

SELECTIVE DIRECTION FINDING FOR CYCLOSTATIONARY SIGNALS BY EXPLOITATION OF NEW ARRAY CONFIGURATION

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ABSTRACT

This paper presents a new cyclic direction-of-arrival (DOA) estimation algorithm. By exploiting a new minimum-redundancy linear-array (MRLA) configuration and cyclostationarity, the proposed algorithm constructs a matrix pencil from the pseudo-data matrix of $2N-1$ virtual sensors, which are the results of temporal and spatial processing with designed M -sensor Sum-MRLA. Theoretical analysis and computer simulations show that, for each cyclic frequency, the algorithm can estimate $2N-2$ DOAs with M -sensor Sum-MRLA ($N > M$).

1. INTRODUCTION

The cyclostationarity exhibited by most communication signals has been applied to direction-of-arrival (DOA) estimation recently, and many extended DOA estimation algorithms including cyclic MUSIC, cyclic ESPRIT [1] and SC-SSF [2] were proposed. As a result of their signal selectivity, the cyclic direction finding algorithms have better performance than the conventional methods in environments where noise and interference characteristics are unknown. Moreover, they can operate correctly even when the overall number of impinging signals is greater than the number of sensors.

In both cyclic and conventional cases, minimum-redundancy linear array (MRLA) configuration together with appropriate matrix augmentation techniques has been proved to provide better performance than the simple uniform linear array (ULA) configuration in terms of ability to resolve much more DOAs [3,4]. The MRLA configuration turns out to be different according to whether the SOI (signal-of-interest) exhibit second-order cyclostationarity (SOC) or second-order conjugate cyclostationarity (SOCC) [4].

Since many analytic signals in communication (e.g., AM signals) only have the property of SOCC, emphasis will be laid on signals with SOCC. In this paper, a new high-resolution conjugate cyclic DOA estimation algorithm is proposed by utilizing a combination of spatial and temporal processing method with Sum-MRLA configuration. The algorithm constructs a matrix pencil from pseudo-data matrix of the $2N-1$ virtual sensors generated from the M -sensor Sum-MRLA

($N > M$), thus extends array aperture and improves DOA estimation performance increasingly. Our method uses not only spatial information but also temporal information, and can estimate more sources with few sensors. Computer simulations are presented to demonstrate the performance of the proposed algorithm.

2. PROBLEM FORMULATION

2.1 Cyclostationarity and Spectral Correlation

A random process $x(t)$ is called to be cyclostationary if its mean $m_x(t)$ and autocorrelation function $R_{xx}(t, \tau)$ are periodic with some period [5]

$$m_x(t) = E\{x(t)\} = m_x(t + k/\alpha) \quad (1)$$

$$R_{xx} = E[x(t + \tau/2)x^*(t - \tau/2)] = R_{xx}(t + k/\alpha, \tau) \quad (2)$$

where α is cyclic frequency, k is an integer and the asterisk denotes complex conjugation. The cyclic autocorrelation function of $x(t)$ is defined as the Fourier series of periodic function $R_{xx}(t, \tau)$

$$R_{xx}^\alpha(\tau) = \langle R_{xx}(t, \tau) e^{-j2\pi\alpha t} \rangle \quad (3)$$

where $\langle \bullet \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \bullet dt$ denotes time average.

Based on the cycloergodicity [5,6], we can substitute the ensemble average with the temporal average, and another equivalent definition of $R_{xx}^\alpha(\tau)$ is:

$$R_{xx}^\alpha(\tau) = \langle x(t + \tau/2)x^*(t - \tau/2) e^{-j2\pi\alpha t} \rangle \quad (4)$$

If we remove the conjugation operation “*” in the above definition, then the above cyclic correlation is called conjugate cyclic correlation function (CCCF)[5,6].

Generally speaking, (conjugate) cyclic frequency α is usually related with the modulation type, carrier frequency and baud rate of the signal, and different signals have different (conjugate) cyclic frequency set $\{\alpha\}$. By selecting an appropriate α , we can extract the SOI from multiple signals and suppress the effect of the other signals, which is often referred to as “signal selectivity”. Because the spectral components spacing at the frequency interval of α are correlated, the cyclostationarity is also referred to as “spectral correlation”.

2.2 Array Model and Assumption

The well-known ULA configuration is redundant, and the redundancy comes from the fact that different sensor pairs can give the same CCCF. The thinned linear array is frequently used to decrease redundancy. Consider an M -sensor thinned array A , let integers $d_1 < d_2 < \dots < d_M$ represent the sensor locations relative to the first sensor, where integer d_i denotes the value of the distance, normalized to half wavelength, between the i th sensor and the reference sensor. Obviously, $d_i = i - 1$. Assume that K narrow-band signals exhibit SOCC with a common cyclic frequency impinging on the array. The output of the array on the i th sensor can be expressed by

$$x_i(t) = \sum_{k=1}^K s_k(t) \exp(j2\pi f_0 d_i \tau_k) + n_i(t) \quad (5)$$

where $\tau_k = d \sin(\theta_k) / c$, which is the half wavelength path delay of the k th analytic signal $s_k(t)$ impinging from direction θ_k , d stands for a half wavelength, $n_i(t)$ is the analytic signal modeling noise and interference at the i th sensor, f_0 and c are carrier frequency and propagation speed, respectively. We make the following assumptions:

[AS1] The SOIs with non-zero second-order CCCFs are not mutually cyclically correlated at the considered conjugate cyclic frequency α .

[AS2] The interference signals and noises do not have the same SOCC as the SOIs.

Under the above assumptions and cyclic-ergodic condition, from Eq. (5), the CCCF between the outputs of the p th and the q th sensor can be represented as [3]

$$\begin{aligned} R_{x_p x_q}^{\alpha}(\tau) &= \langle x_p(t + \tau/2) x_q(t - \tau/2) \exp(-j2\pi\alpha t) \rangle \\ &= \sum_{k=1}^K R_{s_k s_k}^{\alpha}(\tau) \exp(j2\pi f_0 (d_p + d_q) \tau_k) \\ &= R_x^{\alpha}(d_p + d_q, \tau) \end{aligned} \quad (6)$$

where $R_{s_k s_k}^{\alpha}$ is the CCCF of signal $s_k(t)$.

3. THE PROPOSED ALGORITHM

3.1 Sum-MRLA, Virtual Array and Pseudo-data Matrix

Observe that CCCF $R_{x_p x_q}^{\alpha}(\tau)$ in Eq. (6) depends on array configuration only through the sum $d_p + d_q$, then, we utilize the concept of the sum co-array in imaging aperture synthesis to design MRLA being useful for our case at hand. The sum co-array C_{sum} of a thinned array is defined as the following ordered subset of integer [4,7]

$$C_{sum}(A) = \{d_i + d_j, 1 \leq i, j \leq M\} \quad (7)$$

According to imaging aperture synthesis theory, two array A and B are co-array equivalent if $C_{sum}(A) = C_{sum}(B)$ [4], two

equivalent co-arrays have the same aperture. In our case, equivalent co-arrays can generate the same set of CCCFs. Based on the concept of sum co-array, we design a new kind of MRLA called Sum-MRLA. Table 1. List the Sum-MRLA for $4 \leq M \leq 10$ [4]. In the Table, M -sensor Sum-MRLA B is co-array equivalent with N -sensor ULA [4][7], i.e.

Table 1. The configuration of Sum-MRLA for $4 \leq M \leq 9$

M	N	$\{d_i\}$
4	5	$\{0, 1, 3, 4\}$
5	7	$\{0, 1, 3, 5, 6\}$
6	9	$\{0, 1, 3, 5, 7, 8\}$
7	11	$\{0, 1, 2, 5, 8, 9, 10\}$
8	14	$\{0, 1, 2, 5, 8, 11, 12, 13\}$
9	17	$\{0, 1, 2, 5, 8, 11, 14, 15, 16\}$
10	21	$\{0, 1, 3, 4, 9, 11, 16, 17, 19, 20\}$

$$C_{sum}(B_M) = \{0, 1, \dots, 2N-2\} = C_{sum}(ULA_N) \quad (8)$$

Based on Eq. (6), (7) and (8), $2N-1$ independent CCCFs can be computed, then arrange them in a column vector $\mathbf{R}_X^{\alpha}(\tau)$, we obtain the following equation

$$\mathbf{R}_X^{\alpha}(\tau) = \mathbf{A} \mathbf{R}_S^{\alpha}(\tau) \quad (9)$$

where

$$\mathbf{R}_X^{\alpha}(\tau) = [R_x^{\alpha}(0, \tau), R_x^{\alpha}(1, \tau), \dots, R_x^{\alpha}(2N-2, \tau)]^T$$

$$\mathbf{R}_S^{\alpha}(\tau) = [R_{s_1 s_1}^{\alpha}(\tau), R_{s_2 s_2}^{\alpha}(\tau), \dots, R_{s_K s_K}^{\alpha}(\tau)]^T$$

and

$$\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_K)]$$

here, $\mathbf{a}(\theta_k)$ is augmented steering vector associating with the k th signal

$$\mathbf{a}(\theta_k) = [1, \exp(j2\pi f_0 \tau_k), \dots, \exp(j2\pi f_0 (2N-2) \tau_k)]^T \quad (10)$$

In this paper, we regard the $2N-1$ independent CCCFs in vector form as a pseudo snapshot of $2N-1$ virtual sensors at lag τ , and also define the array with $2N-1$ virtual sensors as “virtual array”. For L different lags (lag $\tau \leq (n-1)T_s$) and a given cyclic frequency α , we can compute L column vectors of CCCFs and arrange them in a $(2N-1) \times L$ matrix $\mathbf{X}(\alpha)$ defined as pseudo-data matrix

$$\mathbf{X}(\alpha) = [\mathbf{R}_X^{\alpha}(0), \mathbf{R}_X^{\alpha}(T_s^v), \dots, \mathbf{R}_X^{\alpha}((L-1)T_s^v)] = \mathbf{A} \mathbf{S}(\alpha) \quad (11)$$

where $\mathbf{S}(\alpha) = [\mathbf{R}_S^{\alpha}(0), \mathbf{R}_S^{\alpha}(T_s^v), \dots, \mathbf{R}_S^{\alpha}((L-1)T_s^v)]$, T_s and n are the sampling period and the number of snapshots for the output of M -sensor Sum-MRLA respectively, T_s^v and L are the pseudo sampling period and the number of pseudo snapshots for the output of $(2N-1)$ -sensor virtual array, and T_s^v and L meet $(L-1)T_s^v \leq (n-1)T_s$. The $(2N-1) \times L$ pseudo-data matrix in Eq. (11) is just similar to the equation $\mathbf{X}(t) = \mathbf{A} \mathbf{S}(t) + \mathbf{N}(t)$ in

conventional array processing, which is the output of $(2N-1)$ -sensor ULA in the conventional subspace-based DOA estimation algorithm, i.e., the equivalent aperture of the M -sensor Sum-MRLA is $2N-1$.

3.2 The Algorithm—SC-MRLA-ESPRIT

From Eq. (11), we pick up two sub-matrices

$$\mathbf{X}_1(\alpha) = [\mathbf{R}_X^\alpha(0), \mathbf{R}_X^\alpha(T_s^v), \dots, \mathbf{R}_X^\alpha((L-1)T_s^v)] = \bar{\mathbf{A}}\mathbf{S}(\alpha) \quad (12)$$

$$\mathbf{X}_2(\alpha) = [\mathbf{R}_X^\alpha(0), \mathbf{R}_X^\alpha(T_s^v), \dots, \mathbf{R}_X^\alpha((L-1)T_s^v)] = \bar{\mathbf{A}}\phi\mathbf{S}(\alpha) \quad (13)$$

where

$$\bar{\mathbf{A}} = [\bar{\mathbf{a}}(\theta_1), \bar{\mathbf{a}}(\theta_2), \dots, \bar{\mathbf{a}}(\theta_K)]$$

$$\bar{\mathbf{a}}(\theta_k) = [1, \exp(j2\pi f_0 \tau_k), \dots, \exp(j2\pi f_0 (2N-3)\tau_k)]^T$$

$$\phi = \text{diag}[\exp(j2\pi f_0 \tau_1), \exp(j2\pi f_0 \tau_2), \dots, \exp(j2\pi f_0 \tau_K)]$$

$\mathbf{R}_X^\alpha(\bullet)$ and $\bar{\mathbf{R}}_X^\alpha(\bullet)$ stand for the vector omitting the last element and the first element of the vector $\mathbf{R}_X^\alpha(\bullet)$, respectively. The auto cyclic correlation matrix of $\mathbf{X}_1(\alpha)$ and the cross cyclic correlation matrix between $\mathbf{X}_1(\alpha)$ and $\mathbf{X}_2(\alpha)$ can be expressed as

$$\mathbf{R}_{\mathbf{X}_1, \mathbf{X}_1}(\alpha) = \bar{\mathbf{A}}\mathbf{R}_s(\alpha)\bar{\mathbf{A}}^T \quad (14)$$

$$\mathbf{R}_{\mathbf{X}_1, \mathbf{X}_2}(\alpha) = \bar{\mathbf{A}}\phi\mathbf{R}_s(\alpha)\bar{\mathbf{A}}^T \quad (15)$$

where $\mathbf{R}_s(\alpha)$ is the covariance matrix of the vector $\mathbf{S}(\alpha)$. From Eq. (14) and Eq. (15), DOAs can be estimated through ESPRIT algorithm [8] with the matrix pencil $(\mathbf{R}_{\mathbf{X}_1, \mathbf{X}_1}(\alpha), \mathbf{R}_{\mathbf{X}_1, \mathbf{X}_2}(\alpha))$. Solving for the generalized eigenvalues of the matrix pencil, we obtain

$$\begin{aligned} & |\mathbf{R}_{\mathbf{X}_1, \mathbf{X}_1}(\alpha) - \mathbf{C}\mathbf{R}_{\mathbf{X}_1, \mathbf{X}_2}(\alpha)| = 0 \\ \Rightarrow & |\bar{\mathbf{A}}\mathbf{R}_s(\alpha)\bar{\mathbf{A}}^T - \mathbf{C}\bar{\mathbf{A}}\phi\mathbf{R}_s(\alpha)\bar{\mathbf{A}}^T| = 0 \\ \Rightarrow & \bar{\mathbf{A}}\mathbf{I} - \mathbf{C}\phi\bar{\mathbf{A}}^T\mathbf{R}_s(\alpha)\bar{\mathbf{A}}^T = 0 \end{aligned} \quad (16)$$

from Eq. (16), we can obtain

$$c_{k,k} = 1/\phi(k,k) = \exp(-j2\pi f_0 d \sin(\theta_k)/c) \quad (17)$$

where $c_{k,k}$ is the k th generalized eigenvalue of the matrix pencil. The K DOAs can be estimated from Eq. (17). Based on analysis mentioned above, we propose a new direction finding algorithm summarized as follows:

Step 1: From Table 1, select M -sensor Sum-MRLA, N of the equivalent co-array and array configuration d_i can be determined accordingly.

Step 2: Based on Eq. (6), for given cyclic frequency α , compute $\mathbf{R}_X^\alpha(\tau)$ (total $2N-1$ elements) from output of the selected array for $\tau \leq (n-1)T_s$. Construct the pseudo-data matrix in Eq. (11).

Step 3: Construct the two sub-matrices in Eq. (12) and Eq. (13), and compute the matrix pencil $(\mathbf{R}_{\mathbf{X}_1, \mathbf{X}_1}(\alpha), \mathbf{R}_{\mathbf{X}_1, \mathbf{X}_2}(\alpha))$ in Eq. (14) and Eq. (15).

Step 4: Select ESPRIT algorithm [8] to estimate K ($K \leq 2N-2$) DOAs at given cyclic frequency with Eq. (17).

We call the above algorithm the spectral correlation conjugate cyclic ESPRIT method with MRLA configuration (SC-MRLA-CCE). Obviously, the proposed algorithm extends array aperture by exploiting the Sum-MRLA and a combination of spatial and temporal processing. Under the condition of the same number of sensors, the aperture in the proposed algorithm is nearly twice as large as that in the algorithm presented in [4], and at least twice as large as that in SC-ESPRIT algorithm [2].

4. SIMULATION RESULTS

In this section, computer simulations are presented to verify effectiveness of the proposed algorithm. We adopt 5-sensor Sum-MRLA configuration, spatially and temporally white Gaussian noise is considered, and DOA estimation is implemented with SVD-TLS-ESPRIT algorithm [8].

Experiment 1: In this experiment, seven independent AM narrow-band signals with carrier frequency f_0 impinge on the array from directions $\{60^\circ, 30^\circ, 15^\circ, 0^\circ, -15^\circ, -30^\circ, -40^\circ\}$, SNR=5dB, and a narrow-band BPSK inference signal with carrier frequency $0.9f_0$ impinges from direction $\{45^\circ\}$, SNR=10dB. The number of snapshot $n=1000$, the cyclic frequency $\alpha=2f_0$, the pseudo sample period $T_s^v=10T_s$, and the number of pseudo snapshots $L=50$. 30 independent Monte-Carlo trials are shown in Fig.1. The results are illustrated in polar system (ρ, θ) , where ρ is the norm of the generalized eigenvalue in (13), and θ is the DOA of the SOI. The results demonstrate that the algorithm can selectively estimate seven DOAs of seven SOIs with 5 sensors, since the equivalent aperture of 5-sensor Sum-MRLA is 13, the algorithm can estimate 12 DOAs. Under the condition of the same number of sensor, the equivalent aperture of the algorithm is larger than that of the conventional subspace-based algorithm with ULA configuration, it is predicted undoubtedly that the proposed algorithm has better performance than the conventional subspace-based algorithm. The results also show that the proposed algorithm is featured with the signal selectivity of carrier frequency.

We also investigate DOA estimation performance of the proposed algorithm for SNR= -5, 5 and 15 dB. Means and standard deviations of DOA estimates are computed from 100 independent Monte-Carlo trials for performance evaluation, and summarized in Table 2. The simulation results show that our algorithm is robust. It is reasonable that standard deviation of the estimate increase as SNR decreases since noisy data degrade performance.

Experiment 2: The accuracy of the proposed algorithm is tested as a function of the separation between the DOA of the SOIs. Two AM signals with same carrier frequency f_0 arrive from

direction $\{-\theta, \theta\}$ with SNR -5 dB. The number of snapshots is 1000. DOA estimation is performed at the cyclic frequency $\alpha=2f_0$. For proposed algorithm the pseudo sample period $T_s^v=10T_s$, and the number of pseudo snapshots $L=50$. Fig. 2 illustrates the mean squared error of the estimates of direction, evaluated over 50 independent trials, versus the DOA separation 2θ , ranging from $6^\circ \sim 16^\circ$. The results show that the DOA estimates obtained by means of the proposed algorithm are generally more accurate than the corresponding ones obtain by the algorithm in [2]. It is because that the aperture in the proposed algorithm with 5-sensor Sum-MRLA is nearly twice as large as that in the algorithm presented in [2] with 5-sensor ULA.

5. CONCLUSION

We have proposed a new DOA estimation algorithm called SC-MRLA-CCE. Theoretical analysis shows that the algorithm extends array aperture, resolves more sources with fewer sensors and has signal selectivity of carrier frequency. The equivalent array aperture in the proposed algorithm with M -sensor Sum-MRLA is $2N-1$. Computer simulations demonstrate the effectiveness of the proposed algorithm.

6. REFERENCES

- [1] W. A. Gardner, "Simplification of MUSIC and ESPRIT by Exploitation of Cyclostationarity," in *Proc. IEEE*, vol. 76, pp. 845 – 847, July. 1988.
- [2] G. Xu and T. Kailath, "Direction-of-Arrival Estimation via Exploitation of Cyclostationarity – A Combination of Temporal and Spatial Processing," *IEEE Trans. on Signal Processing*, vol. 40, pp.1775-1785, July. 1992.
- [3] S. U. Pillai, Y. Bar-Ness, F. Haber, "A New Approach to Array Geometry for Improved Spatial Spectrum Estimation," in *Proc. IEEE*, vol. 73, pp. 1522 – 1525, Oct. 1985.
- [4] G. Gelli and L. Izzo, "Minimum-Redundancy Linear Arrays for Cyclostationarity-based Source Location," *IEEE Trans. on Signal Processing*, vol. 45, pp. 2605 - 2608, Oct. 1997.
- [5] W. A. Gardner, "Exploitation of Spectral Redundancy in Cyclostationarity Signal," *IEEE Signal Processing Mag.* vol.8, pp.14-37, Apr.1991.
- [6] W. A. Gardner, W. A. Brown. III, C. K. Chen, "Spectral Correlation of Modulated Signal: Part II — digital Modulation," *IEEE Trans. on Commu.* vol. 35, no. 2, pp. 595-601. June. 1987.
- [7] R. T. Hocht, S. A. Kassam, "Arrays Redundancy for Active Line Arrays," *IEEE Trans. on IP*, vol.5, no. 7,1179-1183. July.1996.
- [8] X. D. Zhang and Y. C. Liang, "Prefilter-based ESPRIT for Estimating Parameters of Sinusoids in Non-Gaussian ARMA noise," *IEEE Trans. on Signal Processing*, vol. 43, pp. 349-353, Jan.1995.

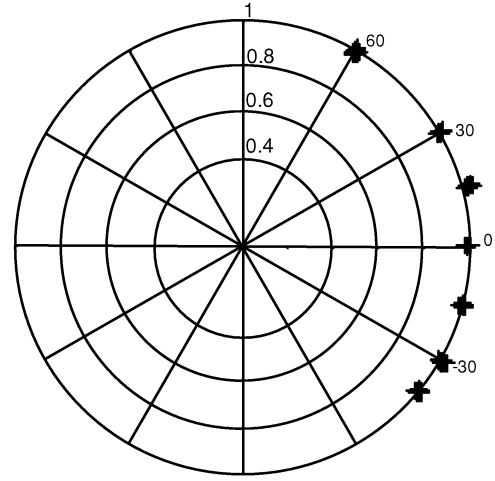


Fig.1 DOA estimation results at cyclic frequency $\alpha=2f_0$

Table 2. Statistics of the DOA estimation results

DOA	SNR=-5dB		SNR=5dB		SNR=15dB	
	mean	Std	mean	std	mean	std
60	60.26	1.558	59.78	0.379	59.84	0.114
30	29.80	0.720	29.88	0.147	29.80	0.064
15	15.30	0.886	14.64	0.224	14.69	0.056
0	0.318	0.686	-0.05	0.098	0.027	0.067
-15	-15.1	0.857	-14.7	0.155	-14.7	0.057
-30	-30.2	0.743	-29.7	0.134	-29.7	0.042
-40	-39.7	1.017	-39.5	0.251	-39.5	0.066

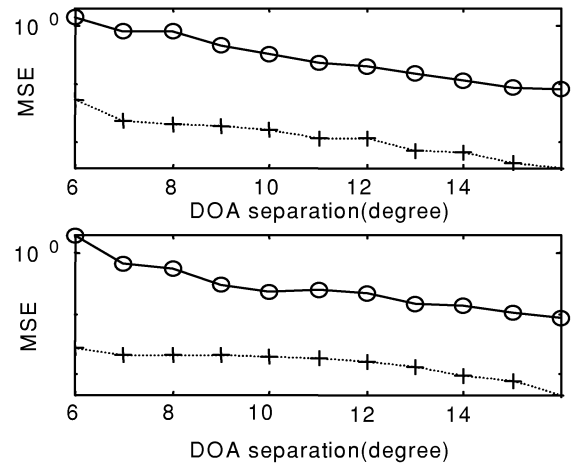


Fig.2 Mean squared error of DOA estimates versus DOA separation for the proposed algorithm ('+') and algorithm in [2] ('o'): DOA $-\theta$ (upper) , DOA θ (lower)