LOSSLESS FILTER BANKS BASED ON TWO POINT TRANSFORM AND INTERPOLATIVE PREDICTION

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ABSTRACT

In this paper, we present a method for designing lossless versions of two-channel FIR filter banks. We demonstrate that equal length PR FIR filter banks can be decomposed into 2-point transforms and unequal length into interpolative predictions. The lossless versions of the filter banks are obtained by replacing every constituent module by the corresponding lossless version. This method allows construction of the lossless versions of filter banks with arbitrary filter length. Lossless versions of several filter banks are designed and they are found to yield good performance for lossless image compression.

1. INTRODUCTION

An unified lossless and lossy image coding system is useful for various applications, since we can reconstruct lossy and lossless images from a part and the whole of an encoded data, respectively. This coding system can be realized by using lossless block transforms or lossless wavelet transforms. In the lossless transforms, integer input signals are transformed into integer transform coefficients and losslessly reconstructed. The mean first order entropy of the integer transform coefficients must be smaller than that of input for image compression. Lossless versions of 8-point discrete cosine transform (DCT) and 8-point Walsh-Hadamard transform (WHT) have been proposed [1][2]. The lossless wavelet transforms consist of lossless versions of filter banks. A lossless version of the symmetric short kernel filter (SSKF) [3] has been proposed [4]. Said and Pearlman proposed the S+P transform (S transform [5] + Prediction) [6]. The TS-transform proposed in [7] is a special case of the S+P transform.

A method for designing lossless versions of general FIR filter banks was proposed [8]. This approach is based on the idea of factoring the filter bank into lifting steps [9]. In this paper, we will propose a design method of lossless versions of 2-channel perfect reconstruction (PR) FIR filter banks based on decomposing them into simpler steps, that is, 2-point transforms and interpolative predictions, than lifting steps. This method will allow us to obtain easily lossless versions of filter banks with arbitrary filter length.

In the following section, lossless versions of a 2-point transform and an interpolative prediction are introduced. In section 3, we will propose methods of decomposing PR FIR filter banks into 2-point transforms or interpolative predictions. Section 4 shows the design examples of lossless filter banks and their performance in lossless image compression. Section 5 concludes this paper.

2. CONSTITUENT MODULES

2.1 Lossless Two-point Transform

A 2×2 matrix $\boldsymbol{A}$ that has scalars a to d is decomposed as follows, if Det($\boldsymbol{A}$) = 1 and $b \neq 0$.

$$\boldsymbol{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ (d-1)/b & 1 \end{bmatrix} \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ (a-1)/b & 1 \end{bmatrix}. \quad (1)$$

The transformation of $[x_0, x_1]$ to $[\theta_0, \theta_1]$ by $\boldsymbol{A}$ is then given by

$$\begin{bmatrix} \theta_b \\ \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} x_1 + c_0 x_0 \\ x_0 + c_1 \theta_b \\ \theta_b + c_2 \theta_0 \end{bmatrix} \quad (2)$$

where $c_0 = (a-1)/b$, $c_1 = b$, $c_2 = (d-1)/b$. The integer version of (2) is given by

$$\begin{bmatrix} \theta_b \\ \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} x_1 + \lfloor c_0 x_0 + 0.5 \rfloor \\ x_0 + \lfloor c_1 \theta_b + 0.5 \rfloor \\ \theta_b + \lfloor c_2 \theta_0 + 0.5 \rfloor \end{bmatrix} \quad (3)$$

where x_0, x_1, θ_b, θ_0 and θ_1 are integer, and c_0, c_1 and c_2 are real. The equation (3) is reversible, that is to say, $[x_0, x_1]$ is losslessly reconstructed from $[\theta_0, \theta_1]$ as

$$\begin{bmatrix} \theta_b \\ x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} \theta_1 - \lfloor c_2 \theta_0 + 0.5 \rfloor \\ \theta_0 - \lfloor c_1 \theta_b + 0.5 \rfloor \\ \theta_b - \lfloor c_0 x_0 + 0.5 \rfloor \end{bmatrix}. \quad (4)$$

The corresponding network is shown in Fig. 1(a). This is called a ladder network [10]. The case of Det($\boldsymbol{A}$) = -1 is shown in Fig. 1(b), where $\boldsymbol{A}$ is decomposed as follows.

$$\boldsymbol{A} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ (1-d)/b & 1 \end{bmatrix} \begin{bmatrix} 1 & -b \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ (a+1)/b & 1 \end{bmatrix}. \quad (5)$$

For example, we can obtain a normalized lossless 2-point WHT by setting $a = b = c = -d = 1/\sqrt{2}$. Unlike the S transform, the dynamic range is uniform.

2.2 Lossless Interpolative Prediction

The 2-point transform is a constituent module of the (2, 2)-FB, where (m, n)-FB indicates the 2-channel filter bank with m-tap

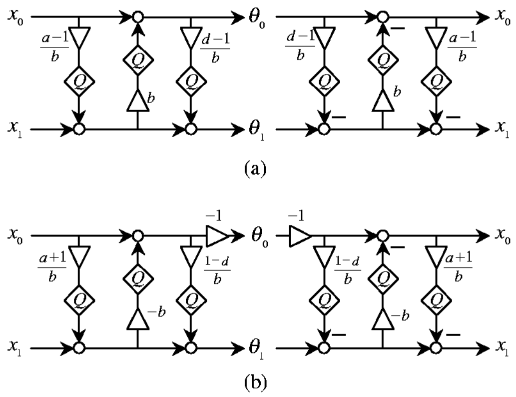

Figure 1. Lossless 2-point transform and its inverse transform. (a)Case of Det($\boldsymbol{A}$)=1. (b)Case of Det($\boldsymbol{A}$)= -1.

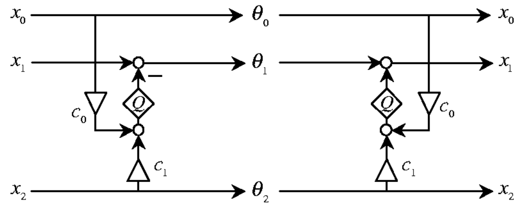

Figure 2. Lossless interpolative prediction and its inverse transform.

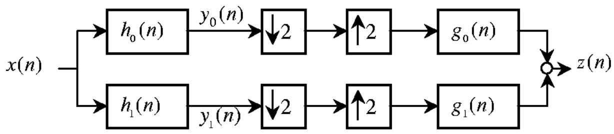

Figure 3. Two-channel analysis/synthesis filter bank.

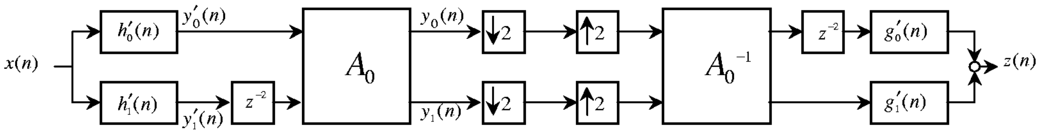

Figure 4. One decomposition of (2L, 2L)-FB.

and n-tap analysis filters. The (1, 3)-FB is as simple as the (2, 2)-FB. An integer version of a constituent module of the (1, 3)-FB is

$$\begin{bmatrix}\theta_0\\ \theta_1\\ \theta_2\end{bmatrix}=\begin{bmatrix}x_0\\ x_1-\lfloor c_0x_0+c_1x_2+0.5\rfloor\\ x_2\end{bmatrix}. \tag{6}$$

It is obvious that the input is losslessly reconstructed. We name it lossless interpolative prediction (LIP). The LIP and its inverse transform are shown in Fig. 2.

3. DECOMPOSITION OF FILTER BANK

3.1 Decomposition of (2L, 2L)-FB

Here, we demonstrate that perfect reconstruction (2L, 2L)-FBs, whose delay is 2L-1, can be decomposed into 2-point transforms. It is known that paraunitary PR filter banks can be decomposed into lattice modules which are special cases of 2-point transforms [11]. We show that non-paraunitary PR filter banks can also be decomposed into 2-point transforms.

The two channel filter bank is shown in Fig. 3. Let's suppose that every filter length is 2L ($L\geq 2$), that is,

$$\begin{aligned}y_0(n)&=\sum_{m=0}^{2L-1}h_0(m)x(n-m)\\ y_1(n)&=\sum_{m=0}^{2L-1}h_1(m)x(n-m)\end{aligned}, \tag{7}$$

the delay of the filter bank is 2L-1, and

$$\begin{aligned}g_0(n)&=(-1)^n h_1(n)\\ g_1(n)&=-(-1)^n h_0(n)\end{aligned}. \tag{8}$$

Then, from theory of filter banks [12], the perfect reconstruction requirement in time domain is

$$\sum_{n=0}^{2L-1}(-1)^{n-1}h_0(n)h_1(2m-n-1)=\delta(m-l) \tag{9}$$

for $1\leq m\leq 2L-1$. The following equations are obtained by setting $m=1$ and $m=2L-1$ in (9),

$$\begin{aligned}&-h_0(0)h_1(1)+h_0(1)h_1(0)=0\\ &h_0(2L-1)h_1(2L-2)-h_0(2L-2)h_1(2L-1)=0\end{aligned}. \tag{10}$$

From (7) and (10), the following equations are obtained.

$$\begin{aligned}y_0'(n)&=h_1(2L-2)y_0(n)-h_0(2L-2)y_1(n)\\ &=\sum_{m=0}^{2L-3}h_0'(m)x(n-m)\\ y_1'(n)&=-s_0h_1(1)y_0(n+2)+s_0h_0(1)y_1(n+2)\\ &=\sum_{m=0}^{2L-3}h_1'(m)x(n-m)\end{aligned} \tag{11}$$

where s_0 is an arbitrary constant. Equation (11) means that the analysis part of the PR (2L, 2L)-FB is decomposed into 2L-2 tap filters and a 2-point transform $\boldsymbol{A}_0$ as shown in Fig. 4, where

$$\boldsymbol{A}_0=\begin{bmatrix}h_1(2L-2) & -h_0(2L-2)\\ -s_0h_1(1) & s_0h_0(1)\end{bmatrix}^{-1}. \tag{12}$$

In the synthesis part in Fig. 3,

$$\begin{aligned}z(n)&=\sum_{m=0}^{L-1}(g_0(2m)y_0(n-2m)+g_1(2m)y_1(n-2m))\\ z(n+1)&=\sum_{m=0}^{L-1}(g_0(2m+1)y_0(n-2m)+g_1(2m+1)y_1(n-2m))\end{aligned} \tag{13}$$

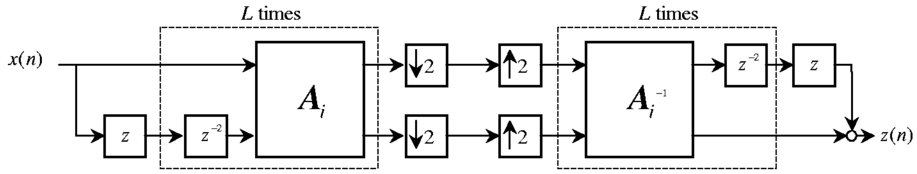

Figure 5. Full decompositions of (2*L*, 2*L*)-FB.

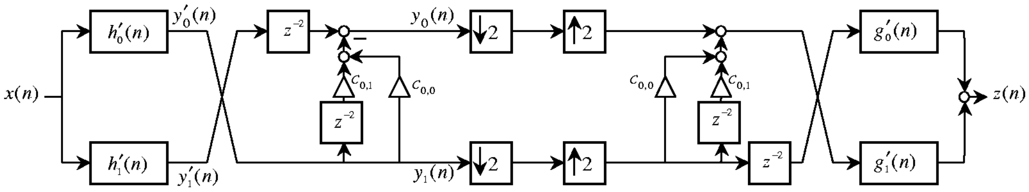

Figure 6. One decomposition of (2*L*+1, 2*L*-*1*)-FB.

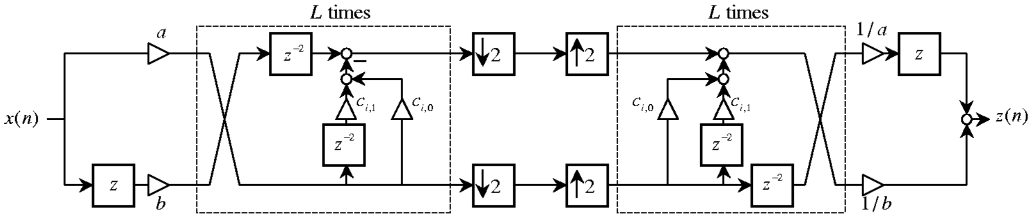

Figure 7. Full decompositions of (2*L*+1, 2*L*-*1*)-FB.

where n is even. From (8), (10), (11) and (13), the following equations are derived.

$$
\begin{aligned}
z(n+2) &= \sum_{m=0}^{L-2}(g'_0(2m)y'_0(n-2m)+g'_1(2m)y'_1(n-2m)) \\
z(n+3) &= \sum_{m=0}^{L-2}(g'_0(2m+1)y'_0(n-2m)+g'_1(2m+1)y'_1(n-2m))
\end{aligned} .(14)
$$

Equation (11), (12) and (14) mean that the synthesis part is decomposed into 2*L*-2 tap filters and a 2-point transform $\boldsymbol{A}_0^{-1}$ as shown in Fig. 4. The filter bank that consists of $h'_0(n)$, $h'_1(n)$, $g'_0(n)$ and $g'_1(n)$ is PR and its delay is 2*L*-3. Thus, the original filter bank is decomposed into 2*L*-layer 2-point transforms as shown in Fig. 5 by repeating the above decompositions. To replace every 2-point transform by the corresponding lossless 2-point transform, we must multiply each matrix by a constant so that the determinant will become 1 or –1. There is *L* degrees of freedom in the decomposition process, that is, s_0, s_1, …, s_{L-2}, where s_i is an arbitrary constant used in $\boldsymbol{A}_i$, and the ratio of dynamic range of $y_0(n)$ to that of $y_1(n)$ in the original filter bank. These parameters should be determined so that the performance of the lossless filter bank, for example, lossless compression efficiency, would become high.

3.2 Decomposition of (2*L*+1, 2*L*-1)-FB

Here, we demonstrate that PR (2*L*+1, 2*L*-1)-FBs, whose delay is 2*L*-1, can be decomposed into interpolative predictions. Let's suppose in Fig. 3 that filter length of $h_0(n)$ is 2*L*+1, that of $h_1(n)$ is 2*L*-1 ($L \geq 2$), that is,

$$
\begin{aligned}
y_0(n) &= \sum_{m=0}^{2L} h_0(m)x(n-m) \\
y_1(n) &= \sum_{m=0}^{2L-2} h_1(m)x(n-m)
\end{aligned} , \tag{15}
$$

the delay of the filter bank is 2*L*-1, and (8) is satisfied. Then, the perfect reconstruction requirement in time domain is

$$
\sum_{n=0}^{2L}(-1)^{n-1}h_0(n)h_1(2m-n-1)=\delta(m-l) \tag{16}
$$

for $1 \leq m \leq 2L-1$. The following equations are obtained by setting $m=1$ and $m=2L-1$ in (16),

$$
\begin{aligned}
-h_0(0)h_1(1)+h_0(1)h_1(0)&=0 \\
h_0(2L)h_1(2L-3)-h_0(2L-1)h_1(2L-2)&=0
\end{aligned} . \tag{17}
$$

From (15) and (17), the following equations are obtained.

$$
\begin{aligned}
y'_0(n) &= y_1(n) = \sum_{m=0}^{2L-2} h'_0(m)x(n-m) \\
y'_1(n) &= y_0(n+2) - \frac{h_0(1)}{h_1(1)} y_1(n+2) - \frac{h_0(2L-1)}{h_1(2L-3)} y_1(n) \\
&= \sum_{m=0}^{2L-4} h'_1(m)x(n-m)
\end{aligned} . \tag{18}
$$

Equation (18) means that the analysis part of this filter bank is decomposed into a 2*L*-1 tap filter $h'_0(n)$, a 2*L*-3 tap filter $h'_1(n)$ and an interpolative prediction as shown in Fig. 6, where $c_{0,0} = -h_0(1)/h_1(1)$ and $c_{0,1} = -h_0(2L-1)/h_1(2L-3)$. We can derive in the same manner as **3.1** that the synthesis part is decomposed into a 2*L*-3 tap filter $g'_0(n)$, a 2*L*-1 tap filter $g'_1(n)$ and an interpolative prediction as shown in Fig. 6. The filter bank

Table I. Coefficients of LIPs in (5, 7)-LFB and 2-point transforms in 12-LFB and 16-LCQF.

	Coefficients of LIPs			
(5, 7) - LFB	$c_{0,0} = c_{0,1} = 39/185$ $c_{1,0} = c_{1,1} = -5/14$ $c_{2,0} = c_{2,1} = 0.2$			
	2 - point transforms			
12 - LFB	$a_0 = -1.066016$	$b_0 = 1.066016$	$c_0 = 0.469036$	$d_0 = 0.469036$
	$a_1 = 1.010085$	$b_1 = 0.142379$	$c_1 = -0.142379$	$d_1 = -1.010085$
	$a_2 = -0.099572$	$b_2 = -1.004945$	$c_2 = 1.004945$	$d_2 = 0.099572$
	$a_3 = -1.052956$	$b_3 = -0.329721$	$c_3 = 0.329721$	$d_3 = 1.052956$
	$a_4 = -0.310223$	$b_4 = 1.047014$	$c_4 = 1.047014$	$d_4 = -0.310223$
	$a_5 = -0.658149$	$b_5 = 1.197147$	$c_5 = 1.197147$	$d_5 = -0.658149$
16 - LCQF	$a_0 = -0.836680$	$b_0 = 0.547692$	$c_0 = 0.547692$	$d_0 = 0.836680$
	$a_1 = 0.919718$	$b_1 = 0.392579$	$c_1 = 0.392579$,	$d_1 = -0.919718$
	$a_2 = 0.862876$	$b_2 = -0.505416$	$c_2 = 0.505416$	$d_2 = 0.862876$
	$a_3 = -0.514990$	$b_3 = 0.857196$	$c_3 = 0.857196$	$d_3 = 0.514990$
	$a_4 = 0.141530$	$b_4 = 0.989934$	$c_4 = 0.989934$	$d_4 = -0.141530$
	$a_5 = 0.679310$	$b_5 = 0.733851$	$c_5 = 0.733851$	$d_5 = -0.679310$
	$a_6 = 0.946089$	$b_6 = 0.323907$	$c_6 = 0.323907$	$d_6 = -0.946089$
	$a_7 = 0.997421$	$b_7 = 0.071776$	$c_7 = 0.071776$	$d_7 = -0.997421$

that consists of $h'_0(n)$, $h'_1(n)$, $g'_0(n)$ and $g'_1(n)$ is PR and its delay is $2L$-3. Thus, the original filter bank is decomposed into $2L$-layer interpolative predictions and four multipliers as shown in Fig. 7 by repeating the above decompositions.

We can delete the four multipliers by shifting them through the interpolative predictions. In this time, the coefficients of the interpolative predictions will change. Thus, the lossless version of the ($2L$+1, $2L$-1)-FB can be obtained by replacing every interpolative prediction by LIP. For example, the lossless version of (5, 3)-SSKF is given by setting L = 2, $c_{0,0} = c_{0,1} = -0.25$ and $c_{1,0} = c_{1,1} = 0.5$ [4].

4. DESIGN EXAMPLES AND LOSSLESS COMPRESSION EFFICIENCY

The lossless versions of 16-tap CQF [13], 12-tap linear phase filter [14] and (5, 7)-FB [15] were constructed, where we name them 16-LCQF, 12-LFB and (5, 7)-LFB, respectively. The coefficients of LIP and 2-point transforms are shown in Table I. To evaluate the resulting lossless versions, six USC test images were used. All of these images are 512×512 and 8bits/pixel grayscale. For comparison purposes, we also provide the results obtained with two leading lossless filter banks: (5, 3)-LFB [4] and TS-transform and the lossless DCT (LDCT) [1]. The lossless filter banks were used for two-dimensional ten-band octave decomposition. The mean first order entropy of transformed images are given in Table II, where the values of PCM are entropies of input images. It is seen that the performances of new lossless filter banks are comparable to those of conventional transforms.

5. CONCLUSIONS

A design method of lossless versions of filter banks has been proposed. This method is based on decomposing filter banks into 2-point transforms or interpolative predictions. This allows construction of lossless versions of PR FIR filter banks with arbitrary filter length. The new lossless filter banks were found to

Table II. Mean first order entropy of transformed images (bit/pel).

	Aerial	Airplane	Baboon	Couple	Lenna	Peppers
PCM	6.99	6.80	7.47	7.20	7.59	7.49
LDCT	5.49	4.54	6.32	4.76	4.79	4.97
(5,3)-LFB	5.28	4.37	6.33	4.62	4.73	4.91
TS	5.41	4.50	6.39	4.78	4.79	4.95
(5,7)-LFB	5.53	4.59	6.43	4.83	4.86	4.96
12-LFB	5.34	4.49	6.31	4.73	4.77	4.96
16-LCQF	5.49	4.67	6.36	4.93	4.85	5.07

yield good performance for lossless image compression. It seems that the decomposition of filter banks is available for not only lossless coding system, but also adaptive wavelet transform.

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