

BLIND CHANNEL ESTIMATION AND EQUALIZATION OF MULTIPLE-INPUT MULTIPLE-OUTPUT CHANNELS

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ABSTRACT

Channel estimation and blind equalization of MIMO (multiple-input multiple-output) communications channels is considered using primarily the second-order statistics of the data. We consider estimation of (partial) channel impulse response and design of finite-length MMSE (minimum mean-square error) blind equalizers. The basis of the approach is the design of a zero-forcing equalizer that whitens the noise-free data. We allow infinite impulse response (IIR) channels. Moreover, the multichannel transfer function need not be column-reduced. Our approaches also work when the “subchannel” transfer functions have common zeros so long as the common zeros are minimum-phase zeros. The channel length or model orders need not be known. The sources are recovered up to a unitary mixing matrix and are further ‘unmixed’ using higher-order statistics of the data. An illustrative simulation example is provided.

1. INTRODUCTION

Consider a discrete-time MIMO system with N outputs and M inputs:

$$\mathbf{y}(k) = \mathcal{F}(z)\mathbf{w}(k) + \mathbf{n}(k) = \mathbf{s}(k) + \mathbf{n}(k) \quad (1-1)$$

where $\mathbf{y}(k) = [y_1(k) \ y_2(k) \ \cdots \ y_N(k)]^T$, similarly for $\mathbf{s}(k)$, $\mathbf{w}(k)$ and $\mathbf{n}(k)$, and z is the $\mathcal{Z}$ -transform variable as well as the backward-shift operator (i.e., $z^{-1}\mathbf{w}(k) = \mathbf{w}(k-1)$, etc.), $\mathbf{s}(k)$ is the noise-free output, $\mathbf{n}(k)$ is the additive measurement noise and the $N \times M$ matrix $\mathcal{F}(z)$ is given by

$$\mathcal{F}(z) := \sum_{l=0}^{\infty} \mathbf{F}_l z^{-l} = \mathcal{A}^{-1}(z)\mathcal{B}(z), \quad (1-2)$$

$$\mathcal{A}(z) = I + \sum_{i=1}^{n_a} \mathbf{A}_i z^{-i} \text{ and } \mathcal{B}(z) = \sum_{i=0}^{n_b} \mathbf{B}_i z^{-i}. \quad (1-3)$$

We allow all of the above variables to be complex-valued.

Such models arise in several useful baseband-equivalent digital communications and other applications [1]-[6],[8],[10]-[12],[14]. In these applications one of the objectives is to recover the inputs $\mathbf{w}(k)$ given the noisy measurements but not given the knowledge of the system transfer function. A large number of papers (see [4],[5],[10],[11],[14]) have concentrated on a two-step procedure: first estimate the channel impulse response (IR) and then design an equalizer using the estimated channel. A fundamental restriction in these works is that the channel is FIR with no common zeros among the various subchannels. A few (see [1] and [12], e.g.) have proposed direct design of the equalizer by-passing channel estimation. Still they assume FIR (SIMO) channels with no common zeros.

In this paper we allow IIR channels. The MIMO channel does not have to be column-reduced. We will also allow common zeros so long as they are minimum-phase. Finally, in the presence of nonminimum-phase common zeros,

our proposed approach equalizes the spectrally-equivalent minimum-phase counterpart of $\mathcal{F}(z)$; it does not “fall apart” unlike quite a few existing approaches. Our proposed approach extends the SIMO results of [1] and [8] to MIMO channels.

2. PRELIMINARIES

2.1. FIR Inverses

Assume the following:

- (H1) $N > M$.
- (H2) $\text{Rank}\{\mathcal{B}(z)\} = M \ \forall z$ including $z = \infty$ but excluding $z = 0$, i.e., $\mathcal{B}(z)$ is irreducible [7, Sec. 6.3].
- (H3) $\det(\mathcal{A}(z)) \neq 0$ for $|z| \geq 1$.

It has been shown in [6] (using some results from [2]) that under (H1)-(H3) there exists a finite degree left-inverse (not necessarily unique) of $\mathcal{F}(z)$:

$$\mathcal{G}(z)\mathcal{F}(z) = I_M \quad (2-1)$$

where $\mathcal{G}(z)$ is $M \times N$ given by

$$\mathcal{G}(z) = \sum_{l=0}^{L_e} \mathbf{G}_l z^{-l} \text{ for any } L_e \geq n_a + (2M-1)n_b - 1. \quad (2-2)$$

2.2. Linear Innovations Representation

Assume further the following:

- (H4) Input sequence $\{\mathbf{w}(k)\}$ is zero-mean, spatially independent and temporally i.i.d. with each of its components having non-zero fourth cumulants. Take $E\{\mathbf{w}(k)\mathbf{w}^H(k)\} = I_M$ where I_M is the $M \times M$ identity matrix and the superscript $\mathcal{H}$ is the Hermitian operation.

Lemma 1. Under (H1)-(H4), $\{\mathbf{s}(k)\}$ may be represented as

$$\mathbf{s}(k) = - \sum_{i=1}^K \mathbf{D}_i \mathbf{s}(k-i) + I_s(k) \quad (2-3)$$

where $K \leq n_a + Mn_b$, $\mathbf{D}_i$'s are some $N \times N$ matrices and $\{I_s(k)\}$ is a zero-mean white $N \times 1$ random sequence (linear innovations for $\{\mathbf{s}(k)\}$) with

$$I_s(k) = \mathbf{F}_0 \mathbf{w}(k). \quad \bullet \quad (2-4)$$

Proof: See [16]. $\square$

It follows from (1-1) and Lemma 1 that

$$\mathcal{D}(z)\mathbf{s}(k) = \mathcal{D}(z)\mathcal{F}(z)\mathbf{w}(k) = \mathbf{F}_0 \mathbf{w}(k) \quad (2-5)$$

where $\mathcal{D}(z) = I_N + \sum_{i=1}^K \mathbf{D}_i z^{-i}$. Since $\mathbf{w}(k)$ is full-rank and white, it follows from (2-5) that

$$\mathcal{D}(z)\mathcal{F}(z) = \mathbf{F}_0 \Rightarrow (\mathbf{F}_0^H \mathbf{F}_0)^{-1} \mathbf{F}_0^H \mathcal{D}(z)\mathcal{F}(z) = I_M. \quad (2-6)$$

Clearly the $M \times N$ polynomial matrix $\mathcal{G}(z) := (\mathbf{F}_0^\mathcal{H} \mathbf{F}_0)^{-1} \mathbf{F}_0^\mathcal{H} \mathcal{D}(z)$ is of degree $K \leq n_a + M n_b$ and it is a left inverse to $\mathcal{F}(z)$. This result is summarized below.

Lemma 2. Under (H1)-(H4), there exists an integer $K \leq n_a + M n_b$ and a polynomial matrix $\mathcal{G}(z) = \sum_{i=0}^K \mathbf{G}_i z^{-i}$ of degree K such that $\mathcal{G}(z)\mathcal{F}(z) = \mathbf{I}_M$. •

Lemma 3. Let $\mathcal{R}_{ssL_e}$ denote a $[N(L_e + 1)] \times [N(L_e + 1)]$ matrix with its ij -th block element as $\mathbf{R}_{ss}(j-i) = E\{\mathbf{s}(k+j-i)\mathbf{s}^\mathcal{H}(k)\}$. Then under (H1)-(H4), $\rho(\mathcal{R}_{ssL_e}) \leq N L_e + M$ for $L_e \geq n_a + M n_b$ where $\rho(A)$ denotes the rank of A . •
Sketch of proof: It follows from Lemma 1 and (2-3) that

$$\begin{bmatrix} \mathbf{I}_N & \mathbf{D}_1 & \cdots & \mathbf{D}_{n_a+Mn_b} & 0 & \cdots & 0 \end{bmatrix} \mathcal{R}_{ssL_e} \\ = \begin{bmatrix} \mathbf{F}_0 \mathbf{F}_0^\mathcal{H} & 0 & \cdots & 0 \end{bmatrix}. \quad (2-7)$$

Apply Sylvester's inequality [7, p. 655] to (2-7) to deduce the desired result. □

3. EQUALIZATION: NO COMMON ZEROS

Assume that (H1)-(H4) hold true. In addition assume the following regarding the measurement noise:

(H5) $\{\mathbf{n}(k)\}$ is zero-mean Gaussian with $E\{\mathbf{n}(k + \tau)\mathbf{n}^\mathcal{H}(k)\} = \sigma_n^2 \mathbf{I}_N \delta(\tau)$.

3.1. Zero-Delay Zero-Forcing Blind Equalizer

Using (1-2), (2-1) and (2-2), we have

$$\sum_{i=0}^{\infty} \mathbf{G}_{m-i} \mathbf{F}_i = \begin{cases} \mathbf{I}_M, & m = 0 \\ 0, & m = 1, 2, \dots, \end{cases} \quad (3-1)$$

leading to

$$\begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \cdots & \mathbf{G}_{L_e} \end{bmatrix} \bar{\mathcal{S}} = \begin{bmatrix} 1 & 0 & \cdots & \cdots \end{bmatrix} \quad (3-2)$$

where $\bar{\mathcal{S}}$ is the $(N(L_e + 1)) \times \infty$ matrix given by

$$\bar{\mathcal{S}} = \begin{bmatrix} \mathbf{F}_0 & \mathbf{F}_1 & \mathbf{F}_2 & \mathbf{F}_3 & \cdots & \cdots & \cdots \\ 0 & \mathbf{F}_0 & \mathbf{F}_1 & \mathbf{F}_2 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & \mathbf{F}_0 & \mathbf{F}_1 & \cdots \end{bmatrix}. \quad (3-3)$$

Let $\bar{\mathcal{S}}^\#$ denote the pseudoinverse of $\bar{\mathcal{S}}$. By [15, Prop. 1], $\bar{\mathcal{S}}^\# = \bar{\mathcal{S}}^\mathcal{H} (\bar{\mathcal{S}} \bar{\mathcal{S}}^\mathcal{H})^\#$. Then the minimum norm solution to the FIR equalizer is given by [15, Sec. 6.11]

$$\begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \cdots & \mathbf{G}_{L_e} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_0^\mathcal{H} & 0 & \cdots & 0 \end{bmatrix} (\bar{\mathcal{S}} \bar{\mathcal{S}}^\mathcal{H})^\#. \quad (3-4)$$

In a fashion similar to $\mathcal{R}_{ssL_e}$ in Lemma 2, let $\mathcal{R}_{yyL_e}$ denote a $[N(L_e + 1)] \times [N(L_e + 1)]$ matrix with its ij -th block element as $\mathbf{R}_{yy}(j-i) = E\{\mathbf{y}(k+j-i)\mathbf{y}^\mathcal{H}(k)\}$; define similarly $\mathcal{R}_{nnL_e}$ pertaining to the additive noise. Carry out an eigendecomposition of $\mathcal{R}_{yyL_e}$. Then the smallest $N - M$ eigenvalues of $\mathcal{R}_{yyL_e}$ equal σ_n^2 because under (H1)-(H4), $\rho(\mathcal{R}_{ssL_e}) \leq N L_e + M$ whereas $\rho(\mathcal{R}_{nnL_e}) = N L_e + N = \rho(\mathcal{R}_{yyL_e})$. Thus a consistent estimate $\hat{\sigma}_n^2$ of σ_n^2 is obtained by taking it as the average of the smallest $N - M$ eigenvalues of $\hat{\mathcal{R}}_{yyL_e}$, the data-based consistent estimate of $\mathcal{R}_{yyL_e}$.

Under (H4) and (H5),

$$(\bar{\mathcal{S}} \bar{\mathcal{S}}^\mathcal{H}) = \mathcal{R}_{ssL_e} = \mathcal{R}_{yyL_e} - \mathcal{R}_{nnL_e} = \mathcal{R}_{yyL_e} - \sigma_n^2 \mathbf{I}. \quad (3-5)$$

Thus, $(\bar{\mathcal{S}} \bar{\mathcal{S}}^\mathcal{H})$ can be estimated from noisy data. However, we don't know $\mathbf{F}_0$. To this end, we seek an $N \times N$ FIR filter $\mathcal{G}_a(z) := \sum_{i=0}^{L_e} \mathbf{G}_{ai} z^{-i}$ satisfying

$$\begin{bmatrix} \mathbf{G}_{a0} & \mathbf{G}_{a1} & \cdots & \mathbf{G}_{aL_e} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_N & 0 & \cdots & 0 \end{bmatrix} \mathcal{R}_{ssL_e}^\# \quad (3-6)$$

Comparing (3-4) and (3-6) it follows that

$$\begin{bmatrix} \mathbf{G}_0 & \mathbf{G}_1 & \cdots & \mathbf{G}_{L_e} \end{bmatrix} = \mathbf{F}_0^\mathcal{H} \begin{bmatrix} \mathbf{G}_{a0} & \mathbf{G}_{a1} & \cdots & \mathbf{G}_{aL_e} \end{bmatrix} \quad (3-7)$$

leading to

$$\sum_{i=0}^{L_e} \mathbf{G}_i z^{-i} =: \mathcal{G}(z) = \mathbf{F}_0^\mathcal{H} \mathcal{G}_a(z). \quad (3-8)$$

In practice, therefore, we apply $\mathcal{G}_a(z)$ to the data leading to

$$\mathbf{v}(k) := \mathcal{G}_a(z)\mathbf{y}(k) = \mathbf{v}_s(k) + \mathcal{G}_a(z)\mathbf{n}(k) \quad (3-9)$$

such that

$$\mathbf{F}_0^\mathcal{H} \mathbf{v}_s(k) = \mathbf{w}(k) \quad (3-10)$$

where

$$\mathbf{v}_s(k) := \mathcal{G}_a(z)[\mathbf{y}(k) - \mathbf{n}(k)] = \mathcal{G}_a(z)\mathbf{s}(k). \quad (3-11)$$

In (3-10) $\{\mathbf{w}(k)\}$ is a white M -vector sequence (by assumption (H4)), however, $\{\mathbf{v}_s(k)\}$ is not necessarily a white vector sequence. Given the second-order statistics of $\{\mathbf{v}_s(k)\}$, how does one estimate $\mathbf{F}_0$ so that $\{\mathbf{w}(k)\}$ satisfying (H4) is recovered? We need to have $\mathbf{R}_{ww}(\tau) := E\{\mathbf{w}(k+\tau)\mathbf{w}^\mathcal{H}(k)\} = 0$ for $|\tau| \neq 0$ and $\rho(\mathbf{R}_{ww}(0)) = M$. By (3-10), $\mathbf{R}_{ww}(\tau) = \mathbf{F}_0^\mathcal{H} \mathbf{R}_{v_s v_s}(\tau) \mathbf{F}_0$ where $\mathbf{R}_{v_s v_s}(\tau) := E\{\mathbf{v}_s(k+\tau)\mathbf{v}_s^\mathcal{H}(k)\}$. Define ($L > 0$ is some large integer)

$$\bar{\mathbf{R}}_{v_s v_s} := [\mathbf{R}_{v_s v_s}^T(-1) \quad \mathbf{R}_{v_s v_s}^T(-2) \quad \cdots \quad \mathbf{R}_{v_s v_s}^T(-L)] : Q^*]^T \quad (3-12)$$

where $Q = [\mathbf{q}_{r+1} : \cdots : \mathbf{q}_N]$, $r = \rho(\mathbf{R}_{v_s v_s}(0))$ with $M \leq r \leq N$, $\mathbf{q}_i$'s ($r+1 \leq i \leq N$) are orthonormal eigenvectors of $\mathbf{R}_{v_s v_s}(0)$ corresponding to zero eigenvalues, and the symbol $*$ denotes the complex conjugation operation. Note that if $r = N$, Q is omitted from (3-12). Let $\mathbf{F}_{0s}$ and $\mathbf{F}_{0n}$ be the orthogonal projections of $\mathbf{F}_0$ onto the r -dimensional 'signal subspace' (range space) and $(N-r)$ -dimensional 'noise subspace' (null space), respectively, of $\mathbf{R}_{v_s v_s}(0)$. By (3-10), $\rho(\mathbf{F}_{0s}) = M$. Then $\mathbf{F}_0 = \mathbf{F}_{0s} + \mathbf{F}_{0n}$ with $\mathbf{F}_{0s}^\mathcal{H} \mathbf{F}_{0n} = 0$ and $\mathbf{R}_{v_s v_s}(0) \mathbf{F}_{0n} = 0$. It then follows that $E\{\mathbf{F}_{0n}^\mathcal{H} \mathbf{v}_s(k) \mathbf{v}_s^\mathcal{H}(k) \mathbf{F}_{0n}\} = \mathbf{F}_{0n}^\mathcal{H} \mathbf{R}_{v_s v_s}(0) \mathbf{F}_{0n} = 0$; hence, $\mathbf{F}_{0n}^\mathcal{H} \mathbf{v}_s(k) = 0$ with probability one (w.p.1) and (cf. (3-10))

$$\mathbf{F}_{0s}^\mathcal{H} \mathbf{v}_s(k) = \mathbf{w}(k). \quad (3-13)$$

Lemma 4. $\bar{\mathbf{R}}_{v_s v_s}$ is rank deficient for any $L \geq 1$ such that $\bar{\mathbf{R}}_{v_s v_s} \mathbf{F}_{0s} = 0$ and $\rho(\mathbf{F}_{0s}) = M$. •

Proof: By construction $Q^\mathcal{H} \mathbf{F}_{0s} = 0$ as columns of Q span the noise-subspace of $\mathbf{R}_{v_s v_s}(0)$ and $\mathbf{F}_{0s}$ is the orthogonal projection of $\mathbf{F}_0$ onto the r -dimensional signal subspace of $\mathbf{R}_{v_s v_s}(0)$. Furthermore, we have

$$\mathbf{R}_{ww}(\tau) = E\{\mathbf{w}(k+\tau)\mathbf{w}^\mathcal{H}(k)\} = 0 \quad \forall \tau \geq 1 \quad (3-14)$$

because $\mathbf{v}_s(k)$ is obtained by causal filtering of $\mathbf{y}(k)$, hence of $\mathbf{w}(k)$. Using (3-13) in (3-14) it then follows that there exists a $N \times M$ $\mathbf{F}_{0s} \neq 0$ such that

$$\mathbf{F}_{0s}^\mathcal{H} \mathbf{R}_{v_s v_s}(\tau) = 0 \quad \forall \tau \geq 1 \Rightarrow \mathbf{R}_{v_s v_s}(-\tau) \mathbf{F}_{0s} = 0 \quad \forall \tau \geq 1. \quad (3-15)$$

The desired result is then immediate. $\square$

Pick a $N \times M$ column-vector $\mathbf{H}_0$ to equal the rightmost M right singular vectors in a singular-value decomposition (SVD) $\bar{\mathbf{R}}_{v_s v_s} = U \Sigma V^H$, i.e. the right singular vectors corresponding to the M smallest singular values. Therefore, $\rho(\bar{\mathbf{H}}_0) = M$. Then since ideally the M smallest singular values of $\bar{\mathbf{R}}_{v_s v_s}$ are zero, we have $\mathbf{H}_0^H \bar{\mathbf{R}}_{v_s v_s}(-\tau) \mathbf{H}_0 = 0$ for $\tau = 1, 2, \dots, L$. This, in turn, implies that

$$(\mathbf{H}_0^H \bar{\mathbf{R}}_{v_s v_s}(-\tau) \mathbf{H}_0)^H = 0 \text{ for } \tau = 1, 2, \dots, L. \quad (3-16)$$

Moreover $\mathbf{Q}^H \mathbf{H}_0 = 0$. An eigendecomposition yields

$$\bar{\mathbf{R}}_{v_s v_s}(0) = \sum_{i=1}^r \sigma_i \mathbf{q}_i \mathbf{q}_i^H = \bar{\mathbf{Q}} \Sigma \bar{\mathbf{Q}}^H \quad (3-17)$$

where $\bar{\mathbf{Q}} = [\mathbf{q}_1 \dots \mathbf{q}_r]$, $\mathbf{q}_i$'s ($1 \leq i \leq r$) are orthonormal eigenvectors of $\bar{\mathbf{R}}_{v_s v_s}(0)$ corresponding to non-zero eigenvalues σ_i 's and $\Sigma = \text{diag}\{\sigma_1, \dots, \sigma_r\}$. Thus columns of $\bar{\mathbf{Q}}$ span the signal subspace of $\bar{\mathbf{R}}_{v_s v_s}(0)$ and the columns of $\bar{\mathbf{Q}}$ span the noise subspace of $\bar{\mathbf{R}}_{v_s v_s}(0)$. Since $\mathbf{Q}^H \mathbf{H}_0 = 0$, we have $\mathbf{H}_0 = \bar{\mathbf{Q}} \mathbf{C}$ where $\mathbf{C}$ is $r \times M$ and $\rho(\mathbf{C}) = M$. Then $\mathbf{H}_0^H \bar{\mathbf{R}}_{v_s v_s}(0) \mathbf{H}_0 = \tilde{\mathbf{C}}^H \tilde{\mathbf{C}}$ where $\tilde{\mathbf{C}} = \Sigma^{1/2} \mathbf{C}$ and $\rho(\tilde{\mathbf{C}}) = M$. Finally, by result R11 on p. 261 of [9], $\rho(\tilde{\mathbf{C}} \tilde{\mathbf{C}}^H) = \rho(\tilde{\mathbf{C}} \tilde{\mathbf{C}}^H)$, and therefore, by Sylvester's inequality [7, p. 655], $\rho(\mathbf{H}_0^H \bar{\mathbf{R}}_{v_s v_s}(0) \mathbf{H}_0) = M$.

The overall system with $\mathbf{w}(k)$ as input and $\mathbf{H}_0^H \mathbf{v}_s(k)$ as output has the transfer function

$$\begin{aligned} & \mathbf{H}_0^H \mathcal{G}_a(z) \mathcal{A}^{-1}(z) \mathcal{B}(z) \\ &= [\det(\mathcal{A}(z)) I_M]^{-1} \mathbf{H}_0^H \mathcal{G}_a(z) \text{adj}(\mathcal{A}(z)) \mathcal{B}(z) \end{aligned} \quad (3-18)$$

and therefore, is an autoregressive moving average (ARMA) model with autoregressive (AR) order no more than $N n_a$ and moving average (MA) order no more than $n_b + L_e + (N-1)n_a$, denoted by ARMA($N n_a, n_b + L_e + (N-1)n_a$). Therefore, it follows from (3-16) that $\mathbf{H}_0^H \mathbf{v}_s(k)$ is zero-mean white if $L \geq n_b + L_e + (N-1)n_a$. Moreover, since $\mathbf{v}_s(k)$ is obtained by causal filtering of $\mathbf{w}(k)$, it follows that $\mathbf{H}_0^H \mathbf{v}_s(k) = \sum_{i=0}^{\infty} \mathbf{P}_i \mathbf{w}(k-i)$ where $M \times M$ $\mathcal{P}(z) = \sum_{i=0}^{\infty} \mathbf{P}_i z^{-i}$ is stable satisfying $\mathcal{P}(e^{j\omega}) \mathcal{P}^H(e^{j\omega}) = I_M \forall \omega$ (allpass). Therefore, we have $E\{\mathbf{H}_0^H \mathbf{v}_s(k+\tau) \mathbf{w}^H(k)\} = \mathbf{P}_\tau$ for $\tau \geq 0$, and using (3-13), we have $E\{\mathbf{H}_0^H \mathbf{v}_s(k+\tau) \mathbf{w}^H(k)\} = \mathbf{H}_0^H \bar{\mathbf{R}}_{v_s v_s}(\tau) \mathbf{F}_{0s} = 0$ (by construction of $\mathbf{H}_0$) for $\tau \geq 1$. Thus $\mathbf{P}_\tau = 0$ for $\tau \geq 1$. Therefore, we have

$$\mathbf{H}_0^H \mathbf{v}_s(k) = \mathbf{P}_0 \mathbf{w}(k) \text{ such that } \rho(\mathbf{P}_0) = M. \quad (3-19)$$

By (3-11) and (3-19), we have

$$\mathbf{H}_0^H \mathcal{G}_a(z) \mathbf{y}(k) = \mathbf{P}_0 \mathbf{w}(k) + \mathbf{H}_0^H \mathcal{G}_a(z) \mathbf{n}(k) \ni \rho(\mathbf{P}_0) = M. \quad (3-20)$$

Since $\mathbf{P}_0 \mathbf{U} \mathbf{U}^H \mathbf{P}_0^H = \mathbf{P}_0 \mathbf{P}_0^H$ for any unitary matrix $\mathbf{U}$, one can not uniquely determine $\mathbf{P}_0$ from (3-20) given second-order statistics of data $\mathbf{y}(k)$, and knowledge (estimates) of $\mathbf{H}_0$, filter $\mathcal{G}_a(z)$ and noise variance σ_n^2 [13]. One has to exploit higher-order statistics (HOS) of data. The model (3-20) is an instantaneous mixture model [13], therefore, any existing method may be applied to estimate $\mathbf{P}_0$ given (3-20). In this paper we have used the joint diagonalization procedure of [13]. The required estimate of $\mathbf{P}_0$ is obtained as

$$\mathbf{P}_0 = \mathbf{W}^{-1} \mathbf{U} \quad (3-21)$$

where $\mathbf{W}$ "diagonalizes" $\mathbf{H}_0^H \bar{\mathbf{R}}_{v_s v_s}(0) \mathbf{H}_0$ into an identity matrix and $\mathbf{U}$ is a unitary matrix obtained via the joint diagonalization procedure of [13] using fourth-order cumulants of $\mathbf{W} \mathbf{H}_0^H \mathcal{G}_a(z) \mathbf{y}(k)$. Let $\mathbf{l}_i$ ($i = 1, 2, \dots, M$) denote the orthonormal eigenvectors of $\mathbf{H}_0^H \bar{\mathbf{R}}_{v_s v_s}(0) \mathbf{H}_0$ with the

corresponding eigenvalues γ_i 's. Set $\mathcal{L} = [\mathbf{l}_1 \dots \mathbf{l}_M]$ and $\Gamma = \text{diag}(\gamma_1, \dots, \gamma_M)$. Then

$$\mathbf{W} = \Gamma^{-1/2} \mathcal{L}^H \quad (3-22)$$

diagonalizes $\mathbf{H}_0^H \bar{\mathbf{R}}_{v_s v_s}(0) \mathbf{H}_0$ to an identity matrix: $\mathbf{W} (\mathbf{H}_0^H \bar{\mathbf{R}}_{v_s v_s}(0) \mathbf{H}_0) \mathbf{W}^H = I_M$. Finally, we have

$$\bar{\mathbf{H}}_0^H \mathbf{v}_s(k) =: \mathbf{w}(k) \text{ where } \bar{\mathbf{H}}_0 = \mathbf{H}_0 \mathbf{W}^H \mathbf{U}. \quad (3-23)$$

Remark 1. Using (3-11) and (3-23) it follows that $\bar{\mathbf{H}}_0^H [\sum_{i=0}^{L_e} \mathbf{G}_{ai} \mathbf{s}(k-i)] = \mathbf{w}(k)$. The filter $\bar{\mathbf{H}}_0^H \mathcal{G}_a(z)$ whitens the noise-free received signal. Moreover, the derivation of this filter was based upon whitening of $\bar{\mathbf{H}}_0^H \mathcal{G}_a(z) \mathbf{s}(k)$. These considerations motivate the name a whitening approach for the proposed technique. Our approach is far more structured and different than that of [12]. $\square$

3.2. MMSE Equalizer with Delay d

Using the orthogonality principle, the MMSE equalizer of length $L_e + 1$ to estimate $\mathbf{w}(k-d)$ ($d \geq 0$) based upon $\mathbf{y}(n)$, $n = k, k-1, \dots, k-L_e$, satisfies

$$\begin{bmatrix} \bar{\mathbf{G}}_{d,0} & \bar{\mathbf{G}}_{d,1} & \dots & \bar{\mathbf{G}}_{d,L_e} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_d^H & \mathbf{F}_{d-1}^H & \dots & \mathbf{F}_0^H & 0 & \dots & 0 \end{bmatrix} \mathcal{R}_{yyL_e}^{-1}. \quad (3-24)$$

Clearly one can obtain a consistent estimate of $\mathcal{R}_{yyL_e}$ from the given data. It remains to estimate $\mathbf{F}_i$'s to complete the design. Here the discussion of Sec. 3.1 becomes relevant. From (3-11) and (3-23) we have

$$\bar{\mathbf{H}}_0^H \mathbf{v}_s(n) = \sum_{i=0}^{L_e} \bar{\mathbf{H}}_0^H \mathbf{G}_{ai} \mathbf{s}(n-i). \quad (3-25)$$

Using (3-25) and taking expectations we have

$$E\{\mathbf{s}(n) \mathbf{v}_s^H(n-\tau)\} \bar{\mathbf{H}}_0 = \sum_{i=0}^{L_e} \mathbf{R}_{ss}(\tau+i) \mathbf{G}_{ai}^H \bar{\mathbf{H}}_0. \quad (3-26)$$

Using (1-1), (1-2) and (3-23) we have

$$E\{\mathbf{s}(n) \mathbf{v}_s^H(n-\tau)\} \bar{\mathbf{H}}_0 = \mathbf{F}_\tau. \quad (3-27)$$

Hence, we have from (3-26) and (3-27)

$$\mathbf{F}_\tau^H = \bar{\mathbf{H}}_0^H \sum_{i=0}^{L_e} \mathbf{G}_{ai} \mathbf{R}_{ss}^H(\tau+i). \quad (3-28)$$

Substitute the results of (3-28) for $\tau = 0, 1, \dots, d$ in (3-24) to complete the design. The MMSE estimate $\hat{\mathbf{w}}(t-d)$ of $\mathbf{w}(t-d)$ is then given by $\hat{\mathbf{w}}(t-d) = \sum_{i=0}^{L_e} \bar{\mathbf{G}}_{d,i} \mathbf{y}(t-i)$.

4. COMMON MINIMUM-PHASE ZEROS

Here the MIMO transfer function is

$$\mathcal{F}(z) = \mathcal{A}^{-1}(z) \mathcal{B}(z) \mathcal{B}_c(z), \quad \mathcal{B}_c(z) = \sum_{i=0}^{n_{bc}} \mathbf{B}_{ci} z^{-i} \quad (4-1)$$

where $\mathcal{B}(z)$ satisfies (H2) and $\mathcal{B}_c(z)$ is a finite-degree $M \times M$ polynomial that collects all the common zeros/factors of the subchannels. Assume that

(H6) Given model (4-1), $\det(B_c(z)) \neq 0$ for $|z| \geq 1$.

Then while $A^{-1}(z)B(z)$ has a finite left-inverse, $B_c^{-1}(z)$ is IIR though causal under (H6). Then (3-2) holds true approximately for “large” L_e , the approximation getting better with increasing L_e . Similarly Lemmas 1 and 2 hold true approximately for “large” K and Lemma 3 also holds true approximately for $L_e \geq K$. Note also that $\rho(F_0) = M$ where $F_0 = B_0 B_{c0}$ since, by (H6), $\rho(B_{c0}) = M$ (evaluate $\det(B_c(z))$ at $z = \infty$). It is then readily seen that the developments of Sec. 3 apply to the current case also.

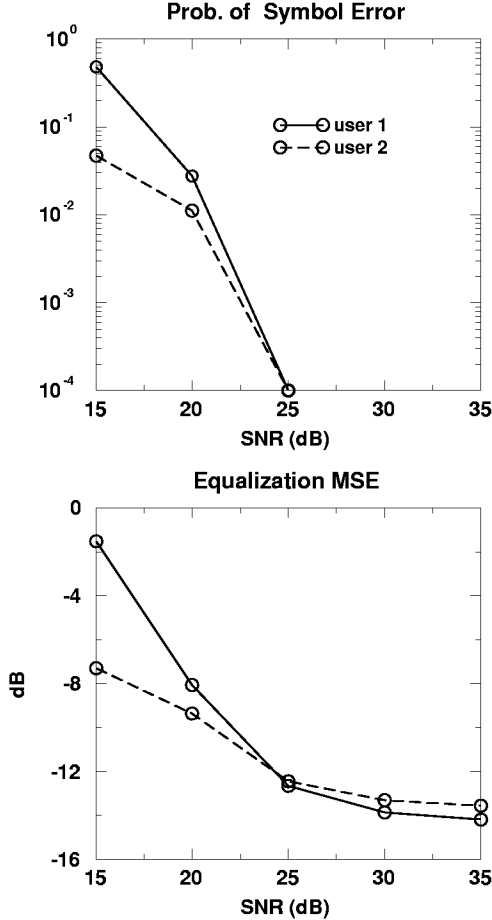


Fig. 1. Normalized MSE and probability of symbol detection error P_e for the two users after MMSE equalization with $d = 3$. Record length $T = 500$ symbols for equalizer design. Averages over 100 Monte Carlo runs. The designed equalizer was applied to record lengths of 3000 symbols for performance evaluation.

5. SIMULATION EXAMPLE

We consider a wireless communications scenario with two ($M = 2$) 4-QAM user signals arriving at a uniform linear array (half-wavelength spacing) of $N = 4$ sensors via a frequency selective multipath channel. The signaling pulse shape for both the users was a raised-cosine pulse with a roll-off factor of 0.2, the pulse being truncated to a length of $4T_s$, where T_s = symbol duration. The array measurements are assumed to be sampled at baud rate with sampling interval T_s seconds and the two sources have the same baud rate. The relative time delay τ (relative to the first arrival), the angle of arrival θ (in degrees w.r.t. the array broadside) and the relative attenuation factor (amplitude) α , (τ, θ, α) ,

for the two sources were selected as:

$$w_1 : (0T_s, 40^\circ, 1.0), (0.3T_s, 20^\circ, 1.0), (0.6T_s, -20^\circ, 1.0)$$

$$w_2 : (0T_s, 10^\circ, 1.0), (1.1T_s, -15^\circ, 1.0), (1.6T_s, -1^\circ, 1.0).$$

Sampling of received signal at the array leads to a discrete-time MIMO FIR model $B(z)$ with $N = 4$, $M = 2$ and $n_b = 4$ such that $B_0 \neq 0$ and $B_4 \neq 0$ (see Sec. 1). The effective MIMO channel was taken to be

$$F(z) = B(z)B_c(z) \text{ where } B_c(z) = (1 - 0.5z^{-1})I_2. \quad (5-2)$$

The part $B_c(z)$ in (5-2) may represent some filtering at the transmitter or receiver, and it leads to a system with common zeros in the two subchannels.

An MMSE equalizer of length $L_e = 7$ (8 taps per sub-channel, totaling 32 taps) was designed with a delay $d = 3$ for each of the two sources. Fig. 1 shows the results of simulations for a record length of $T = 500$ symbols. It is seen that the proposed approach works quite well. Also presence of a common (minimum-phase) zero has not caused any problems.

6. REFERENCES

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