LOCALIZATION OF A DISTRIBUTED SOURCE WHICH IS "PARTIALLY COHERENT" - MODELING AND CRAMER RAO BOUNDS

Raviv Raich, Jason Goldberg, and Hagit Messer

Department of Electrical Engineering–Systems
Tel Aviv University, Tel Aviv 69978, Israel
{raviv,jason,messer}@eng.tau.ac.il

ABSTRACT

The problem of using antenna array measurements to estimate the bearing of a mobile communications user surrounded by local scatterers is considered. The concept of "partial coherence" is introduced to account for the *temporal* as well as spatial correlation effects often encountered in mobile radio propagation channels. A simple, intuitive parametric model for temporal channel correlation is presented. The result is an overall spatio-temporal channel model which is more realistic than formerly proposed models (which assume either full or zero temporal channel correlation). Thus, previously posed bearing estimation problems for a "distributed" or "scattered" source are generalized to a joint spatio-temporal parameter estimation problem. A study of the associated Cramer-Rao Bound for the case of known transmitted signal of constant modulus indicates that the inherent accuracy limitations associated with this generalized problem lie somewhere between the cases of zero and full temporal correlation and become more severe as temporal channel correlation increases.

1. INTRODUCTION

Most source bearing estimation research over the past several years has focused on sources which are modeled as single points in space e.g., [1]. However, in applications such as mobile communications where scatters in the vicinity of the mobile user give rise to multipath effects, a so-called "distributed" or "scattered" source model is more appropriate [2]. A distributed source can be thought of as possessing spatial extent over some continuum of directions. This spatial extent is typically characterized by a parametric spatial density function, see e.g., [3] and references therein.

1.1. Previously Proposed Distributed Source Models

Several types of distributed sources have recently appeared in the literature. For a so-called "incoherently distributed" (ID) source, the time varying channel formed between the source and the elements of the antenna array is described by a random, stationary vector process with zero temporal correlation [3]. That is, the channel vectors are completely uncorrelated from one measurement snapshot to the next. The instantaneous spatial correlation of the channel vectors is described by a spatial correlation matrix which is typically assumed to be known to within a set of spatial parameters such as angular mean and standard deviation e.g., [3]-[5]. This model is appropriate for scenarios in which the mobile and/or scatterers move very rapidly with respect to the rate at which measurements are taken.

In contrast, for a "coherently distributed" (CD) source, the channel formed between the source and the array elements is modeled as a deterministic, time invariant vector typically assumed known to within a set of spatial parameters [3]. This model is not suitable for mobile communications applications, but, as argued in [3], may be suitable for active bearing estimation problems where the transmitted signal is reflected by different parts of a "large" object.

The "generalized array manifold" (GAM) source is related to both the ID and CD source [6]. Like the ID source, the channel vector is again random with spatial correlation described by a parametric correlation matrix. However, in contrast to the ID source, the GAM channel vectors are *fully correlated* in time from one snapshot to the next. That is, the channel vectors are all equal, consisting of a *single* realization of the random vector and (like the CD source) are time-invariant. Moreover, the GAM source model, is appropriate for communications applications where the mobile and scatterers are essentially static over the entire interval during which the received signals at the antenna array are measured.

1.2. The "Partially Coherent" Distributed (PCD) Concept

The ID and GAM cases represent the two extreme scenarios of rapidly fluctuating (zero temporal channel correlation) and static channel (full temporal channel correlation) conditions, respectively. In practice, however, some intermediate scenario with partial temporal channel correlation is more likely to be encountered.

This paper addresses this intermediate case by proposing a class of stationary spatio-temporally random channel vector models which in general possess *some* form of temporal correlation, with the ID and GAM sources included as limiting cases. In keeping with the terminology proposed in [3], this class of models will be referred to as "partially coherent distributed" (PCD) sources.

After introducing a simple, intuitive temporal correlation model for the temporal channel correlation effects, the associated problem of bearing estimation for a PCD source is examined. The Cramer-Rao Bound (CRB) for this problem (for the case of known transmitted signal of constant modulus) is derived and compared to the ID and GAM cases as key scenario parameters such as signal-to-noise-ratio (SNR), temporal correlation effects, angular spreading, and observation interval are varied.

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2. MATHEMATICAL MODELING OF THE PARTIALLY COHERENT DISTRIBUTED SOURCE

Consider a single narrow band, far field, distributed source centered at frequency, ω_o, with wave-fields that impinge on a co-planar array of M antenna elements. The discrete time complex envelope of the received $M \times 1$ dimensional array snapshot vector may thus be modeled as:

$$\mathbf{y}(t_k) = \mathbf{b}(t_k)s(t_k) + \mathbf{n}(t_k) \tag{1}$$

where $\mathbf{b}(t_k)$, $s(t_k)$, and $\mathbf{n}(t_k)$ respectively, are the $M \times 1$ channel vector formed between the user and the M antenna elements at the array, the (scalar) signal transmitted by the mobile user, and $M \times 1$ additive noise vector all at discrete time, t_k. Note that this model corresponds to a *flat fading* multipath scenario e.g., [7]. The additive noise $\mathbf{n}(t_k)$ is a zero mean, spatio-temporally white, stationary, complex, circular, Gaussian random vector process with covariance matrix $\sigma_n^2\mathbf{I}_M$, where $\mathbf{I}_\cdot$ denotes the identity matrix of specified dimension.

For an ideal point source, the channel vector is the deterministic $M \times 1$ array steering vector: $\mathbf{a}(\theta) = [e^{-j\omega_o\tau_1(\theta)}, \cdots, e^{-j\omega_o\tau_M(\theta)}]^T$, where $\{\tau_m(\theta)\}_{m=1}^{M}$ is the set of M differential delays across the array, θ is the source bearing and $(\cdot)^T$ denotes the transpose operation.

For a distributed source the channel vector is modeled as a *random* vector such that:

$$\mathbf{b}(t_k) = \int_{-\pi}^{\pi} f(\theta, t_k|\boldsymbol{\psi})\mathbf{a}(\theta)d\theta \tag{2}$$

where $f(\theta, t_k|\boldsymbol{\psi})$ is a complex, random spatio-temporal weighting function which represents the local scattering about the source and $\boldsymbol{\psi}$ represents the spatial source parameters (e.g., its mean direction and its spread about it). The channel vector is, by application of the central limit theorem to (2), modeled as a zero mean, circular Gaussian random vector process.

Under the ID model, there is no correlation between successive measurements of the channel vector so it is modeled as an independent identically distributed ($i.i.d.$) complex Gaussian vector of zero mean and covariance:

$$\begin{aligned} E[\mathbf{b}(t_k)\mathbf{b}^H(t_{k'})] &= \delta_{kk'}\mathbf{R}_b(\boldsymbol{\psi}) \\ \mathbf{R}_b(\boldsymbol{\psi}) &= \int_{-\pi}^{\pi} r(\theta|\boldsymbol{\psi})\mathbf{a}(\theta)\mathbf{a}^H(\theta)d\theta \end{aligned} \tag{3}$$

where $r(\cdot|\boldsymbol{\psi})$ is an arbitrary probability density type function which represents the spatial distribution of the source (e.g., Gaussian, uniform, etc.). For the GAM modeling it is assumed that all the available measurements, $\mathbf{y}(t_k),\ k \in \{1, \cdots, K\}$, are related to *the same* channel vector, $\mathbf{b}(t_k) = \mathbf{b}$, which is a zero mean complex Gaussian vector of covariance $\mathbf{R}_b(\boldsymbol{\psi})$.

The idea behind the PCD model is to allow for some partial correlation between successive samples of the channel vector. A simple, intuitive model for (partial) temporal channel correlation is that of the first order, stationary auto-regressive (AR) model with a correlation between two adjacent time samples of α. This is a space-time generalization of the purely temporal first order AR channel model presented in eg., [8]. The suggested model for the channel vector in the partially coherent case is:

$$\mathbf{b}(t_k) = \alpha\mathbf{b}(t_{k-1}) + \sqrt{1-\alpha^2}\mathbf{b}_d(t_k) \tag{4}$$

where $\mathbf{b}_d(t_k)$ is a zero mean complex $i.i.d$ driving term of a zero mean, circular Gaussian random vector process:

$$\mathbf{b}_d(t_k) \sim \mathcal{CN}(\mathbf{0}_{M\times 1}, \mathbf{R}_b(\boldsymbol{\psi}))\,, \tag{5}$$

$$E[\mathbf{b}_d(t_k)\mathbf{b}_d^H(t_{k'})] = \delta_{kk'}\mathbf{R}_b(\boldsymbol{\psi}) \tag{6}$$

where the notation $\sim \mathcal{CN}(\cdot,\cdot)$ means "is a complex, circular Gaussian random vector of specified mean and covariance". The above implies that $\mathbf{b}(t_k)$ is a temporal correlated Gaussian random process with first and second order statistics given as:

$$\begin{aligned} E[\mathbf{b}(t_k)] &= \mathbf{0}_{M\times 1} \\ E[\mathbf{b}(t_k)\mathbf{b}^H(t_{k'})] &= \alpha^{|k-k'|}\mathbf{R}_b(\psi) \end{aligned}$$

The suggested model for the PCD source may be over simplified in some cases. However, it serves as an initial model for a practical scenario of collecting measurements in a random channel of some given coherence time. Special cases of the above model are the ID case, where $\alpha = 0$, and the GAM, for which $\alpha = 1$. The parameters which characterize the overall model are the signal "parameters," $\mathbf{s}(t_k)$, the noise variance σ_n^2, the channel parameter, α, and the source location parameters, $\boldsymbol{\psi}$.

With all parameters unknown, the overall parameter estimation problem is very complicated. To reduce complexity, in the following, the problem of source location (i.e., estimating $\boldsymbol{\psi}$) where the channel parameter and the noise parameters are unknown, but the signal wave-shape is known, is referred. To make it realistic it is assumed that the complex attenuation of the received signal is unknown. Such a scenario may be realistic when a known training sequence is transmitted.

3. PROBLEM FORMULATION FOR KNOWN CONSTANT MODULUS TRANSMITTED SIGNAL

Consider the case where the transmitted signal is of constant modulus, known up to a complex scale factor:

$$s(t_k) = \sigma_s e^{j\phi}u(t_k); \quad u(t_k) = e^{j\gamma(t_k)} \tag{7}$$

where $u(t_k)$ is known and the signal strength and phase parameters, σ_s and ϕ, respectively, are unknown. In this case, a new set of preprocessed measurements can be defined as:

$$\begin{aligned} \tilde{\mathbf{y}}(t_k) &= e^{-j\gamma(t_k)}\mathbf{y}(t_k) = \sigma_s\tilde{\mathbf{b}}(t_k) + \tilde{\mathbf{n}}(t_k) \\ \tilde{\mathbf{b}}(t_k) &= e^{j\phi}\mathbf{b}(t_k), \quad \tilde{\mathbf{n}}(t_k) = e^{-j\gamma(t_k)}\mathbf{n}(t_k) \end{aligned} \tag{8}$$

The preprocessing does not in any way alter the statistical properties of the channel and the noise components which, using the new notation above, can be expressed as:

$$\begin{aligned} \tilde{\mathbf{b}}(t_k) &= \alpha\tilde{\mathbf{b}}(t_{k-1}) + \sqrt{1-\alpha^2}\tilde{\mathbf{b}}_d(t_k) \\ \tilde{\mathbf{b}}_d(t_k) &= e^{j\phi}\mathbf{b}_d(t_k) \sim \mathcal{CN}(\mathbf{0}_{M\times 1}, \mathbf{R}_b(\boldsymbol{\psi})) \\ \tilde{\mathbf{n}}(t_k) &\sim \mathcal{CN}(\mathbf{0}_{M\times 1}, \sigma_n^2\mathbf{I}_M). \end{aligned}$$

If K snapshots are available, the set of K preprocessed vector measurements can be concatenated into a single vector of length KM:

$$\tilde{\mathbf{y}} = \left[\tilde{\mathbf{y}}^T(t_1), \tilde{\mathbf{y}}^T(t_2), \cdots, \tilde{\mathbf{y}}^T(t_K)\right]^T \tag{9}$$

Since the channel vector and noise processes are mutually independent Gaussian random processes, the concatenated data vector, $\tilde{\mathbf{y}}$, is also Gaussian:

$$\tilde{\mathbf{y}} \sim \mathcal{CN}(\mathbf{0}_{KM\times 1}, \mathbf{R}_{\tilde{y}}) \tag{10}$$

$$\mathbf{R}_{\tilde{y}} = \sigma_s^2 \mathbf{V} \otimes \mathbf{R}_b + \sigma_n^2 \mathbf{I}_{KM} \tag{11}$$

$$\mathbf{V} = \begin{bmatrix} 1 & \alpha & \alpha^2 & \cdots & \alpha^{K-1} \\ \alpha & \ddots & & & \\ \alpha^2 & \ddots & & & \vdots \\ \vdots & & & & \alpha \\ \alpha^{K-1} & \cdots & \alpha^2 & \alpha & 1 \end{bmatrix}$$

where $\cdot \otimes \cdot$ denotes the Kronecker matrix product.

The bearing estimation problem to be solved may be formulated as that of using the set of K array snapshot measurements, $\{\mathbf{y}(t_k)\}_{k=1}^{K}$, to estimate the mobile user's spatial parameters, $\boldsymbol{\psi}$.

4. THE CRAMER RAO BOUND

Assume that the distributed source is spatially characterized by two parameters: its mean direction, θ_0, and its spread about this direction, Δ. As such, $\boldsymbol{\psi} = [\theta_0, \Delta]^T$. Under this model, the vector of all unknown parameters in the problem is:

$$\boldsymbol{\Theta} = [\theta_0, \Delta, \sigma_s^2, \sigma_n^2, \alpha]^T. \tag{12}$$

Due to the Gaussianity of the data (10), the Fisher Information Matrix (FIM) can be obtained using e.g., [9]:

$$\mathbf{J}_{ij} = \mathrm{tr}\left\{ \mathbf{R}_{\tilde{y}}^{-1} \frac{\partial \mathbf{R}_{\tilde{y}}}{\partial \Theta_i} \mathbf{R}_{\tilde{y}}^{-1} \frac{\partial \mathbf{R}_{\tilde{y}}}{\partial \Theta_j} \right\}. \tag{13}$$

It is shown in [10] that the FIM for the PCD is expressed in terms of the 4×4 FIM of the ID with a single snapshot, denoted by $\mathbf{J}^{\mathrm{ID}}$:

$$\mathbf{J} = \sum_{k=1}^{K} \mathbf{T}_k \mathbf{J}^{\mathrm{ID}} \left(\theta_0, \Delta, \sigma_s^2, \frac{\sigma_n^2}{\lambda_t^{(k)}} \right) \mathbf{T}_k^T,$$

$$\mathbf{T}_k = \begin{bmatrix} \mathrm{diag}\left[1, 1, 1, \frac{1}{\lambda_t^{(k)}} \right] \\ 0,\ 0,\ \sigma_s^2\ \frac{\dot{\lambda}_t^{(k)}}{\lambda_t^{(k)}}, 0 \end{bmatrix} \tag{14}$$

where $\lambda_t^{(k)}$ and $\mathbf{u}_t^{(k)}$ are, respectively, the k'th eigenvalue and the k'th eigenvector in the eigen-decomposition of $\mathbf{V}$ given in:

$$\mathbf{V} = \sum_{k=1}^{K} \lambda_t^{(k)} \mathbf{u}_t^{(k)} \mathbf{u}_t^{(k)^H} \tag{15}$$

and $\dot{\lambda}_t^{(k)}$ is the first derivative of $\lambda_t^{(k)}$ with respect to α.Note that $\lambda_t^{(k)}$, $\dot{\lambda}_t^{(k)}$, and $\mathbf{u}_t^{(k)}$ are α dependent.

The bearing of the distributed source is defined to be its mean angle, θ_0. To obtain the CRB for the mean angle, one needs to evaluate the five-dimensional FIM of (14) and to invert it. The 1×1 entry of $\mathbf{J}^{-1}$ is the CRB on the mean direction of a PCD source of unknown spreading, where the source signal is a known constant modulus signal of unknown complex attenuation, and where the level of the additive noise and the temporal correlation coefficient of the propagation channel are unknown.

5. NUMERICAL STUDY

To illustrate the results consider a distributed source with a Gaussian shape spreading, such that:

$$r(\theta|\theta_0, \Delta) = \frac{1}{\Delta\sqrt{2\pi}} e^{\frac{-(\theta-\theta_o)^2}{2\Delta^2}} \tag{16}$$

Assume an equally spaced four sensors linear array of inter sensor separation of $\Lambda/2$, where Λ is the wavelength of the transmitting source. Further, consider the case where $\theta_0 = 0^\circ$ and $\sigma_s^2 = 1$.

The CRB on θ_0 has been evaluated numerically, using the results of section 4, for different values of α, the spreading, Δ, the SNR$=\frac{\sigma_s^2}{\sigma_n^2}$.

In Fig. 1 the bound for different values of α is depicted as a function of the SNR, where the spreading is fixed to $\Delta = 10^\circ$ and the number of snapshots is $K = 50$. It shows that the bound for a distributed source with random channel vector does not converge to zero when the SNR goes to infinity. However, for $\alpha = 0$ (ID) it converges to a value $\sqrt{K}$ smaller than for the case where $\alpha = 1$ (GAM). With a partial coherent channel ($0 < \alpha < 1$) the bound behaves as that of the GAM for low SNR, and as that of the ID for high SNR. The threshold SNR where the transition occurs is directly related to α.

Fig. 2 depicts the bound for different values of α as a function of the observation time, or the number of snapshots, K. The SNR is fixed to 15dB and the spreading is fixed to 10°, as before. Moreover, it can be shown that for any $\alpha < 1$ the bound converges to zero asymptotically, with a rate which is inversely proportional to K [10].

Fig. 3 depicts the bound for different values of α as a function of the the spreading, Δ. The SNR is fixed to 15dB and $K = 50$. Both under the ID and the GAM models, for $\Delta = 0^\circ$ the bound converges to that of a point source. The same is true under the PCD model with any α. However, the bound is an increasing function of both Δ and α.

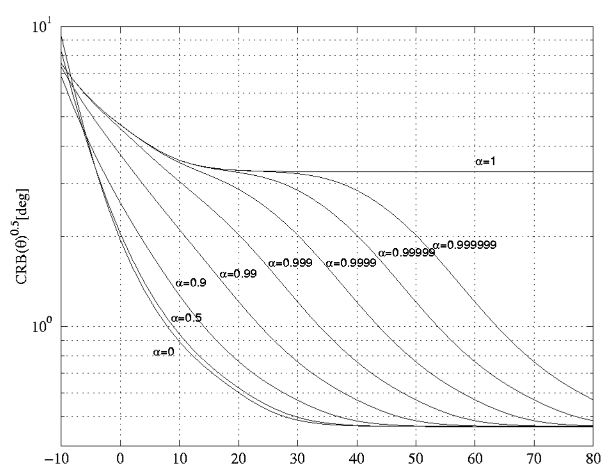

Figure 1: The CRB on the mean direction of a PCD source as a function of the signal to noise ratio. $\theta_0 = 0^\circ$, $K = 50$, $\Delta = 10^\circ$

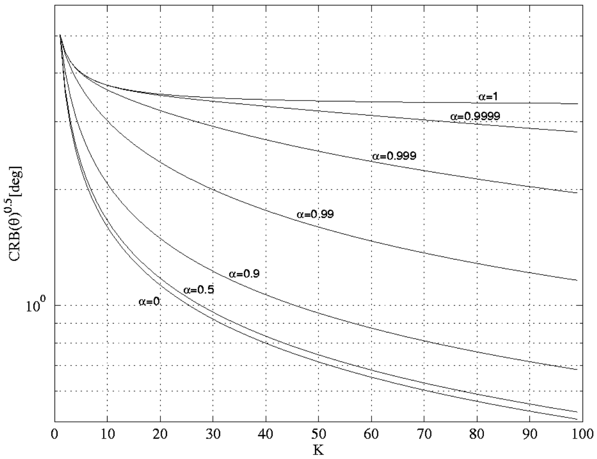

Figure 2: The CRB on the mean direction of a PCD source as a function of the number of snapshots, K. $\theta_0 = 0°$, SNR=15[dB], $\Delta = 10°$

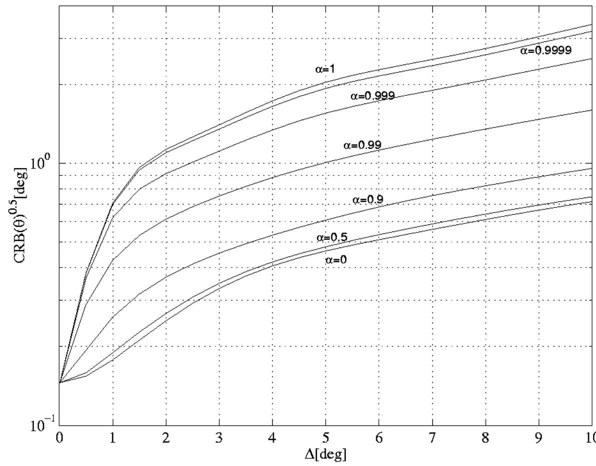

Figure 3: The CRB on the mean direction of a PCD source as a function of the spatial spreading, Δ. $\theta_0 = 0°$, SNR=15[dB], $K = 50$

6. CONCLUSIONS

This paper presents a model which can be used for localizing a source in a random propagation channel, as in mobile communications. According to the model, the source is a PCD type source since successive samples of the channel are partially correlated. The existing incoherent model (ID) and GAM model, which is fully coherent, are special cases of the suggested model.

A parametric model for the data received by an array of sensors is derived and the CRB on the source bearing estimation error for the case of a known, constant modulus source signal is evaluated. It is shown that the ID model results in optimistic bound. Other features of the bound as a function of SNR, the spreading and the observation time are also studied for different amounts of temporal channel correlation.

7. REFERENCES

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