

ESTIMATION OF AIRCRAFT ALTITUDE AND ALTITUDE RATE WITH OVER-THE-HORIZON RADAR

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ABSTRACT

In previous work, a matched-field estimate of aircraft altitude which uses multiple over-the-horizon radar dwells was presented. This approach exploited the altitude dependent structure of the micro-multipath rays which result from reflections local to the aircraft. While it was shown that the multi-dwell matched-field estimate is able to accurately estimate altitude while using typical radar parameters, the estimate was derived assuming that the aircraft altitude is constant for the duration of the track. In this paper, a matched-field method for jointly estimating altitude and altitude rate is presented which extends the micro-multipath model to include effects of constant altitude rate on the micro-multipath Doppler frequencies. Simulation results illustrate that altitude and altitude rate can be jointly estimated while achieving an altitude estimation accuracy of ± 2500 feet using 10 radar dwells.

1. INTRODUCTION

Over-the-horizon radar (OTH) is currently used in remote sensing and wide area surveillance applications where the deployment of a conventional line-of-sight radar is impractical [1, 2]. A more recent use of OTH radar is the detection and tracking of small private aircraft for counter drug efforts [3] where track information is used to classify an aircraft as a possible target of interest. Altitude estimation is an important tool in the counter drug effort as a criterion for aircraft classification and, if interdiction is required, an accurate estimate of altitude simplifies the task of intercepting the aircraft. Furthermore, the detection of aircraft ascent or descent can aid in determining the locations of possible origination and destination airfields.

In this paper, the joint estimation of altitude and altitude rate is solved using a matched field approach where the observed target return in complex range-Doppler space is modeled as a function of aircraft altitude and altitude rate. Matched-field processing (MFP) is a well known estimation method in the underwater source localization problem [4], but is relatively new in radar applications. MFP has been

proposed for altitude estimation [5, 6] where it has been assumed that the target is flying at a constant altitude. It has been shown that by using multiple radar dwells, altitude estimation can be achieved using typical radar parameters [6]. If the altitude rate is known *a priori*, the multi-dwell MFP altitude estimation approach is able to estimate altitude to within ± 2500 feet using as few as 5 radar dwells. In this paper, the multi-dwell matched-field approach is extended to estimate altitude rate as well as aircraft altitude by incorporating the effects of altitude rate into the model of the observed range-Doppler signal.

The signal model includes multipath propagation which results from reflections of the radar rays off of the ground near the aircraft. Although these micro-multipath returns are in general not resolvable in range-Doppler space, the slight micro-multipath ray differences in slant range and Doppler frequency result in dwell to dwell shape variations of the target return in complex range-Doppler space. The observed variations in the complex range-Doppler return are compared to those predicted by the model in order to estimate altitude and altitude rate. The comparison is accomplished by evaluating the likelihood function of the observation conditioned on the hypothesized altitude and altitude rate. To handle slowly changing amplitude variations due to ionospheric effects and which are not dependent on aircraft altitude or altitude rate, the complex coefficients of the micro-multipath reflections are treated as a first order Markov model across radar revisits.

2. MICRO-MULTIPATH MODEL INCLUDING ALTITUDE RATE

In [6], the model for the complex range-Doppler surface including micro-multipath propagation was derived for an aircraft traveling along a horizontal flight path. In this section, the micro-multipath model is extended to include the case of an ascending or descending aircraft.

Due to ray reflections off of the ground local to the aircraft, there are actually two possible ray paths from the transmitter to the aircraft and two paths from the aircraft to the receiver, illustrated in Figure 1. The time delays for

the four ray paths are $\tau_{t,d}$ for the direct transmit ray, $\tau_{t,r}$ for the ground reflected transmit ray, $\tau_{r,d}$ for the direct receive ray, and $\tau_{r,r}$ for the ground reflected receive ray. Likewise, the elevation angles of these rays are denoted by $\beta_{t,d}$, $\beta_{t,r}$, $\beta_{r,d}$, and $\beta_{r,r}$. The target velocity which is projected onto the four rays is written as

$$v_{t,d} = v_r \cos \beta_{t,d} + v_z \sin \beta_{t,d} \quad (1)$$

$$v_{t,r} = v_r \cos \beta_{t,r} - v_z \sin \beta_{t,r} \quad (2)$$

$$v_{r,d} = v_r \cos \beta_{r,d} + v_z \sin \beta_{r,d} \quad (3)$$

and

$$v_{r,r} = v_r \cos \beta_{r,r} - v_z \sin \beta_{r,r} \quad (4)$$

where v_r and v_z are the horizontal and vertical components of the aircraft velocity vector, respectively. The negative signs in (2) and (4) are due to the ground reflected rays illuminating the target from below.

There are a total of four possible combinations of transmit and receive rays with slant ranges g_l and Doppler frequencies ω_l , for $l = 1, \dots, 4$, which contribute to the radar return from an aircraft target. For example, letting $l = 1$ denote the direct transmit and direct receive ray combination, $g_1 = c_0 (\tau_{t,d} + \tau_{r,d}) / 2$ and $\omega_1 = 2\pi f (v_{t,d} + v_{r,d}) / c_0$ where f is the radar operating frequency and c_0 is the speed of light.

At zero altitude, the four micro-multipath ray combinations are identical and are referred to as the “baseline rays.” As altitude increases, the slant ranges and Doppler frequencies of the micro-multipath rays become more spread about the baseline values. Furthermore, the Doppler frequencies are also a function of the altitude rate, as indicated in equations (1) to (4). This dependence is shown in Figures 2 and 3, which illustrate the Doppler frequencies of the micro-multipath rays as a function of aircraft altitude. The four lines in these figures correspond to the four different possible combinations of transmit and receive rays: direct transmit, direct receive (D-D), direct transmit, reflected receive (D-R), reflected transmit, direct receive (R-D), and reflected transmit, reflected receive (R-R). For the examples in Figure 2 and 3, the aircraft radial velocity is -190 m/s and the ground range is 1200 km.

Notice that in Figure 2 and 3, there is a qualitative similarity between the micro-multipath Doppler frequencies between an ascending aircraft at an altitude of 40000 feet (Figure 2) and a descending aircraft at 10000 feet (Figure 3). Although the Doppler frequencies are not exactly the same for these two cases, the similarity will result in a near ambiguity in the altitude-altitude rate log-likelihood surface. In general, a similarity in the micro-multipath Doppler frequencies exists between higher altitude aircraft with greater altitude rates and lower altitude aircraft with lower altitude rates.

3. JOINT MATCHED FIELD ESTIMATION OF ALTITUDE AND ALTITUDE RATE

In this section, the maximum likelihood estimate of altitude and altitude rate is derived. If the altitude rate is constant, $\dot{z} = dz/dt = C$, then the altitude sequence $Z_k = \{z_0, z_1, \dots, z_K\}$ can be entirely parameterized by the initial altitude and the altitude rate, $\{z_0, \dot{z}\}$, by the relation $z_k = \dot{z}t_k + z_0$ where t_k is the time of the k^{th} radar revisit.

For the matched-field estimate developed here, the target return in complex range-Doppler space must be modeled as a function of altitude and altitude rate. Since OTH radar is successfully used to estimate aircraft ground range, velocity, and azimuth [7], these target parameters are assumed to be known *a priori*. Furthermore, measurements of the ionosphere are typically readily available, so the parameters of a deterministic propagation model are also known.

The signal model is developed for an $N \times M$ block of the complex range-Doppler surface centered around the slant range and Doppler frequency for the target return of interest. By writing the signal model as an $NM \times 1$ vector, the component of the target return along the l^{th} micro-multipath ray is $c_{l,k} \mathbf{h}_{l,k} e^{j\omega_{l,k} t_k}$ where $\mathbf{h}_{l,k}$ is the complex transfer function of the radar in range-Doppler space, θ_k is an unknown phase term that rapidly varies from dwell to dwell, and $c_{l,k}$ is a random complex reflection coefficient due to the unknown reflections off of the aircraft and ground which is assumed to vary slowly over multiple dwells. Note that both $\mathbf{h}_{l,k}$ and $\omega_{l,k}$ are functions of altitude and altitude rate. The return due to a target consists of a sum of the individual multipath returns and noise,

$$\mathbf{x}_k = e^{j\theta_k} \mathbf{H}_k(z_0, \dot{z}) \mathbf{D}_k(z_0, \dot{z}) \mathbf{c}_k + \mathbf{n}_k \quad (5)$$

where the l^{th} column of $\mathbf{H}_k(z_0, \dot{z})$ is $\mathbf{h}_{l,k}$, $\mathbf{D}_k(z_0, \dot{z})$ is a diagonal matrix of the Doppler phase terms, and \mathbf{c}_k is a vector of the unknown complex reflection coefficients which is assumed to be zero mean Gaussian distributed with covariance $E[\mathbf{c}_k \mathbf{c}_k^H] = \sigma_a^2 \mathbf{\Lambda}_c$. The term \mathbf{n}_k represents additive noise which is assumed to be zero mean Gaussian distributed with covariance $\sigma_n^2 \mathbf{I}$.

The maximum likelihood estimate of altitude and altitude rate is made by maximizing the joint distribution of the sequence of $K + 1$ data snapshots conditioned on altitude, altitude rate, and the $K + 1$ unknown phase terms $\theta_0, \dots, \theta_K$. By treating the complex ray coefficients as a first order Markov process over revisits, the coefficients for consecutive dwells are correlated with $E[\mathbf{c}_k \mathbf{c}_{k-\ell}^H] = \sigma_a^2 \mathbf{\Lambda}_c$ for $|\ell| \leq 1$ and $E[\mathbf{c}_k \mathbf{c}_{k-\ell}^H] = \mathbf{0}$ for $|\ell| > 1$. From the first order Markov model assumption, the conditional distribution can be written as a product of first order conditional probabilities which are each a function of the unknown phase differences $\Delta\theta_k = \theta_k - \theta_{k-1}$. The maximum

likelihood estimate can then be written as

$$\begin{aligned} \{\hat{z}_0, \hat{\dot{z}}\}_{ML} = \arg \max_{z_0, \dot{z}} & \left[\log p(\mathbf{x}_0 | z_0, \dot{z}) \right. \\ & \left. + \sum_{k=1}^K \max_{\Delta \theta_k} \log p(\mathbf{x}_k | \mathbf{x}_{k-1}, z_0, \dot{z}, \Delta \theta_k) \right] \end{aligned} \quad (6)$$

where each term in the summation over k can be independently maximized over $\Delta \theta_k$. This maximization over $\Delta \theta_k$ has a closed form solution which leads to the joint estimate $\{\hat{z}_0, \hat{\dot{z}}\}_{ML} = \arg \max_{z_0, \dot{z}} \bar{L}_K(z_0, \dot{z})$ where $\bar{L}_K(z_0, \dot{z})$ is

$$\bar{L}_K(z_0, \dot{z}) = -\log \pi^{MN} |\mathbf{R}_{0,0}| - \mathbf{x}_0^H \mathbf{R}_{0,0}^{-1} \mathbf{x}_0 + \sum_{k=1}^K L_k(z_0, \dot{z}) \quad (7)$$

and

$$\begin{aligned} L_k(z_0, \dot{z}) = & -\log \pi^{MN} |\mathbf{Q}_k| \\ & - \mathbf{x}_k^H \mathbf{Q}_k^{-1} \mathbf{x}_k + 2|\mathbf{x}_k^H \mathbf{Q}_k^{-1} \mathbf{R}_{k,k-1} \mathbf{R}_{k-1,k-1}^{-1} \mathbf{x}_{k-1}| \\ & - \mathbf{x}_{k-1}^H \mathbf{R}_{k-1,k-1}^{-1} \mathbf{R}_{k,k-1}^H \mathbf{Q}_k^{-1} \mathbf{R}_{k,k-1} \mathbf{R}_{k-1,k-1}^{-1} \mathbf{x}_{k-1}. \end{aligned} \quad (8)$$

The log-likelihood function in (7) can be easily updated each time a new dwell becomes available by $\bar{L}_{K+1}(z_0, \dot{z}) = \bar{L}_K(z_0, \dot{z}) + L_{K+1}(z_0, \dot{z})$.

4. SIMULATION RESULTS

In this section, the matched-field estimate of altitude and altitude rate presented in the previous section is evaluated by simulation. The simulation scenario models a level flying aircraft at 5000 feet with a radial velocity of -190 m/s and a ground range of roughly 1200 km. The ionosphere is modeled with a triple layer quasi-parabolic electron density profile [8, 9] with parameters derived from real ionospheric measurements. The signal consists of the samples in the complex range-Doppler surface contained in a 7 slant range bins by 1 Doppler frequency bin neighborhood around the true slant range and Doppler frequency. The radar operating frequency is 10 MHz, the bandwidth is 16.7 kHz, and the CIT is nominally 2.5 seconds.

The log-likelihood surface vs. altitude and altitude rate at revisit $k = 10$ for an SNR of 30 dB is shown in Figure 5. The matched-field estimate of altitude for this example is 5100 ft and the estimate of altitude rate is 0.55 ft/s, which is marked by an 'x'. The true altitude and altitude rate pair of 5000 ft and 0 ft/s is indicated by an 'o'. Notice that in Figure 5, there is a near ambiguity along a line in the altitude-altitude rate log-likelihood function. This results from similarities in the micro-multipath Doppler frequencies for different combinations of altitude and altitude rate, as illustrated in Figures 2 and 3. It should be noted, however, that the uncertainty is not completely ambiguous since the Doppler frequencies are not identical. Therefore,

increased SNR or observation time can be used to mitigate this near ambiguity.

Figure 5 illustrates the distribution of the matched-field estimate of altitude and altitude rate at revisit $k = 10$ (5 minutes). The distribution is estimated over 100 random simulation trials. From Figure 5, it is clear that the altitude estimate is mostly within ± 2500 feet of the true aircraft altitude, and the estimated altitude rate is within ± 1 ft/s of the true altitude rate. The effect of estimating altitude rate in addition to altitude is to increase the number of radar dwells required for the same altitude estimation accuracy compared to the case where the altitude rate is known *a priori*.

5. CONCLUSIONS

In this paper, a joint matched-field estimate of target altitude and altitude rate using OTH radar was presented. The joint estimate was developed by extending the model for micro-multipath ray propagation to include the effects of altitude rate on Doppler frequency. Through simulation it was shown that, for typical radar operating parameters, altitude estimation accuracy of ± 2500 feet and altitude rate estimation accuracy of ± 1 ft/s can be achieved with as few as 10 radar revisits. Although similarities in the micro-multipath ray model exist for different combinations of altitude and altitude rate, the near ambiguity which results from these similarities can be mitigated with larger SNR or observation time.

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6. REFERENCES

- [1] D. E. Barrick, J. M. Headrick, R. W. Bogle, and D. D. Crombie, "Sea backscatter at HF: Interpretation and utilization of the echo," *Proceedings of the IEEE*, vol. 62, pp. 673–680, June 1974.
- [2] D. B. Trizna, J. C. Moore, J. M. Headrick, and R. W. Bogle, "Directional sea spectrum determination using HF Doppler radar techniques," *IEEE Transactions on Antennas and Propagation*, vol. AP-25, pp. 4–11, January 1977.
- [3] E. Ferraro and D. Ganter, "Cold war to counter drug [ROTHR radar system]," *Microwave Journal, Euro-Global Edition*, vol. 41, pp. 82–92, March 1998.
- [4] A. B. Baggeroer, W. A. Kipperman, and H. Schmidt, "Matched-field processing: Source localization in correlated noise as an optimum parameter estimation problem," *Journal of the Acoustical Society of America*, vol. 83, pp. 571–587, February 1988.

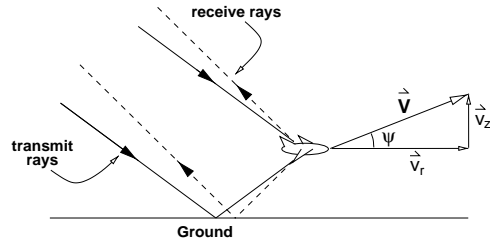


Figure 1: Micro-multipath raypaths due to reflections local to the aircraft target for a bistatic radar.

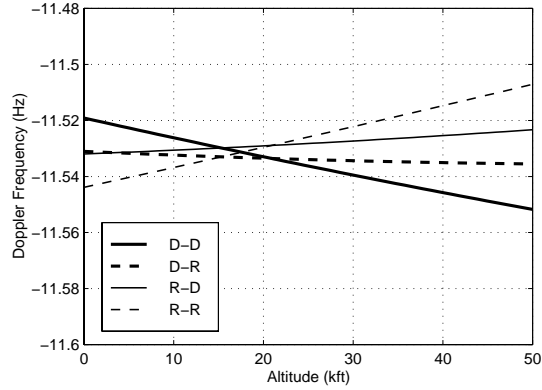


Figure 2: Doppler frequencies of the micro-multipath rays as a function of altitude for an ascending target with an altitude rate of +2.2 ft/s.

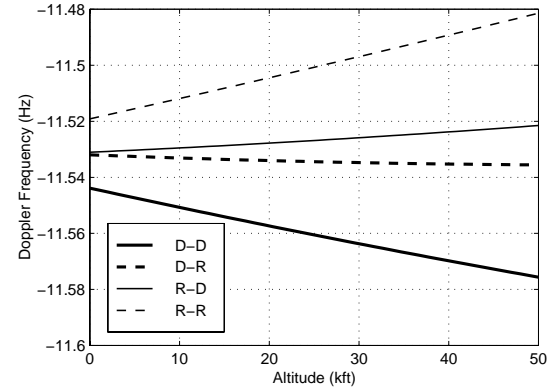


Figure 3: Doppler frequencies of the micro-multipath rays as a function of altitude for a descending target with an altitude rate of -2.2 ft/s.

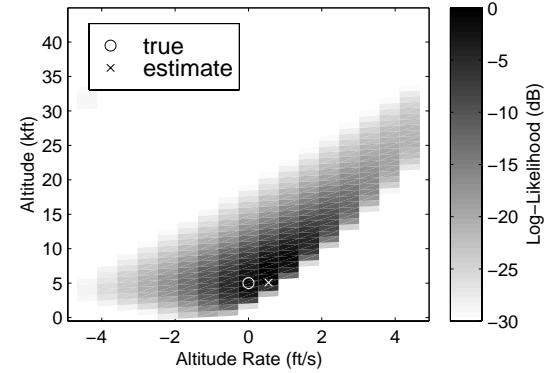


Figure 4: Simulation log likelihood vs. altitude and altitude rate at revisit 10 (5 minutes) for level flying aircraft with an initial altitude of 5000 ft.

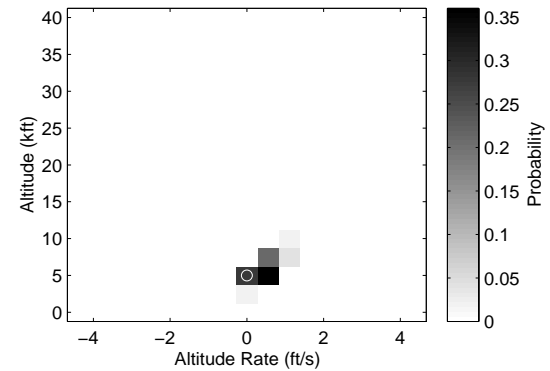


Figure 5: Probability density of the matched-field estimate of altitude and altitude rate at revisit 10 (5 minutes) estimated over 100 random simulation trials.

- [5] M. Papazoglou and J. L. Krolik, "Electromagnetic matched-field processing for over-the-horizon radar in an uncertain environment," *Submitted to IEEE Transactions on Antennas and Propagation*, May 1997.
- [6] M. Papazoglou and J. L. Krolik, "Multi-dwell matched-field altitude estimation for over-the-horizon radar," *Submitted to IEEE Transactions on Signal Processing*, June 1998.
- [7] L. F. McNamara, *The Ionosphere: Communications, Surveillance, and Direction Finding*. Orbit, a foundation series, Malabar, FL: Krieger Publishing Co., 1991.
- [8] T. A. Croft and H. Hoogasian, "Exact ray calculations in a quasi-parabolic ionosphere with no magnetic field," *Radio Science*, vol. 3, pp. 69–74, January 1968.
- [9] P. L. Dyson and J. A. Bennett, "A model of the vertical distribution of the electron concentration in the ionosphere and its application to oblique propagation studies.," *Journal of Atmospheric and Terrestrial Physics*, vol. 50, no. 3, pp. 251–262, 1988.