A NOVEL MULTIRATE ADAPTIVE FIR FILTERING ALGORITHM AND STRUCTURE

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ABSTRACT

A new class of FIR filtering algorithms and VLSI architectures based on the multirate approach were recently proposed. They not only reduce the computational complexity in FIR filtering, but also retain attractive implementation-related properties such as regularity and multiply-and-accumulate (MAC) structure. In addition, the multirate feature can be applied to low-power/high-speed VLSI implementation. These properties make the multirate FIR filtering very attractive in many DSP and communication applications. In this paper, we propose a novel adaptive filter based on this new class of multirate FIR filtering structures. The proposed adaptive filter inherits the advantages of the multirate structures such as low computational complexity and low-power/high-speed applications. Moreover, the multirate feature helps to improve the convergence property of the adaptive filters.

1. INTRODUCTION

The finite-impulse response (FIR) filter is the fundamental processing element in many digital signal processing (DSP) and communication systems. Many algorithms have been studied to reduce the computational complexity of FIR filtering. Recently, a new class of fast FIR filtering algorithms based on the multirate approach were proposed [1][2]. It is a multirate parallel filtering structure with decimation factor equal to M. The input signal at sampling rate f_s is first decimated into M interleaved sequences $x_i(n)$, $i = 0, 1, 2, \cdots, M-1$. After the pre-processing network, the generated output data streams are fed into the sub-filters running in parallel at a low rate of f_s/M. The outputs are then converted back to the filtering output signal, $y(n)$, through the post-processing network and up-sampling circuit. The special cases for $M = 2, 3$ are depicted in Fig. 1(a) and (b), respectively.

The advantages of the multirate filtering structure are as follows. First, the required multiplication operations per unit sample time (abbreviated as MPU) decreases as decimation factor, M, increases. This feature is preferable in reducing the million-instructions-per-second (MIPS) count in running programmable DSP processors (DSPs). Second, in contrast to the overlap-and-add/overlap-and-save approaches [3], the multirate FIR filtering is performed only in the real domain without using FFT/IFFT operations. It also retains the multiply-and-accumulate (MAC) structure which is optimized in most programmable DSPs. Moreover,
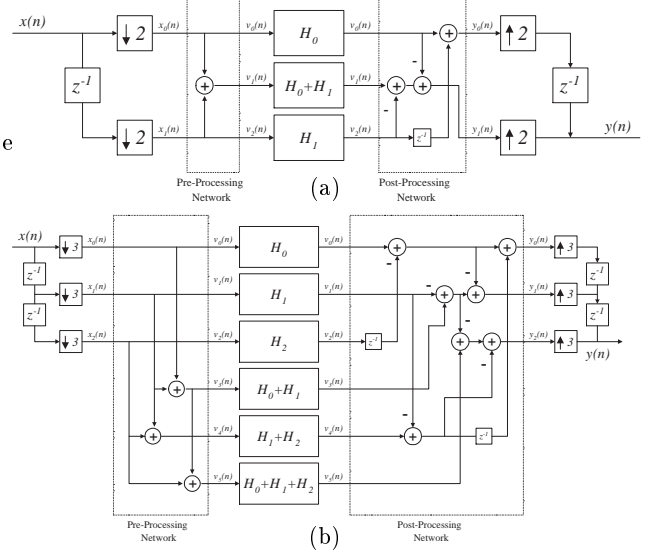

Figure 1: Multirate FIR filters with decimation factor (a) $M=2$, (b) $M=3$.

for hardware implementation, the VLSI structures are more regular and require fewer intermediate memories compared with the overlap-based approaches. Third, the multirate FIR is a parallel processing structure in nature. Hence, it can be readily applied to high-speed/low-power applications [4][5].

Due to the vast advantages of the multirate FIR filtering algorithm and architecture, we are motivated to study a novel adaptive filtering scheme based on the multirate approach. Figure 2 shows our idea. Part (a) is the block diagram of a conventional LMS-type adaptive filter, where error signal $e(n)$ is used to update the coefficients of the FIR filter so as to minimize the mean-squared error function, $E[e^2(n)]$. In our approach, we replace the transversal filter with the multirate FIR filter. As a result, the new adaptive filter inherits the advantages of the multirate FIR structures such as low computational complexity, regularity, and low-power/high-speed applications. Also, the multirate feature can help to improve the convergence properties of the adaptive filters. The detailed algorithm and architecture are discussed in the following section.
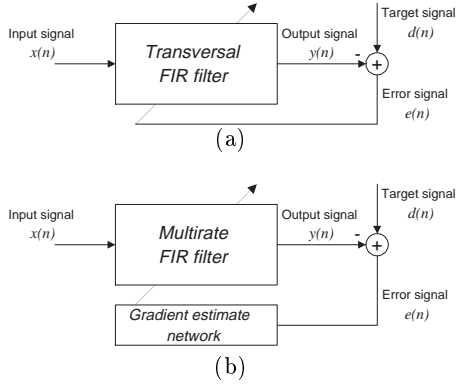
Figure 2: (a) Conventional adaptive filter. (b) The proposed adaptive filter based on the multirate FIR structure.

2. UPDATING ALGORITHM AND VLSI ARCHITECTURE

In this section, we derive the updating equations and architecture of the proposed multirate adaptive filter. Mathematically, an N-th order LMS adaptive FIR filter can be described by the following equations:

$$
\begin{aligned}
y(n) &= \sum_{k=0}^{N-1} w_k(n)x(n-k), \\
e(n) &= d(n) - y(n), \\
w_k(n+1) &= w_k(n) + \mu e(n)x(n-k),
\end{aligned} \tag{1}
$$

for $k = 0, 1, \ldots, N-1$, where $x(n)$ is filter input signal, $w_k(n)$ is the kth filter coefficient, $d(n)$ is the desired response, and μ is the step size.

Due to the characteristics of the proposed multirate adaptive filter, the updating equations in Eq. (1) need to be modified. First, as can be seen from Fig. 1, we can treat the central part of the multirate FIR filter that operates at the frequency of f_s/M as a block-based FIR system. We may then employ the updating scheme in *block LMS (BLMS)* [6] and rewrite Eq. (1) as

$$
w_k(n+M) = w_k(n) + \mu \sum_{m=0}^{M-1} e(n+m)x(n-k-m). \tag{2}
$$

Moreover, in the multirate FIR filtering scheme, the filter weights, w_k for $0 \le k \le M-1$, are decimated and grouped into M sub-filters with tap length equals to $N' \triangleq N/M$ (assume that N is multiple of M.) The ith sub-filter, W_i, is composed of $w_{i,j}(n)$, for $0 \le j \le N'-1$. They can be related to $w_k(n)$ as

$$
w_{i,j}(n) \triangleq w_{i+Mj}(n) \text{ for } 0 \le i \le M-1, \text{ and } 0 \le j \le N'-1,
$$

and the subscripts i, j are used to denote the jth coefficient in the ith decimated sub-filter. Since Eq. (2) is a block-based update operated at an M-times lower sampling rate, it will be convenient to define a new time index l. Single

increment of l corresponds to M increments of the original index n. Besides, we also define the decimated signals as

$$
\begin{aligned}
e_m(l) &\triangleq e(Ml+m) = d(Ml+m) - y(Ml+m), \\
x_i(l) &\triangleq x(Ml+i).
\end{aligned}
$$

By applying above definitions and substituting $n = Ml$ into Eq. (2), we can derive the new weight updating equation for $w_{i,j}(n)$ as

$$
\begin{aligned}
&w_{i+Mj}(Ml+M) \\
&= w_{n+Mj}(Ml) + \mu \sum_{m=0}^{M-1} e(Ml+m)x(Ml-i-Mj+m) \\
&= w_{n+Mj}(Ml) + \mu \sum_{m=0}^{M-1} e_m(l)x_{m-i}(l-j). \tag{3}
\end{aligned}
$$

Furthermore, by using the fact of $x_{m-i}(l) = x_{m-i+M}(l-1)$ for $m-i < 0$, the new updating equation of the proposed multirate adaptive filter can be rewritten as

$$
\begin{aligned}
w_{i,j}(l+1) &= w_{i,j}(l) + \mu \left[\sum_{m=0}^{i-1} e_m(l)x_{m-i+M}(l-j-1) \right. \\
&\quad \left. + \sum_{m=i}^{M-1} e_m(l)x_{m-i}(l-j) \right] \\
&\triangleq w_{i,j}(l) + \mu \nabla_{i,j} \tag{4}
\end{aligned}
$$

for $0 \le i \le M-1$ and $0 \le j \le N'$. $\nabla_{i,j}$ is defined as the estimated gradient of jth weight of the ith sub-filter.

A direct implementation of Eq. (4) is depicted in Fig. 3. It shows a regular realization of the proposed new updating algorithm with example of $M = 3$. By substituting Fig. 3 and Fig. 1(b) into Fig. 2(b), we can have the overall structure (including pre-, post-processing networks, multirate filtering block, and the weight updating block) of the proposed adaptive filter in Fig 4. As can be shown in Fig. 3 and Fig. 4, both weight updating and multirate filtering block can be implemented in a very regular way. Besides, we can also show that the updating equation in (4) can be applied for other choices of M and N.

3. COMPLEXITY ANALYSIS AND COMPARISON

Table 1 lists the required computational complexity of the filtering operation, error calculation, and weight updating among the standard LMS and multirate adaptive filters with $M = 2$ and $M = 3$ [1]. Note that both the MPU and addition operations per unit sample (abbreviated as APU) are about the same in error calculation and weight updating operation for all approaches. The computational complexity saving comes from the multirate filtering operations. The overall computational complexity of the multirate adaptive algorithm is less than the one of conventional LMS. As M

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[1] $M = 2$ and 3 are the most applied configuration in practical implementation. We may also regard the standard LMS as a special case of multirate adaptive filter with $M = 1$.
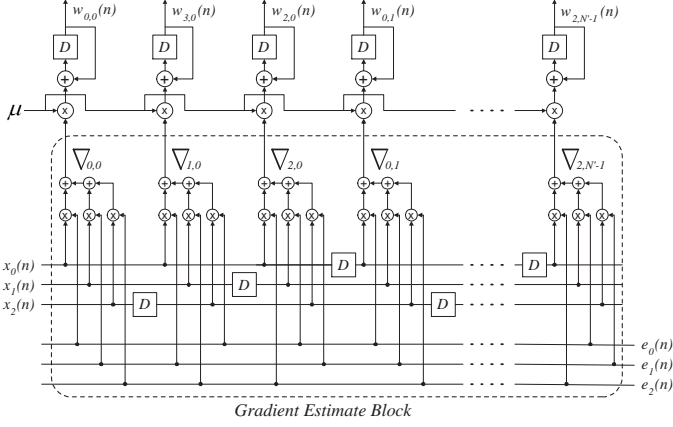
	Standard LMS (M=1)		Multirate approach (M=2)		Multirate approach (M=3)	
	MPU	APU	MPU	APU	MPU	APU
Filtering	N	$N-1$	$0.75\,N$	$N+0.5$	$0.67\,N$	$N+1.33$
Error calculation	---	1	---	1	---	1
Weight updating	N	N	N	N	N	N
Total	$2N$	$2N$	$1.75\,N$	$2N+1.5$	$1.67\,N$	$2N+2.33$
Supply voltage (V'_{dd})	$3V$		$2.04\,V$		$1.70\,V$	
Power consumption (P)	$P_0=C_{eff}V_{dd}f_s$		$0.41\,P_0$		$0.27\,P_0$	

Table 1: The comparison of computational complexity and power for standard LMS and multirate adaptive filters.

Figure 3: Direct implementation of weight updating block (WUB) with $M = 3$.

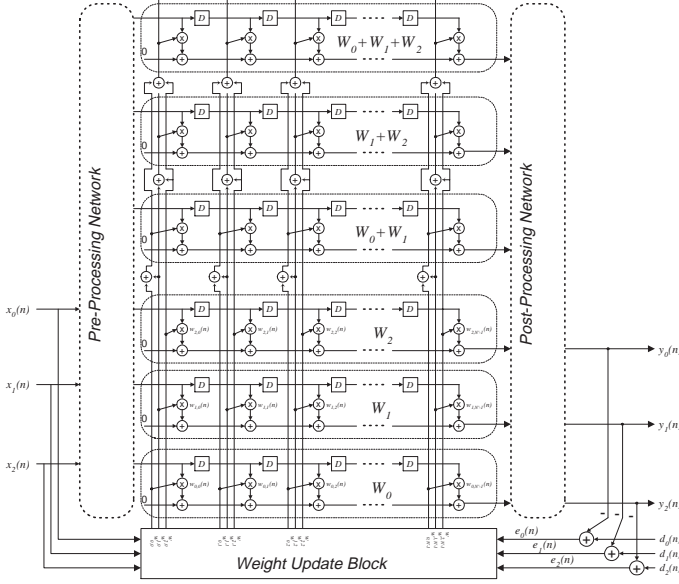Figure 4: The overall VLSI structure of the multirate adaptive filter with decimation factor of $M = 3$.

increases, the saving is more significant. In addition, the proposed approach still retains the MAC operations, which is preferable in programmable DSP implementation.

Moreover, by following the arguments in [5], we know that the multirate system is very suitable for low-power/high-speed applications. It can be shown that the lowest possible supply voltage V'_{dd} for a device running at an M-times slower clock rate can be approximated by

$$\frac{V'_{dd}}{(V'_{dd} - V_t)^2} = M\frac{V_{dd}}{(V_{dd} - V_t)^2}, \qquad (5)$$

where V_t is the threshold voltage of the device. Assume the $V_{dd} = 3V$ and $V_t = 0.7V$ in the original system (standard LMS). Provided that the capacitance due to the multipliers is dominant in the circuit and is roughly proportional to the number of multipliers, we can estimate the power consumption of multirate adaptive filter as

$$P = \left(\frac{MPU_{total} \cdot M}{2N}\right) \left(\frac{V'_{dd}}{V_{dd}}\right)^2 \left(\frac{1}{M}\right) P_0, \qquad (6)$$

where P_0 denotes the estimated power consumption of the standard LMS adaptive filter. The required supply voltage and power consumption for multirate approaches with $M = 2, 3$ are listed in the last two rows of Table 1, where C_{eff} is the effective capacitance of a single multiplier. It shows that the power consumption is greatly reduced compared with the standard LMS, and the saving is more significant as M increases.

4. APPLICATION TO DELAYED-LMS

In the VLSI implementation of Eq. (1), the long feedback path of the error signal imposes a critical limitation on its high-speed implementation. In applications which require high sampling rate or large number of filter taps, the direct implementation may not be applicable. To overcome the aforementioned speed constraint, the *delayed LMS (DLMS)* is usually adopted [7]. It uses a delayed estimation error to update the filter weights, *i.e.*, the weight updating equation in Eq. (1) becomes

$$w_k(n+1) = w_k(n) + \mu e(u-D)x(u-k-D). \qquad (7)$$

The extra D can help to relax speed constraint within the feedback path of $e(n)$. Hence, the transversal filter can be
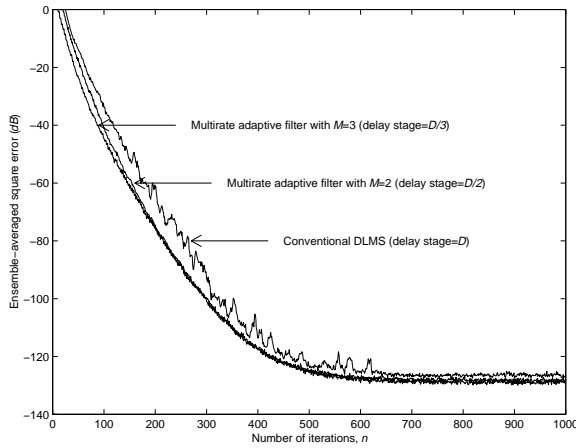
Figure 5: The learning curves of the conventional DLMS and multirate adaptive filters with $M = 2, 3$ (tap length $N = 18$ and eigenvalue-spread $\chi(R) = 6.08.$)
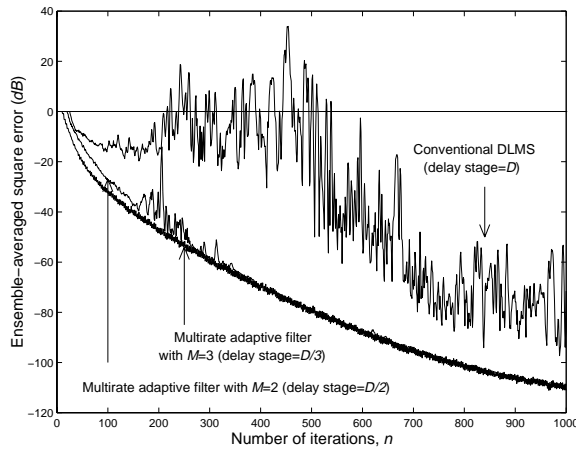

Figure 6: The learning curves of the conventional DLMS and multirate adaptive filters with $M = 2, 3$ (tap length $N = 18$ and eigenvalue-spread $\chi(R) = 21.71.$)

implemented as a D-stage pipelined FIR filter so as to handle the high-sampling input signal. One major disadvantage of the DLMS algorithm is its **slow convergence rate** [7]. That is, the optimum step size decreases as D increases, so does the convergence rate.

In the proposed adaptive filter, the tap length is only $N' = N/M$. As a result, for fully-pipelined designs [8][9], the delay stage is reduced from N of the standard DLMS architecture to N', which leads to improvement in the convergence rate.

To verify our observations, we compare the ensemble-averaged error between the conventional DLMS and the proposed multirate adaptive filter in the application of channel equalization [10, Chap.9]. Figure 5 and 6 show the learning curves for these two approaches in two different channels, where the eigenvalue spread, $\chi(\mathbf{R})$, of the received signal are 6.08 and 21.71, respectively. Based on the results presented in Fig. 5 and Fig. 6, we can make the following observations:

- The conventional DLMS behaves worst in terms of convergence rate and the steady state mean-squared error.

- The multirate adaptive filters with $M = 2$ and $M = 3$ have smoother convergence curves (less fluctuations.) The estimated gradient is averaged over M sample periods. Hence, the gradient estimation is more accurate.

- The multirate approach performs better in both convergence rate and steady state mean-squared error as M increases. It is due to the fact that the delay stage D is smaller than the conventional implementation. The phenomenon becomes more clear in more severe environment (larger eigenvalue spread.)

5. CONCLUSIONS

In this paper, a new adaptive structure based on the multirate filter is proposed. By virtue of the advantages of multirate FIR filtering algorithm, the proposed scheme can reduce the required computational complexity and reserve the MAC structure. It also improves the convergence rate and steady state error in running delayed LMS.

6. REFERENCES

[1] Z. J. Mou and D. Duhamel, "Fast FIR filtering: Algorithm and implementations," *Signal Processing*, vol. 13, pp. 377–384, 1987.

[2] Z. J. Mou and P. Duhamel, "Short-length FIR filters and their use in fast nonrecursive filtering," *IEEE Trans. on Signal Processing*, vol. 39, pp. 1322–1332, June 1991.

[3] A. V. Oppenheim and R. W. Schafer, *Discrete-time Signal Processing*. Prentice Hall, 1989.

[4] D. A. Parker and K. K. Parhi, "Low-area/power parallel FIR digital filter implementations," *Signal Processing*, no. 17, pp. 75–92, 1997.

[5] K. J. R. Liu, A.-Y. Wu, A. Raghupathy, and J. Chen, "Algorithm-based low-power and high-performance multimedia signal processing," *Proceedings of the IEEE, Special Issue on* Multimedia Signal Processing, vol. 86, pp. 1155–1202, June 1998.

[6] J. J. Shynk, "Frequency-domain and multirate adaptive filtering," *IEEE Signal Processing Magazine*, pp. 14–37, Jan. 1992.

[7] G. Long, F. Ling, and J. Proakis, "The LMS algorithm with delayed coefficient adaptation," *IEEE Trans. Acoust. Speech, Signal Processing*, vol. 37, pp. 1397–1405, Sep. 1989.

[8] H. Herzberg, R. Haimi-Cohen, and Y. Be'ery, "A systolic array realization of an LMS adaptive filter and the effects of delayed adaption," *IEEE Trans. Signal Processing*, vol. 40, pp. 2799–2803, Nov. 1992.

[9] M. D. Meyer and D. P. Agrawal, "A high sampling rate delayed LMS fitler architecture," *IEEE Trans. Circuits Syst. II*, vol. 40, pp. 727–729, Nov. 1993.

[10] S. Haykin, *Adaptive Filter Theory*. Prentice-Hall, Englewood Cliffs, N.J., 2nd ed., 1991.