transform, based on the mother wavelet of Equation 2, provides a new method for detecting the characteristic scale or periods T of radar pulse sequences in interleaved

pulse streams. It improves on other established methods, including TDOA histogramming, the periodogram and folding.

The transform provides a scale-invariant method for extracting T. It depends only on one dimensionless integer parameter, M. The parameter M controls the detector's resolution and sets an optimum window length MT for all scales of T.

The transform is suited to searching large ranges of T, rapidly. It solves the problem of separating the fundamental peak of the detection function from harmonics or sub-multiples of the peak. The detector function suppresses background and it is robust against missing pulses, jitter and cross interference between interleaved pulse sequences.

The wavelet detector can be used in conjunction with parameter estimation methods to improve the robustness, speed and accuracy of algorithms used to process interleaved radar pulse sequences for electronic surveillance purposes.

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