MINIMIZATION OF WEIGHTED SENSITIVITY FOR 2-D STATE-SPACE DIGITAL FILTERS DESCRIBED BY THE FORNASINI-MARCHESINI SECOND MODEL

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ABSTRACT

This paper considers the problem of minimizing the weighted coefficient sensitivity for 2-D state-space digital filters described by the Fornasini-Marchesini (F-M) second model. First, a simple technique is presented for obtaining a set of filter structure with very low weighted L_1/L_2-sensitivity. Next, an iterative procedure is applied to obtain the optimal coordinate transformation that minimizes the weighted L_2-sensitivity measure. This is based on the matrix Riccati differential equation. Finally, a numerical example is given to illustrate the utility of the proposed technique.

1. INTRODUCTION

Over the past decade, several techniques have been proposed to synthesize 2-D state-space filter structures that minimize the coefficient sensitivity [1]-[6]. Here, 2-D state-space digital filters are represented by either the Roessor model [1]-[3] or the F-M second model [4]-[6]. In [1],[4] all the frequency regions are treated uniformly, whereas the others are interested in the sensitivity behavior of a transfer function within a specified frequency range. To evaluate the weighted sensitivity, a pure L_2 norm is used in [3],[6] instead of a mixture of L_1/L_2 norms. The L_2-sensitivity minimization is more natural and reasonable but technically more challenging than the conventional L_1/L_2 mixed sensitivity minimization. In [6], Li has employed a gradient-flow-based optimization technique to minimize the weighted L_2-sensitivity. However, the drawback of this algorithm is that the convergence rate is very slow.

In this paper, the problem of minimizing the coefficient sensitivity within a specified frequency range is treated for 2-D state-space digital filters described by the F-M second model. Unlike the method reported in [5], no constraint on the weights of the various terms of the mesure is imposed. A simple technique is presented for synthesizing the 2-D filter structures with very low weighted L_1/L_2-sensitivity. This will serve as an initial estimate in the iteration process. An iterative procedure empolyed for minimizing the weighted L_2-sensitivity is based on the matrix Riccati differential equation that was initiated in [7] for the 1-D case.

2. WEIGHTED SENSITIVITY ANALYSIS

Let a 2-D stable, locally controllable and locally observable state-space digital filter be described by

$$\begin{aligned} \boldsymbol{x}(i+1,j+1) &= \boldsymbol{A}_1\boldsymbol{x}(i,j+1)+\boldsymbol{A}_2\boldsymbol{x}(i+1,j) \\ &\quad +\boldsymbol{b}_1 u(i,j+1)+\boldsymbol{b}_2 u(i+1,j) \\ y(i,j) &= \boldsymbol{c}\boldsymbol{x}(i,j)+du(i,j) \end{aligned} \tag{1}$$

where $\boldsymbol{x}(i,j)$ is an $n\times 1$ local state vector, $u(i,j)$ is a scalar input, $y(i,j)$ is a scalar output, and $\boldsymbol{A}_1$, $\boldsymbol{A}_2$, $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, $\boldsymbol{c}$, d are real constant matrices of appropriate dimensions. The transfer function of (1) is given by

$$\begin{aligned} H(z_1,z_2) &= \boldsymbol{c}\left(\boldsymbol{I}_n - z_1^{-1}\boldsymbol{A}_1 - z_2^{-1}\boldsymbol{A}_2\right)^{-1} \\ &\quad \cdot(z_1^{-1}\boldsymbol{b}_1 + z_2^{-1}\boldsymbol{b}_2)+d. \end{aligned} \tag{2}$$

The weighted sensitivity functions are then defined as [3]

$$\begin{aligned} \frac{\delta H(z_1,z_2)}{\delta \boldsymbol{A}_k} &= W_A(z_1,z_2)\frac{\partial H(z_1,z_2)}{\partial \boldsymbol{A}_k} \\ \frac{\delta H(z_1,z_2)}{\delta \boldsymbol{b}_k} &= W_B(z_1,z_2)\frac{\partial H(z_1,z_2)}{\partial \boldsymbol{b}_k} \qquad k=1,2 \\ \frac{\delta H(z_1,z_2)}{\delta \boldsymbol{c}^t} &= W_C(z_1,z_2)\frac{\partial H(z_1,z_2)}{\partial \boldsymbol{c}^t} \end{aligned} \tag{3}$$

where $W_A(z_1,z_2)$, $W_B(z_1,z_2)$ and $W_C(z_1,z_2)$ are three stable, causal, scalar rational functions of the complex variables z_1 and z_2. Let

$$W_A(z_1,z_2)=W_1(z_1,z_2)W_2(z_1,z_2) \tag{4}$$

be a factorization of $W_A(z_1,z_2)$.

Let $\boldsymbol{X}(z_1,z_2)$ be an $m\times n$ complex matrix valued function of the complex variables z_1 and z_2. The L_p-norm of $\boldsymbol{X}(z_1,z_2)$ is defined as

$$||\boldsymbol{X}||_p = \left[\frac{1}{(2\pi j)^2}\oint\oint_{\Gamma^2} ||\boldsymbol{X}(z_1,z_2)||_F^p \frac{dz_1dz_2}{z_1z_2}\right]^{\frac{1}{p}} \quad (5)$$

where $\Gamma^2=\{(z_1,z_2):|z_1|=1,|z_2|=1\}$ and

$$||\boldsymbol{X}(z_1,z_2)||_F = \left[\sum_{p=1}^{m}\sum_{q=1}^{n}|x_{pq}(z_1,z_2)|^2\right]^{\frac{1}{2}}.$$

Then, the overall weighted sensitivity measure can be defined by either of

$$\begin{aligned} m_{1/2} &= \sum_{k=1}^{2}\left|\left|\frac{\delta H(z_1,z_2)}{\delta \boldsymbol{A}_k}\right|\right|_1^2 + \sum_{k=1}^{2}\left|\left|\frac{\delta H(z_1,z_2)}{\delta \boldsymbol{b}_k}\right|\right|_2^2 \\ &\quad + \left|\left|\frac{\delta H(z_1,z_2)}{\delta \boldsymbol{c}^t}\right|\right|_2^2 \end{aligned} \quad (6)$$

and

$$\begin{aligned} m_2 &= \sum_{k=1}^{2}\left|\left|\frac{\delta H(z_1,z_2)}{\delta \boldsymbol{A}_k}\right|\right|_2^2 + \sum_{k=1}^{2}\left|\left|\frac{\delta H(z_1,z_2)}{\delta \boldsymbol{b}_k}\right|\right|_2^2 \\ &\quad + \left|\left|\frac{\delta H(z_1,z_2)}{\delta \boldsymbol{c}^t}\right|\right|_2^2. \end{aligned} \quad (7)$$

Defining

$$\begin{aligned} \boldsymbol{F}(z_1,z_2) &= \left(\boldsymbol{I}_n - z_1^{-1}\boldsymbol{A}_1 - z_2^{-1}\boldsymbol{A}_2\right)^{-1} \\ &\quad \cdot(z_1^{-1}\boldsymbol{b}_1 + z_2^{-1}\boldsymbol{b}_2) \\ \boldsymbol{G}(z_1,z_2) &= \boldsymbol{c}\left(\boldsymbol{I}_n - z_1^{-1}\boldsymbol{A}_1 - z_2^{-1}\boldsymbol{A}_2\right)^{-1}, \end{aligned}$$

the upper bound of (6) is written as

$$\begin{aligned} M_{1/2} &= 2\mathrm{tr}[\boldsymbol{K}_{o1}]\mathrm{tr}[\boldsymbol{K}_{c2}] \\ &\quad +2\mathrm{tr}[\boldsymbol{K}_{oB}] + \mathrm{tr}[\boldsymbol{K}_{cC}] \end{aligned} \quad (8)$$

where $m_{1/2}\le M_{1/2}$. Alternatively, (7) is expressed as

$$m_2 = 2\mathrm{tr}[\boldsymbol{K}_A] + 2\mathrm{tr}[\boldsymbol{K}_{oB}] + \mathrm{tr}[\boldsymbol{K}_{cC}]. \quad (9)$$

Here $\boldsymbol{K}_{o1}$, $\boldsymbol{K}_{c2}$, $\boldsymbol{K}_{oB}$, $\boldsymbol{K}_{cC}$ and $\boldsymbol{K}_A$ are called the weighted Gramians and can be obtained by the following general expression :

$$\boldsymbol{K} = \frac{1}{(2\pi j)^2}\oint\oint \boldsymbol{Y}(z_1,z_2)\boldsymbol{Y}^*(z_1,z_2)\frac{dz_1dz_2}{z_1z_2} \quad (10)$$

where

$$\begin{aligned} \boldsymbol{Y}(z_1,z_2) &= W_1^*(z_1,z_2)\boldsymbol{G}^*(z_1,z_2) \\ \boldsymbol{Y}(z_1,z_2) &= W_2(z_1,z_2)\boldsymbol{F}(z_1,z_2) \\ \boldsymbol{Y}(z_1,z_2) &= W_B^*(z_1,z_2)\boldsymbol{G}^*(z_1,z_2) \\ \boldsymbol{Y}(z_1,z_2) &= W_C(z_1,z_2)\boldsymbol{F}(z_1,z_2) \\ \boldsymbol{Y}(z_1,z_2) &= W_A(z_1,z_2)\boldsymbol{G}^t(z_1,z_2)\boldsymbol{F}^t(z_1,z_2), \end{aligned}$$

respectively.

3. WEIGHTED L_1/L_2-SENSITIVITY REDUCTION

Applying the coordinate transformation $\overline{\boldsymbol{x}}(i,j) = \boldsymbol{T}^{-1}\boldsymbol{x}(i,j)$ to (1), we obtain new coefficients

$$\overline{\boldsymbol{A}}_k = \boldsymbol{T}^{-1}\boldsymbol{A}_k\boldsymbol{T},\ \ \overline{\boldsymbol{b}}_k = \boldsymbol{T}^{-1}\boldsymbol{b}_k,\ \ \overline{\boldsymbol{c}} = \boldsymbol{c}\boldsymbol{T} \quad (11)$$

and new weighted Gramians

$$\begin{aligned} \overline{\boldsymbol{K}}_{o1} &= \boldsymbol{T}^t\boldsymbol{K}_{o1}\boldsymbol{T}, \quad \overline{\boldsymbol{K}}_{c2} = \boldsymbol{T}^{-1}\boldsymbol{K}_{c2}\boldsymbol{T}^{-t} \\ \overline{\boldsymbol{K}}_{oB} &= \boldsymbol{T}^t\boldsymbol{K}_{oB}\boldsymbol{T}, \quad \overline{\boldsymbol{K}}_{cC} = \boldsymbol{T}^{-1}\boldsymbol{K}_{cC}\boldsymbol{T}^{-t} \end{aligned} \quad (12)$$

This makes it possible to write (8) as

$$\overline{M}_{1/2}(\boldsymbol{P}) = J(\boldsymbol{P}) + L(\boldsymbol{P}) \quad (13)$$

where $\boldsymbol{P}=\boldsymbol{T}\boldsymbol{T}^t$ and

$$\begin{aligned} J(\boldsymbol{P}) &= 2\mathrm{tr}[\boldsymbol{K}_{o1}\boldsymbol{P}]\mathrm{tr}[\boldsymbol{K}_{c2}\boldsymbol{P}^{-1}] \\ L(\boldsymbol{P}) &= 2\mathrm{tr}[\boldsymbol{K}_{oB}\boldsymbol{P}] + \mathrm{tr}[\boldsymbol{K}_{cC}\boldsymbol{P}^{-1}]. \end{aligned}$$

The extrema of $J(\boldsymbol{P})$ satisfies

$$\begin{aligned} \frac{\partial J(\boldsymbol{P})}{\partial \boldsymbol{P}} &= 2(\mathrm{tr}[\boldsymbol{K}_{c2}\boldsymbol{P}^{-1}]\boldsymbol{K}_{o1} \\ &\quad -\mathrm{tr}[\boldsymbol{K}_{o1}\boldsymbol{P}]\boldsymbol{P}^{-1}\boldsymbol{K}_{c2}\boldsymbol{P}^{-1}) = \boldsymbol{0}. \end{aligned} \quad (14)$$

All the solution of (14) take the form

$$\boldsymbol{P} = \rho\boldsymbol{P}_b \quad (15)$$

where $\boldsymbol{P}_b$ is the unique solution of $\boldsymbol{P}\boldsymbol{K}_{o1}\boldsymbol{P} = \boldsymbol{K}_{c2}$ given by

$$\boldsymbol{P}_b = \boldsymbol{K}_{o1}^{-\frac{1}{2}}[\boldsymbol{K}_{o1}^{\frac{1}{2}}\boldsymbol{K}_{c2}\boldsymbol{K}_{o1}^{\frac{1}{2}}]^{\frac{1}{2}}\boldsymbol{K}_{o1}^{-\frac{1}{2}} \quad (16)$$

and ρ is an arbitrary positive number. Moreover, $J(\boldsymbol{P})$ has the single extremum described by

$$\begin{aligned} J^o &= J(\rho\boldsymbol{P}_b) = 2\left(\mathrm{tr}[\boldsymbol{K}_{o1}\boldsymbol{P}_b]\right)^2 = 2\left(\mathrm{tr}[\boldsymbol{K}_{c2}\boldsymbol{P}_b^{-1}]\right)^2 \\ &= 2\left(\mathrm{tr}[\boldsymbol{K}_{c2}\boldsymbol{K}_{o1}]^{\frac{1}{2}}\right)^2 = 2\left(\sum_{i=1}^{n}\sigma_i\right)^2 \end{aligned} \quad (17)$$

where $\sigma_i^2, i=1,2,\cdots,n$ are the eigenvalues of $\boldsymbol{K}_{c2}\boldsymbol{K}_{o1}$.

Substituting (15) into (13) and using (17) gives

$$\begin{aligned} \overline{M}_{1/2}(\boldsymbol{P}) &= 2\left(\sum_{i=1}^{n}\sigma_i\right)^2 + 2\rho\,\mathrm{tr}[\boldsymbol{K}_{oB}\boldsymbol{P}_b] \\ &\quad +\rho^{-1}\mathrm{tr}[\boldsymbol{K}_{cC}\boldsymbol{P}_b^{-1}]. \end{aligned} \quad (18)$$

Here, the arithmetic-geometric inequality says that

$$\begin{aligned} &2\rho\,\mathrm{tr}[\boldsymbol{K}_{oB}\boldsymbol{P}_b] + \rho^{-1}\mathrm{tr}[\boldsymbol{K}_{cC}\boldsymbol{P}_b^{-1}] \\ &\quad \ge 2\sqrt{2\mathrm{tr}[\boldsymbol{K}_{oB}\boldsymbol{P}_b]\mathrm{tr}[\boldsymbol{K}_{cC}\boldsymbol{P}_b^{-1}]} \end{aligned} \quad (19)$$

where equality is valid if and only if

$$\rho = \sqrt{\frac{\mathrm{tr}[\boldsymbol{K}_{cC}\boldsymbol{P}_b^{-1}]}{2\mathrm{tr}[\boldsymbol{K}_{oB}\boldsymbol{P}_b]}}. \tag{20}$$

Substituting (20) into (15) yields

$$\boldsymbol{P} = \sqrt{\frac{\mathrm{tr}[\boldsymbol{K}_{cC}\boldsymbol{P}_b^{-1}]}{2\mathrm{tr}[\boldsymbol{K}_{oB}\boldsymbol{P}_b]}}\boldsymbol{P}_b. \tag{21}$$

Moreover, substituting (20) into (18) yields equality in (19) and

$$\begin{aligned}&\overline{M}_{1/2}(\boldsymbol{P})\\ &= 2\left[(\sum_{i=1}^{n}\sigma_i)^2 + \sqrt{2\mathrm{tr}[\boldsymbol{K}_{oB}\boldsymbol{P}_b]\mathrm{tr}[\boldsymbol{K}_{cC}\boldsymbol{P}_b^{-1}]}\right].\end{aligned} \tag{22}$$

4. WEIGHTED L_2-SENSITIVITY MINIMIZATION

Applying Parseval's relation to (9), we obtain

$$\begin{aligned} m_2 &= 2\,\mathrm{tr}\left[\sum_{k=0}^{\infty}\sum_{i+j=k}\boldsymbol{M}_A(i,j)\boldsymbol{M}_A^t(i,j)\right]\\ &\quad +2\,\mathrm{tr}\,[\boldsymbol{K}_{oB}] + \mathrm{tr}\,[\boldsymbol{K}_{cC}] \end{aligned} \tag{23}$$

where $\boldsymbol{M}_A(i,j)$ can be derived from (10). By carrying out the state-space coordinate transformation, (23) is changed to

$$\begin{aligned} \overline{m}_2(\boldsymbol{P}) &= 2\mathrm{tr}\left[\sum_{k=0}^{\infty}\sum_{i+j=k}\boldsymbol{P}\boldsymbol{M}_A(i,j)\boldsymbol{P}^{-1}\boldsymbol{M}_A^t(i,j)\right]\\ &\quad +2\,\mathrm{tr}\,[\boldsymbol{P}\boldsymbol{K}_{oB}] + \mathrm{tr}\left[\boldsymbol{K}_{cC}\boldsymbol{P}^{-1}\right] \end{aligned} \tag{24}$$

where $\boldsymbol{P}$ is defined as in (13).

Differentiating (24) w.r.t. $\boldsymbol{P}$ yields

$$\frac{\partial\overline{m}_2(\boldsymbol{P})}{\partial\boldsymbol{P}} = \boldsymbol{E}(\boldsymbol{P}) - \boldsymbol{P}^{-1}\boldsymbol{F}(\boldsymbol{P})\boldsymbol{P}^{-1} \tag{25}$$

where

$$\begin{aligned} \boldsymbol{E}(\boldsymbol{P}) &= 2\sum_{k=0}^{\infty}\sum_{i+j=k}\boldsymbol{M}_A(i,j)\boldsymbol{P}^{-1}\boldsymbol{M}_A^t(i,j)\\ &\quad +2\boldsymbol{K}_{oB}\\ \boldsymbol{F}(\boldsymbol{P}) &= 2\sum_{k=0}^{\infty}\sum_{i+j=k}\boldsymbol{M}_A^t(i,j)\boldsymbol{P}\boldsymbol{M}_A(i,j) + \boldsymbol{K}_{cC}. \end{aligned}$$

An iterative method for computing the limiting solution of the matrix Riccati differential equation was originally employed to minimize the L_2-sensitivity for 1-D case [7]. Applying it to the minimization of (24) provides

$$\begin{aligned} \boldsymbol{P}_{i+1} &= \boldsymbol{P}_i + 2\boldsymbol{F}(\boldsymbol{P}_i)/\alpha - [\boldsymbol{P}_i + \boldsymbol{F}(\boldsymbol{P}_i)/\alpha]\\ &\quad \cdot[2\boldsymbol{P}_i + \boldsymbol{F}(\boldsymbol{P}_i)/\alpha + \alpha\boldsymbol{E}^{-1}(\boldsymbol{P}_i)]^{-1}\\ &\quad \cdot[\boldsymbol{P}_i + \boldsymbol{F}(\boldsymbol{P}_i)/\alpha] \end{aligned} \tag{26}$$

where $\alpha > 0$ is any scalar constant and $\boldsymbol{P}_i$ is the solution of the previous iteration and the initial estimate $\boldsymbol{P}_0$ is given by (21). This iteration process continues until

$$|\,\overline{m}_2(\boldsymbol{P}_{i+1}) - \overline{m}_2(\boldsymbol{P}_i)\,| < \varepsilon \tag{27}$$

where $\varepsilon > 0$ is a prescribed tolerance.

Once the optimal $\boldsymbol{P}$ is obtained, the optimal coordinate transformation matrix can be constructed as

$$\boldsymbol{T} = \boldsymbol{P}^{\frac{1}{2}}\boldsymbol{U} \tag{28}$$

where $\boldsymbol{U}$ is any $n \times n$ orthogonal matrix.

5. AN ILLUSTRATIVE EXAMPLE

Let the LSS model (1) be specified by

$$\boldsymbol{A}_1 = \begin{bmatrix} -0.05441 & -0.30243 & -0.17978\\ 0.24801 & 0.17827 & -0.61119\\ -0.12387 & 0.19389 & 0.86071 \end{bmatrix}$$

$$\boldsymbol{A}_2 = \begin{bmatrix} 0.67444 & 0.02136 & -0.09449\\ -0.07846 & 0.87476 & 0.73312\\ 0.03742 & -0.26273 & -0.00523 \end{bmatrix}$$

$$\boldsymbol{b}_1 = \begin{bmatrix} 6.59099\\ -18.72488\\ 12.57083 \end{bmatrix} \quad \boldsymbol{b}_2 = \begin{bmatrix} 2.82259\\ -10.17446\\ 7.35410 \end{bmatrix}$$

$$\boldsymbol{c} = \begin{bmatrix} 1.07459 & 1.53783 & 1.80520 \end{bmatrix}$$

and let

$$\begin{aligned} W_A(z_1,z_2) &= \sum_{0\le i+j\le 20}\sum w(i,j)z_1^{-i}z_2^{-j}\\ W_B(z_1,z_2) &= \frac{0.1 + 0.15z_1^{-1} + 0.15z_2^{-1}}{1 - 0.3z_1^{-1} - 0.3z_2^{-1}}\\ W_C(z_1,z_2) &= \frac{0.1 + 0.05z_1^{-1} + 0.05z_2^{-1}}{1 - 0.4z_1^{-1} - 0.4z_2^{-1}} \end{aligned}$$

where

$$w(i_1,i_2) = 0.256322\exp\left[-0.103203\sum_{k=1}^{2}(i_k - 4)^2\right]$$

and it is assumed that $W_1(z_1,z_2) = W_A(z_1,z_2)$ and $W_2(z_1,z_2) = 1$.

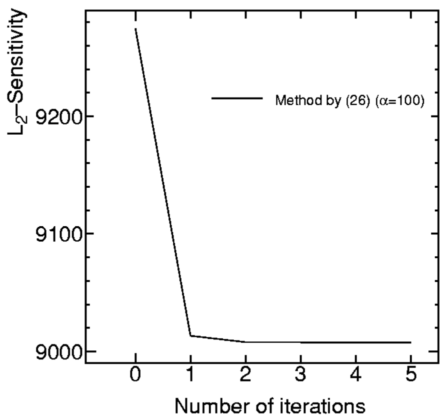

Fig.1. Performace of the convergence by applying (26).

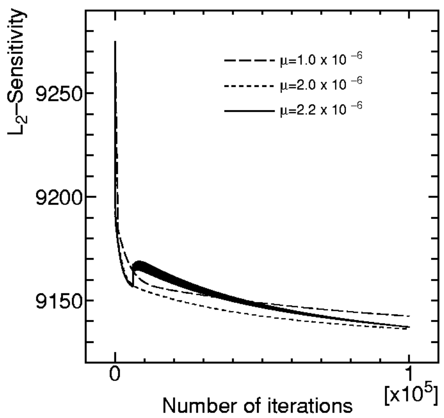

Fig.2. Performace of the convergence by applying the method reported in [6].

TABLE I
NUMERICAL RESULTS IN THE APPLICATION OF (26).

(a) $\alpha = 50$

0	9274.9705
1	9019.4565
2	9009.0366
3	9007.7977
4	9007.6074
5	9007.5750

(b) $\alpha = 100$

0	9274.9705
1	9013.2938
2	9007.8728
3	9007.6001
4	9007.5728
5	9007.5687

The intial estimate $\boldsymbol{P}_0$ was analytically obtained by using (21). Then (26) was used to minimize (24) iteratively. The performance of convergence is drawn in Fig.1 and the relations between i and $\overline{m}_2$ are summarized in Table I numerically. In addition, Fig.2 is given to compare it with the case of using the method reported in [6] where the same intial estimate was used. From these figures it is observed that the convergence rate of (26) is much faster than the case of using the method reported in [6].

6. CONCLUSION

For 2-D state-space digital filters described by the F-M second model, an analytical method has been presented to synthesize the 2-D filter structures with very low weighted L_1/L_2 mixed sensitivity. Then an iterative procedure based on the matrix Riccati differential equation has been applied to synthesize the 2-D filter structures with minimum weighted L_2-sensitivity. The results of a numerical example have shown that the convergence rate of the above iterative procedure is much faster than that reported in [6].

7. REFERENCES

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