MMSE MULTIUSER DETECTION IN MULTIPATH FADING CHANNELS

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ABSTRACT

In this work, we propose an MMSE multiuser detector for asynchronous DS/CDMA systems operating over frequency-selective fading channels. It is shown that computation of a conditional MMSE estimate of the bit to be decoded may be carried out with a computational burden linear in the processing gain N. We also give a closed-form formula for the Error Probability and the Near-far Resistance of the proposed detector, and curves of such performance measures, showing that the new receiver is near-far resistant and outperforms the previously derived decorrelating detector for frequency-selective fading channels.

1. INTRODUCTION

The problem of Multiaccess Interference (MAI) suppression for Direct Sequence Code Division Multiple Access (DS/CDMA) systems over frequency-selective fading channels is a very hot research topic. Indeed, on one hand, DS/CDMA techniques are now emerging as stronger and stronger candidates for the realization of the future wireless networks air-interfaces, on the other, wireless networks are widely supported by urban and indoor channels, which typically introduce frequency-selective fading on wideband CDMA signals.

While the optimum multiuser detector for the fading channel is derived in [1], in [2] and [3] it is shown that the generalization of the decorrelating detector to the case of multipath faded channels - for the case of a synchronous and an asynchronous system, respectively - permits achieving very satisfactory performance with a complexity only linear in the users number. However, it is well known that, for large users number, the decorrelating strategy incurs a dramatic performance impairment, due to the effect of the so-called noise enhancement. Conversely, the MMSE multiuser detector, presented in [4] for unfaded channels, overcomes these difficulties in that it processes the whole signals subspace, and approaches the behavior of the decorrelating detector only in the limiting case of arbitrarily large MAI energy and/or of vanishingly small thermal noise level, so that it is expected to outperform the decorrelating detector also for the case of frequency-selective fading channels.

Accordingly, in what follows we derive an MMSE-based multiuser detector for frequency-selective fading channels which is shown to outperform the decorrelating detector. The paper is organized as follows. In the next Section, we illustrate the system model, while Section 3 is devoted to the synthesis of the proposed detector. Section 4 contains the performance analysis along with the discussion of the numerical results, while in Section 5 we draw the conclusions.

2. SYSTEM MODEL

Let us consider a DS/CDMA network wherein K users asynchronously transmit their information bits. It is assumed that the propagation channel has a coherence bandwidth much narrower than the CDMA signals bandwidth, W say, and a coherence time much larger than the bit signaling time T_b. These hypotheses imply that, upon transmission of a certain signal, the received signal is composed by the superposition of a number, L say, of replicas of the transmitted signal, spaced of integer multiples of $1/W$ apart, and affected by a complex random gain. Accordingly, assuming a BPSK modulation format, the baseband equivalent of the received signal is written as:

$$\begin{aligned} r(t) = \sum_{m=-P}^{P} \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} c_{k,l}(m) A_k b_k(m) \\ s_k\left(t - \tau_k - \frac{l}{W} - mT_b\right) + n(t) \end{aligned} \quad (1)$$

wherein $c_{k,l}(m)$ and $b_k(m) \in \{+1, -1\}$ are the random gain affecting the l-th replica of the k-th user signal and the k-th information bit in the m-th signaling interval, respectively, A_k is the amplitude of the k-th user, $\{\tau_k\}_{k=0}^{K-1}$ is the set of the relative delays, while $s_k(\cdot)$ is the *signature* of the k-th user, whose expression is given by:

$$s_k(t) = \sum_{n=0}^{N-1} \beta_{nk} \psi_{T_c}(t - nT_c)$$

In the above expression, $\psi_{T_c}(\cdot)$ is a unit-height rectangular waveform supported on the interval $[0, T_c[$, T_c is the chip-interval, N the processing gain, while $\{\beta_{nk}\}_{n=0}^{N-1}$ is the spreading sequence assigned to the k-th user. Finally, $(2P+1)$ is the transmitted packet length and $n(t)$ is the white Gaussian thermal noise, with Power Spectral Density $2\mathcal{N}_0$.

Since it is convenient to process digitally the received signal, we are to convert $r(t)$ into a discrete-time signal. To this end, we feed $r(t)$ to a filter matched to a rectangular waveform of duration T_c/M (with M an integer positive number), and sample its output at rate M/T_c. Notice that, if $M = 1$, this operation amounts to the classical chip-matched filtering, which does not yield any loss of information only if the delays $\{\tau_k\}_{k=1}^{K-1}$ and $1/W$ are all integer multiples of T_c. Viceversa, for arbitrary values of the above parameters, the said projection with $M > 1$ allows a reduction of the information loss incurred in the signal discretization, and, ultimately, as will be subsequently shown, a system performance improvement.

3. DETECTOR DESIGN

After the said discretization operation, in the p-th signaling interval we have the following NM-dimensional vector:

$$\begin{aligned} \boldsymbol{r}(p) = A_0 b_0(p) \sum_{l=0}^{L-1} c_{0,l}(p) \boldsymbol{s}_{0,l} + \boldsymbol{m}(p) + \boldsymbol{n}(p) = \\ A_0 b_0(p) \boldsymbol{S}_0 \boldsymbol{c}_0(p) + \boldsymbol{m}(p) + \boldsymbol{n}(p) \end{aligned} \quad (2)$$

In the above expression, $\boldsymbol{s}_{0,l}$ represents the contribution from the waveform $s_0(t - l/W - pT_b)$, and $\boldsymbol{m}(p)$ and $\boldsymbol{n}(p)$ represent the discrete-time versions of the MAI and of the thermal noise, respectively. Additionally, we have let $\boldsymbol{S}_0 = [\boldsymbol{s}_{0,0}, \ldots, \boldsymbol{s}_{0,L-1}]$, $\boldsymbol{c}_0(p) = [c_{0,0}(p), \ldots, c_{0,L-1}(p)]^T$, with $(\cdot)^T$ denoting transpose, and, for simplicity, we have neglected the Inter-Symbol Interference (ISI) contribution caused by the frequency-selective fading [2]. This is a commonly encountered hypothesis, since, for spread-spectrum signals, we have $(L-1)/W << T_b$.

Now, in order to decode the bit $b_0(p)$, we choose to adopt an MMSE optimality criterion. Notice that, since the cross-correlation between the vector $\boldsymbol{r}(p)$ and $b_0(p)$ is zero, we cannot state our detection problem as that of estimating with minimum mean square error the bit $b_0(p)$, as it is customary when dealing with unfaded channels [4]. Thus, in order to overcome this difficulty and to rule out the trivial solution, here we seek the linear MMSE estimate of $b_0(p)$ conditioned upon the fading vector $\boldsymbol{c}_0(p)$. Otherwise stated, we adopt here the following decision rule

$$\widehat{b}_0(p) = \text{sgn}\left\{\Re\left(\boldsymbol{d}_0^H(p)\boldsymbol{r}(p)\right)\right\} \quad (3)$$

wherein $(\cdot)^H$ denotes conjugate transpose, sgn$(\cdot)$ and $\Re(\cdot)$ denote the signum function and real part, respectively, and the NM-dimensional vector $\boldsymbol{d}_0(p)$ is chosen so as to minimize the conditional mean square error

$$E\left[\left|b_0(p) - \boldsymbol{d}_0^H(p)\boldsymbol{r}(p)\right|^2 \Big| \boldsymbol{c}_0(p)\right]$$

Of course, the solution to such a problem is written as:

$$\boldsymbol{d}_0(p) = \boldsymbol{R}_{\boldsymbol{rr}}^{-1}(p)\boldsymbol{R}_{\boldsymbol{r}b_0}(p) \quad (4)$$

wherein we have:

$$\begin{aligned} \boldsymbol{R}_{\boldsymbol{rr}}(p) &= A_0^2 \boldsymbol{S}_0 \boldsymbol{c}_0(p) \boldsymbol{c}_0^H(p) \boldsymbol{S}_0^H + \widetilde{\boldsymbol{R}} \\ \boldsymbol{R}_{\boldsymbol{r}b_0}(p) &= A_0 \boldsymbol{S}_0 \boldsymbol{c}_0(p) \end{aligned} \quad (5)$$

with $\widetilde{\boldsymbol{R}}$ representing the statistical correlation matrix of the term $\boldsymbol{m}(p) + \boldsymbol{n}(p)$: this matrix is independent of the temporal index p, in that we have assumed that the fading vector sequences are wide-sense-stationary (WSS).

Some remarks are now in order about the found solution. First of all, notice that, as it is obvious for any coherent signaling scheme, the vector $\boldsymbol{d}_0(p)$ depends upon the realizations of the fading vector $\boldsymbol{c}_0(p)$. Here, since the emphasis is on the problem of MAI suppression in DS/CDMA systems over frequency-selective fading channels, we do not dwell on the problem of the channel estimation, and, in keeping with [1, 2, 3], we assume that $\boldsymbol{c}_0(p)$ is perfectly estimated. Notice however, that the receiver requires knowledge of only the fading coefficients affecting the signal from the user to be decoded. Moreover, it is seen that, due to the term $\boldsymbol{c}_0(p)$, the conditional covariance matrix $\boldsymbol{R}_{\boldsymbol{rr}}(p)$ depends upon the temporal index p. As a consequence, from (4) it might appear that implementation of the decision rule (3) entails an *on-line* (e.g. at each signaling interval for the case of fast fading) inversion of the $NM \times NM$-dimensional matrix $\boldsymbol{R}_{\boldsymbol{rr}}(p)$, which is a prohibitive computational burden for practical implementation of the proposed detector. Luckily enough, this difficulty can be easily overcome. Indeed, from (5) it is seen that the vector $\boldsymbol{c}_0(p)$, which originates the time-varying nature of $\boldsymbol{R}_{\boldsymbol{rr}}(p)$, introduces a rank-one modification over the full-rank time invariant matrix $\widetilde{\boldsymbol{R}}$. As a consequence, substituting expressions (5) into (4), and applying the matrix inversion lemma [5], we have

$$\boldsymbol{d}_0(p) = \frac{A_0 \widetilde{\boldsymbol{R}}^{-1} \boldsymbol{S}_0 \boldsymbol{c}_0(p)}{1 + A_0^2 \boldsymbol{c}_0^H(p) \boldsymbol{S}_0^H \widetilde{\boldsymbol{R}}^{-1} \boldsymbol{S}_0 \boldsymbol{c}_0(p)} \quad (6)$$

Additionally, since the decision rule (3) is invariant to any positive scaling of the test statistic, we can neglect the denominator in (6) and consider the following simplified solution:

$$\boldsymbol{d}_0'(p) = \widetilde{\boldsymbol{R}}^{-1} \boldsymbol{S}_0 \boldsymbol{c}_0(p) \quad (7)$$

As a consequence, it is evident that computation of our solution does not require a matrix inversion with O$\left((NM)^3\right)$ computational complexity at each signaling interval, but just the product of the $NM \times L$-dimensional matrix $\widetilde{\boldsymbol{R}}^{-1}\boldsymbol{S}_0$ (which may be computed *una tantum*) by the L-dimensional fading vector $\boldsymbol{c}_0(p)$.

4. PERFORMANCE ANALYSIS

It is straightforward to show that, conditioned upon the fading vectors, the other-users delays $\{\tau_k\}_{k=1}^{K-1}$, and the bit from the other users falling into the p-th signalling interval, the test statistic is a Gaussian random variate, whence the conditional error probability is written as:

$$P(e) = \frac{1}{2}\text{erfc}\left\{\frac{\Re\left\{\boldsymbol{c}_0^H(p)\boldsymbol{S}_0^H\widetilde{\boldsymbol{R}}^{-1}(A_0\boldsymbol{S}_0\boldsymbol{c}_0(p) + \boldsymbol{m}(p))\right\}}{\sqrt{2\mathcal{N}_0 \boldsymbol{c}_0^H(p)\boldsymbol{S}_0^H\widetilde{\boldsymbol{R}}^{-2}\boldsymbol{S}_0\boldsymbol{c}_0(p)}}\right\} \quad (8)$$

(with erfc$\{\cdot\}$ the complementary error function) and the unconditional error probability can be obtained by numerically averaging expression (8).

Besides the Error-Probability, a widely adopted performance measure in the analysis of CDMA Systems is the Near-Far Resistance, which gives an insight into the system behavior as the thermal noise level vanishes and/or the other-users amplitudes increase arbitrarily. It can be shown that, as $2\mathcal{N}_0 \to 0$, the contribution to the test statistic from the interference $\boldsymbol{m}(p)$ is nullified, in that the solution $\boldsymbol{d}_0(p)$ ends up orthogonal to the subspace spanned by the interference [4]. As a consequence, for $2\mathcal{N}_0 << 1$, conditioned upon the vector $\boldsymbol{c}_0(p)$, the error probability admits the following asymptotic expression:

$$P_0(e|\boldsymbol{c}_0(p)) \approx \frac{1}{2}\text{erfc}\left\{\frac{A_0\sqrt{\boldsymbol{c}_0^H(p)\boldsymbol{S}_0^H\boldsymbol{Q}\boldsymbol{S}_0\boldsymbol{c}_0(p)}}{\sqrt{2\mathcal{N}_0}}\right\} \quad (9)$$

wherein the $NM \times NM$-dimensional matrix $\boldsymbol{Q}$ is the projector along the orthogonal complement of the subspace spanned by the interference. Its expression, in turn, is given by

$$\boldsymbol{Q} = \boldsymbol{I} - \boldsymbol{\Phi}\boldsymbol{\Phi}^H$$

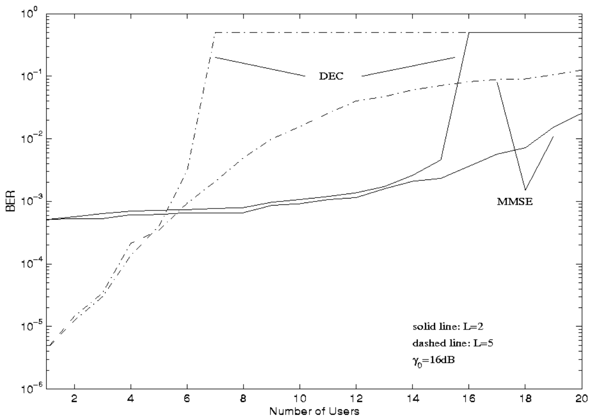

Figure 1: BER of the proposed detector and of the decorrelating detector versus the users number, for $L = 2$ and $L = 5$ and $\gamma_0 =$ 16dB.

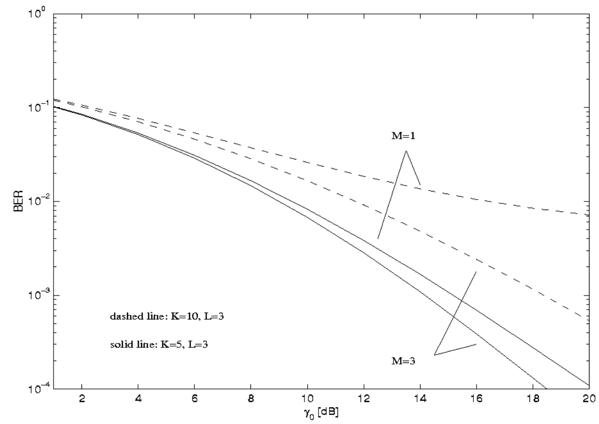

Figure 2: BER of the proposed detector versus the average received energy contrast for two values of M and K, and for $L = 3$.

Obviously, $\boldsymbol{\Phi}$ is a matrix containing on its columns an orthonormal basis for the vector space spanned by the interference $\boldsymbol{m}(p)$. Such a basis, which, due to the stationarity of the fading, is one and the same $\forall p$, can be found by means of a Singular Value Decomposition (SVD) of the covariance matrix of the vector sequence $\boldsymbol{m}(\cdot)$. Finally, $\boldsymbol{I}$ is the identity matrix of order NM.

Now, if one consider the relevant case of Rayleigh distributed fading, i.e. if one assumes that the fading coefficients $c_{k,l}(\cdot)$ are complex zero-mean Gaussian variates, expression (9) can be averaged with respect to $\boldsymbol{c}_0(\cdot)$; following the same steps as in [1], it is easy to show that the system Near-Far resistance is written as:

$$\eta = \left(\frac{\det \left(\boldsymbol{S}_0^H \boldsymbol{Q} \boldsymbol{S}_0 \right)}{\det \left(\boldsymbol{S}_0^H \boldsymbol{S}_0 \right)} \right)^{1/L} \tag{10}$$

Of course, even if we have not explicited such a dependence, it is understood that the Near-Far Resistance (10) depends - through the matrix $\boldsymbol{\Phi}$ - upon the realizations of the other users delays $\{\tau_k\}_{k=1}^{K-1}$, whereby, in general, a certain set of such parameters results in a given value of Near-far Resistance.

In subsequent plots we have adopted Gold codes as spreading sequences, with processing gain $N = 31$, and have assumed, as it is common practice, that the fading coefficients $c_{k,l}(\cdot)$ are uncorrelated with respect to k and l. In figure 1 we have represented the system Bit-Error-Rate (BER) versus the network users number, for an average received energy contrast $\gamma_0 = 16$dB, and for $L = 2$ and 5. We have considered a synchronous system with $A_k/A_0 = 5$dB for $k = 1, \ldots, K-1$, and we have let $W = 1/T_c$. For comparison purposes, we also show the performance of the decorrelating (DEC) detector [2]. As expected, for large K the MMSE detector largely outperforms the DEC receiver: indeed, the receiver in [2] suffers the effects of the noise enhancement, and, as the product $L(K-1)$ exceeds the processing gain N (i.e. the dimensionality of the signals subspace in a synchronous system) it totally fails and is unable to reliably decode the transmitted bits stream. Conversely, the MMSE detector, while slightly outperforming the DEC receiver for small K, achieves satisfactory performance also as K increases, thus enabling a remarkable improvement in the system capacity. Moreover, the claim, made in [2], that large values of the multipath order L are beneficial for the system BER, is only partially true: indeed, figure 1 shows that this consideration holds only for small K; conversely, as the network load increases, larger and larger values of L are detrimental for the system BER, in that they cause a sort of multiaccess interference multiplication.

With reference to an asynchronous system, in figure 2 we have represented the system BER versus γ_0 for two values of K and M and $L = 3$. The curves are the result of an average over 100 random realizations of the delays $\{\tau_k\}_{k=1}^{K-1}$. As anticipated, it is seen that letting $M > 1$ yields a noticeable performance improvement in asynchronous systems, especially for $K = 10$. Notice that we have not shown here the performance of the DEC receiver for asynchronous systems derived in [3] for two reasons. First, the detector in [3] considers an infinitely-extended processing window, and is thus practically unfeasible, whereas the proposed receiver processes only the waveform received in the current signaling interval. Second, in [3] the front-end section of the receiver amounts to a bank of KL filters (which are matched to the waveforms of all of the active users and to all of their replicas), whereas here we have adopted just one matched filter and one sampler. As a consequence, a comparison between our system and the one in [3] is greatly unfair, in that a remarkable complexity gap exists between the two receivers.

In figure 3 we have represented the system Near-Far Resistance, averaged over 100 realizations of the other users delays, versus the users number, for $L = 3$ and several values of the oversampling factor M. As expected, also this performance measure ends up larger and larger as M increases. Moreover, in figure 4 we have represented the system Near-Far resistance versus the network users number for $M = 3$ and several values of the multipath diversity order L. It is seen that the system Near-Far Resistance degrades as L increases. This behavior may be easily justified by noticing that the rank of the projection matrix $\boldsymbol{Q}$ decreases with L, as higher and higher values of L cause an increment in the dimensionality of the interference subspace.

Even if we have seen that increasing M permits achieving better and better performance, we finally notice that such a parameter should not be chosen very high: indeed, since the vector $\boldsymbol{r}(\cdot)$ is NM-dimensional, it is apparent that increasing M may pose

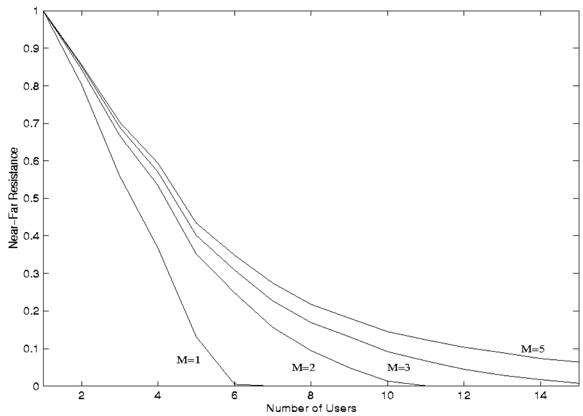

Figure 3: System Near-Far Resistance versus the network users number, for $L = 3$ and several values of the oversampling factor M.

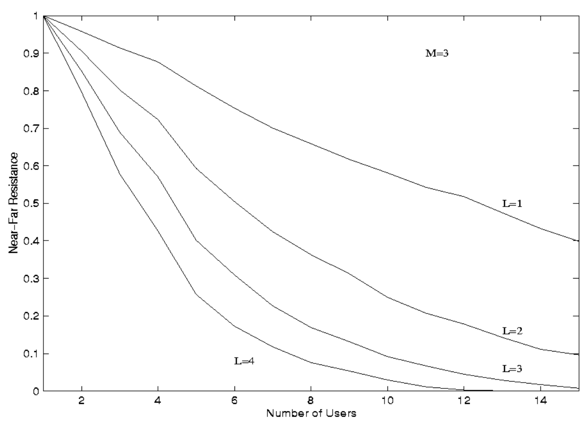

Figure 4: System Near-Far Resistance versus the network users number, for $M = 3$ and several values of the multipath diversity order L.

practical problems due to the receiver complexity. However, our results confirm that even moderate values of M permit achieving a remarkable performance improvement, as well as that, as M increases, the system performance gains end up smaller and smaller.

5. CONCLUSIONS

In this work we have introduced an MMSE multiuser detector for asynchronous DS/CDMA systems over frequency-selective Rayleigh fading channels. The front-end section of the proposed receiver amounts to a filter matched to a rectangular waveform whose duration equals a fraction of the chip-interval. Such a choice, while avoiding the need for a bank of KL filters matched to all of the signatures of the active users and to their replicas, allows a noticeable performance improvement, with respect to the case of pure chip-matched filtering, as the above fraction is reduced. Additionally, by applying the matrix inversion lemma, we have shown that the proposed MMSE multiuser detector for Rayleigh multipath fading channels may be implemented with a computational burden linear in the processing gain N. As to the performance assessment, we have provided closed-form formulas for the system Error Probability and Near-Far Resistance, showing the superiority of the new receiver as compared to the previously derived decorrelating detector for frequency-selective fading channels.

Ongoing research on this topic is focused on the issue of adaptive implementation of the proposed receiver, as well as on its extension to the case that, besides the MAI, also a strong narrowband communication interfering signal corrupts the received waveform.

6. REFERENCES

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