SYNCHRONIZATION BY PILOT SIGNAL

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ABSTRACT

In this paper, we propose a novel approach for code acquisition in the code-division multiple access (CDMA) communication system. The essential assumption is that the pilot signal of the desired user is available. If the codes are exactly orthogonal, the method can be derived from an optimization criterion. Using the pilot signal, the performance can be greatly improved without increasing computational complexity. Simulations show that our method clearly outperforms minimum variance method, eigenvector-based MUSIC, and matched filter.

1. INTRODUCTION

Code Division Multiple Access (abbreviated CDMA) is a technique for third generation wireless radio communications. In the CDMA system a good bandwidth efficiency is achieved by simultaneous use of the same frequency band by several users. The problem of identifying each user is dealt with codes, unique to each one and having possibly low cross-correlation to the others' one. Code design alone, however, is not sufficient to prevent users to interfere with each other in reception, and due to the nonorthogonality of the codes, multiple-access interference (MAI) arise. Another problem is that the downlinks, i.e. mobile phones, far from the uplink, i.e. base station, are in a lot worse situation than those near to one because of fading. This problem called near-far effect is currently solved by making it possible for a uplink to control the transmitting powers of downlinks. Power control, however, is a difficult task, and it wastes the battery lifes. Currently developed near-far resistant methods are either computationally very complex (e.g. maximum likelihood method [4]), or yet too suboptimal to yield desirable results (e.g. subspace-based MUltiple SIgnal Classifiation, MUSIC [1, 3, 5, 6]).

Traditional code acquisition methods rely on minimization or maximization of objective functions like system output power, variance, or projection of the test code vectors to the signal subspace. Another way to synchronize the codes is to use known pilot symbols. In this paper, we approach the problem of code acquisition of the desired user by using the known pilot signal. We remember that in the method of minimum output energy, known code with the known delay can be used to estimate the corresponding symbols of the desired user. We show that the dual method can be used to estimate the code and delay of the desired user by using the known symbol stream.

Experiments with simulated CDMA uplink data are included in the paper, as a goal to estimate the delays of a user in the system. As reference methods we use traditional matched filter, eigenvector based minimum variance method [2] and MUSIC-estimator. In addition, we use a straightforward method, where the pilot signal is applied directly to the obreseved data matrix.

According to our tests, the proposed method performs clearly better than the others.

2. SIGNAL MODEL

The signal model studied in this paper can be either uplink or downlink model with AWGN channel. The data have the form

$$r(t) = \sum_{m=1}^{N} \sum_{k=1}^{K} b_{km} \sum_{l=1}^{L_k} a_{kl} s_k(t - mT - d_{kl}) + n(t), \quad (1)$$

where a_{kl} is the complex factor of the kth user's lth path, b_{km} is kth user's mth symbol, $s_k(\cdot)$ is kth user's chip sequence, $s_k(t) \in \{-1, +1\}$, $t \in [0, T)$, $s_k(t) = 0$, $t \notin [0, T)$, and d_{kl} is the delay of the kth user's lth path. Each delay is assumed to change sufficiently slowly for most of the time. $n(t)$ denotes noise. The chip sequence length is C, and N is the number of bits.

Collect C-vectors $\mathbf{r}_m$ from subsequent discretized equispaced data samples $r[n]$:

$$\mathbf{r}_m = \begin{bmatrix} r[mC] & r[mC+1] & \cdots & r[(m+1)C-1] \end{bmatrix}^T \quad (2)$$

They have the form [1, 3, 6]

$$\mathbf{r}_m = \sum_{k=1}^{K} \left[b_{k,m-1} \sum_{l=1}^{L} a_{kl} \mathbf{g}_{kl}^{E} + b_{km} \sum_{l=1}^{L} a_{kl} \mathbf{g}_{kl}^{L} \right] + \mathbf{n}_m \quad (3)$$

where $\mathbf{n}_m$ denotes noise vector, and the "early" and "late" parts of the code vectors are

$$\mathbf{g}_{kl}^{E} = [\; s_k[C - d_{kl} + 1] \;\; \cdots \;\; s_k[C] \;\; 0 \;\; \cdots \;\; 0 \;]^T \quad (4)$$

$$\mathbf{g}_{kl}^{L} = [\; 0 \;\; \cdots \;\; 0 \;\; s_k[1] \;\; \cdots \;\; s_k[C - d_{kl}] \;]^T \quad (5)$$

Here d_{kl} is a discretized delay index, $d_{kl} \in \{0, \ldots, (C-1)/2\}$. The model (3) can be represented in more compact form

$$\mathbf{r}_m = \mathbf{G}\tilde{\mathbf{b}}_m + \mathbf{n}_m \quad (6)$$

where $C \times 2K$ matrix $\mathbf{G} = [\mathbf{g}_1, \ldots, \mathbf{g}_{2K}]$ contains the basis vectors and fading terms

$$\begin{aligned} \mathbf{G} &= \left[\sum_{l=1}^{L} a_{1l}\mathbf{g}_{1l}^{E}, \sum_{l=1}^{L} a_{1l}\mathbf{g}_{1l}^{L}, \ldots, \sum_{l=1}^{L} a_{Kl}\mathbf{g}_{Kl}^{E}, \sum_{l=1}^{L} a_{Kl}\mathbf{g}_{Kl}^{L} \right] \\ &= \left[\mathbf{G}_1^E \mathbf{a}_1, \mathbf{G}_1^L \mathbf{a}_1, \ldots, \mathbf{G}_K^E \mathbf{a}_K, \mathbf{G}_K^L \mathbf{a}_K \right] \end{aligned} \quad (7)$$

and $2K$-vector $\tilde{\mathbf{b}}_m$ contains the symbols

$$\tilde{\mathbf{b}}_m = [b_{1,m-1}, b_{1m}, \ldots, b_{K,m-1}, b_{Km}]^T \quad (8)$$

In the pure matrix form the representation of the data is

$$\mathbf{X} = \mathbf{G}\mathbf{B} + \mathbf{N} \quad (9)$$

where

$$\mathbf{X} = [\mathbf{r}_1, \ldots, \mathbf{r}_N] \quad (10)$$

$$\mathbf{B} = [\tilde{\mathbf{b}}_1, \ldots, \tilde{\mathbf{b}}_N] \quad (11)$$

$$\mathbf{N} = [\mathbf{n}_1, \ldots, \mathbf{n}_N] \quad (12)$$

Notice that

$$\mathbf{G}_k^E \mathbf{a}_k + \mathbf{G}_k^L \mathbf{a}_k = \mathbf{G}_k \mathbf{a}_k = \mathbf{y}_k \quad (13)$$

where $\mathbf{a}_k$ contains kth user's fading terms, and $C \times L$ matrix $\mathbf{G}_k$ contains the delayed codes corresponding to the user k:

$$\mathbf{G}_k = \begin{bmatrix} s_k[C - d_{k1} + 1] & \cdots & s_k[C - d_{kL} + 1] \\ \vdots & \ddots & \vdots \\ s_k[C] & \cdots & s_k[C] \\ s_k[1] & \cdots & s_k[1] \\ \vdots & \ddots & \vdots \\ s_k[C - d_{k1}] & \cdots & s_k[C - d_{kL}] \end{bmatrix} \quad (14)$$

3. THE PILOT ALGORITHM

Our new algorithm estimates the desired user's delays by exploiting the knowledge of the pilot signal. The performance of the algorithm is dual with the algorithm of Wang and Poor [7], which is derived from an optimization criterion. That algorithm, on the contrary to our algorithm, assumes that the codes and delays of the first user are known, and it estimates the first user's fadings and symbols blindly, without knowledge of the disturbing users' codes. For illustrative reasons, we derive here our algorithm "directly", without using any optimization criterion. However, the new algorithm could be derived in the same way as in [7]. First, we derive the algorithm of Wang and Poor directly using the matrix properties. Our model differs slightly from the model of [7]. In our formalism, the first column of $\mathbf{G}$ is known, and it is denoted by $\mathbf{g}_1$. Assume also that the symbols are strictly orthogonal. Without loss of generality, it can be assumed that they are scaled in a such a way that $\mathbf{B}\mathbf{B}^T = \mathbf{I}$. For notational simplicity, assume that no noise exist. Under these assumptions, we now show that we can *exactly* estimate the binary symbols of the desired user without any knowledge of the disturbing users' codes. Moreover, the other codes do not need to be orthogonal against the desired code. The proof is as follows:

$$\mathbf{X} = \mathbf{G}\mathbf{B} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad (15)$$

is a Singular Value Decomposition (SVD) of $\mathbf{X}$. Here dimensions of $\mathbf{U}$, $\mathbf{\Sigma}$, and $\mathbf{V}$ are (define $M = 2K$) $C \times M$, $M \times M$, and $N \times M$, respectively, and orthonormalities $\mathbf{U}^T\mathbf{U} = \mathbf{V}^T\mathbf{V} = \mathbf{I}$ hold. $\mathbf{\Sigma}$ is a diagonal matrix. Data is whitened by

$$\mathbf{Y} = \mathbf{V}^T = \mathbf{\Sigma}^{-1}\mathbf{U}^T\mathbf{X} = \mathbf{\Sigma}^{-1}\mathbf{U}^T\mathbf{G}\mathbf{B} = \mathbf{T}\mathbf{B} \quad (16)$$

Because

$$\mathbf{Y}\mathbf{Y}^T = \mathbf{V}^T\mathbf{V} = \mathbf{I} = \mathbf{T}\mathbf{B}\mathbf{B}^T\mathbf{T}^T = \mathbf{T}\mathbf{T}^T \quad (17)$$

then the $M \times M$ matrix $\mathbf{T} = \mathbf{\Sigma}^{-1}\mathbf{U}^T\mathbf{G}$ is orthonormal. Denote $\mathbf{T} = [\mathbf{t}_1, \ldots, \mathbf{t}_M]$ and $\mathbf{B}^T = [\mathbf{b}_1, \ldots, \mathbf{b}_M]$. Here $\mathbf{b}_{2(k-1)+1}$ and $\mathbf{b}_{2(k-1)+2}$ are N-vectors containing kth user's symbols in such a way that the first one contains the symbols $1, \ldots, N$, and the second one contains the delayed symbols $2, \ldots, N+1$. Because we know $\mathbf{g}_1$ and the SVD matrices, we can select the filter as

$$\mathbf{c}_1 = \mathbf{t}_1 = \mathbf{\Sigma}^{-1}\mathbf{U}^T\mathbf{g}_1 \quad (18)$$

Then, due to the orthonormality of $\mathbf{T}$, we obtain the desired symbols

$$\mathbf{c}_1^T\mathbf{Y} = \mathbf{c}_1^T\mathbf{T}\mathbf{B} = [1, 0, \ldots, 0][\mathbf{b}_1, \ldots, \mathbf{b}_M]^T = \mathbf{b}_1^T \quad (19)$$

On the other hand, direct substitution yields

$$\begin{aligned} \mathbf{c}_1^T\mathbf{Y} &= \mathbf{g}_1^T\mathbf{U}\mathbf{\Sigma}^{-1}\mathbf{\Sigma}^{-1}\mathbf{U}^T\mathbf{G}\mathbf{B} \\ &= \mathbf{g}_1^T\mathbf{U}\mathbf{\Lambda}^{-1}\mathbf{U}^T\mathbf{G}\mathbf{B} = \mathbf{g}_1^T\mathbf{U}\mathbf{\Lambda}^{-1}\mathbf{U}^T\mathbf{X} \end{aligned} \quad (20)$$

due to the fact that for eigenvalue matrix in the eigendecomposition $\mathbf{X}\mathbf{X}^T\mathbf{U} = \mathbf{U}\mathbf{\Lambda}$ the equality $\mathbf{\Lambda} = \mathbf{\Sigma}^2$ holds. But Eq. (20) is just the method of Wang and Poor. In practice, noise corrupts the output estimate.

The above approach can be modified to the case where duality arises. More precisely, assume that $\mathbf{G}^T\mathbf{G} = \mathbf{I}$ (dual with the case $\mathbf{B}^T\mathbf{B} = \mathbf{I}$). In our simulations, this assumption only roughly holds. Assume also that the desired symbol $\mathbf{b}_1$ is known (dual with the case that $\mathbf{g}_1$ is known). No orthogonality assumption of $\mathbf{B}$ need to be made. Because

$$\mathbf{X}^T = \mathbf{B}^T\mathbf{G}^T = \mathbf{V}\mathbf{\Sigma}\mathbf{U}^T \tag{21}$$

then the whitened data is

$$\mathbf{Y} = \mathbf{U}^T = \mathbf{\Sigma}^{-1}\mathbf{V}^T\mathbf{X}^T = \mathbf{\Sigma}^{-1}\mathbf{V}^T\mathbf{B}^T\mathbf{G}^T = \mathbf{C}\mathbf{G}^T \tag{22}$$

where $\mathbf{C} = \mathbf{\Sigma}^{-1}\mathbf{V}^T\mathbf{B}^T$. Because we assumed that $\mathbf{G}$ is orthogonal, then $\mathbf{C}$ is also orthogonal. Then we can apply similar approach than Wang and Poor. The filter is now

$$\mathbf{c}_1 = \mathbf{\Sigma}^{-1}\mathbf{V}^T\mathbf{b}_1 \tag{23}$$

and

$$\mathbf{c}_1^T\mathbf{Y} = \mathbf{g}_1^T \tag{24}$$

Notice that if $\mathbf{G}$ is not orthogonal (which is in practice the case), the filter does produce $\mathbf{g}_1$ only approximatively. However, when we have obtained the estimate $\hat{\mathbf{g}}_1$, we can finally improve it by projecting the true code vector with different test codes to $\hat{\mathbf{g}}_1$, and by selecting that code vector which yields largest projection. More formally, remember from (13) that $\mathbf{y}_1 = \mathbf{g}_1 + \mathbf{g}_2 = \mathbf{G}_1\mathbf{a}_1$. We obtain the estimate of $\mathbf{y}$ from Eqs. (23) and (24)

$$\hat{\mathbf{y}}_1 = \hat{\mathbf{g}}_1 + \hat{\mathbf{g}}_2 = \mathbf{U}\mathbf{\Sigma}^{-1}\mathbf{V}^T(\mathbf{b}_1 + \mathbf{b}_2) \tag{25}$$

where $\mathbf{b}_1$ and $\mathbf{b}_2$ contain the first user's known symbols $1, \ldots, N$ and $2, \ldots, N+1$, respectively. Because $\mathbf{G}_1$ contains the shifted code vectors of the first user (see Eq. (14)), and because these vectors are roughly orthogonal, the matched filter type delay estimator can be finally used:

$$\hat{d} = \arg\max_d |\hat{\mathbf{y}}_1^H \mathbf{g}_1(d)| \tag{26}$$

where $\mathbf{g}_1(d)$ is the first user's circularly shifted code vector:

$$\mathbf{g}_1(d) = [s_1[C-d+1], \ldots, s_1[C], s_1[1], \ldots, s_1[C-d]]^T \tag{27}$$

4. EXPERIMENTS

We compared our method in the uplink environment to the minimum variance method, eigenvector-based MUSIC, matched filter, and the straightforward pilot method. In the pilot method, $\hat{\mathbf{y}}_1$ is simply $\hat{\mathbf{y}}_1 = \mathbf{X}(\mathbf{b}_1 + \mathbf{b}_2)$. The parameters were as follows: $K = 5$, $L = 3$, Multiple Access Interference (MAI) of each disturbing user with respect to the first user is 20 dB. 100 simulations for each Signal-to-Noise Ratio (SNR) were performed. SNR was varied from 0 to 30 dB. Total number of delays in the hundred simulations was $100 \times L = 300$. Figure 1 shows the number of correct delay estimates/300. The curves are as follows: new (solid), dot (MUSIC), solid-dot (minimum variance, MV), dashed (straightforward pilot, PILOT), plus (matched filter, MF). The new method clearly outperforms other methods. Figure 2 shows the case, where SNR is 5 dB. Histogram shows the cases where 0, 1, 2, or 3 delays were correctly estimated. The order is following from up to bottom: new, PILOT, MUSIC, MV, MF. Again, the performance of the new method is best.

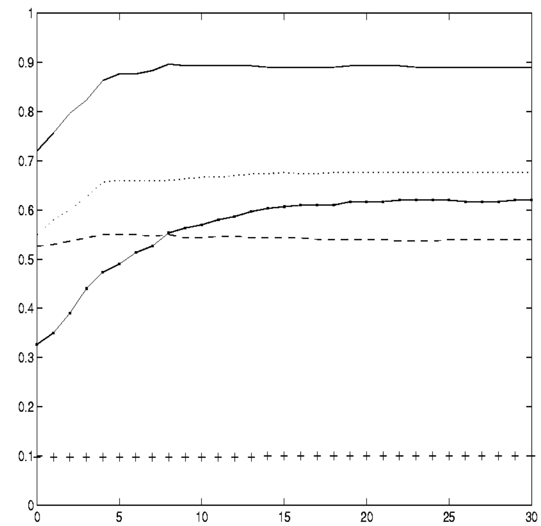

Figure 1: Relative number of correct delay estimates. SNR varies from 0 to 30 dB. New (solid), dot (MUSIC), solid-dot (MV), dashed (PILOT), plus (MF). .

5. REFERENCES

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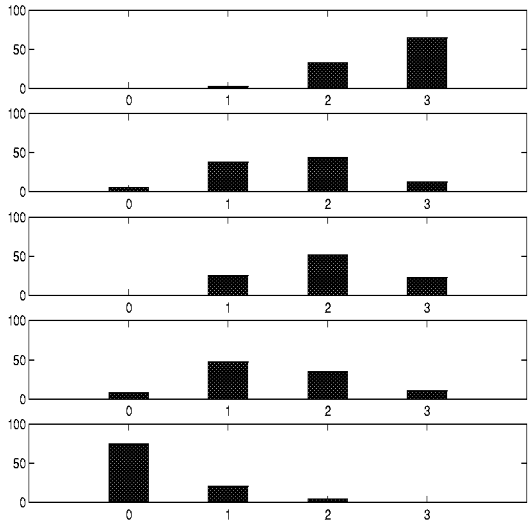

Figure 2: SNR is 5 dB. Histogram shows the cases where 0, 1, 2, or 3 delays were correctly estimated. The order is following from up to bottom: new, PILOT, MUSIC, MV, MF.

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