EFFICIENT METHOD FOR CARRIER OFFSET CORRECTION IN OFDM SYSTEM

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ABSTRACT

In this work, we present a simple approach to estimate and correct the carrier offset in an orthogonal frequency division multiplexing (OFDM) system. The approach leads to a computationally and statistically efficient estimator for the carrier offset. Computer simulations verify that the estimation accuracy is comparable to the Cramer-Rao bound (CRB). We demonstrated that by incorporating the estimated carrier offset (obtained using reasonable frames of OFDM data) in the demodulation process, the bit-error-rate (BER) can approach that of the ideal OFDM system with no carrier offset.

1. INTRODUCTION

OFDM is a bandwidth efficient signaling scheme for digital communications [1]. Renewed interests in the OFDM systems are mainly due to its potential applications in mobile wireless communications and wide-band multi-carrier code-division multiple-access (MC-CDMA) systems. In an OFDM system, a stream of information bits is first segmented into blocks of finite length, say P. Each block of data is then *simultaneously* transmitted over P orthogonal frequency channels, which are specified by different sub-carriers. In doing so, the high-rate data can be transmitted through *multiple* low-rate narrowband channels. Therefore, the symbol interval for streams of data out of each narrowband sub-channels is enlarged, making the transmission less sensitive to dispersive channel. What makes the OFDM different from the traditional FDM system is that in the OFDM systems, all the sub-channels are actually overlapping in spectrum but are still *orthogonal.* This allows us to create a large number of orthogonal FDM channels without difficulty of building non-overlapping sub-band filters with brick wall frequency response. In addition, each OFDM sub-channel can be treated as narrowband channel, hence reducing the overall effect of frequency-selective fading on the data transmission, especially for future MC-CDMA systems. It has been noticed that the IFFT/FFT processor provides an efficient implementation for OFDM modulation/demodulation [2].

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However, when the transmission channel introduces an offset on the major carrier of the RF OFDM signals. This carrier frequency offset will be passed along to each sub-carriers after the first stage of RF demodulation. When the pre-defined group of sub-carriers used in the transmitter are used in the receiver to demultiplex information bits out of the OFDM signals, the inter-carrier-interference (ICI) occurs. This ICI will cause serious performance degradation of the OFDM system [1⇔6]. It has been shown in [3] that to maintain signal-to-interference ratio (SIR) of 20dB or greater in an OFDM system, the carrier offset should be limited to less than 4% of the intercarrier spacing. A carrier offset correction algorithm based on transmitting repeated symbols was proposed in [3]. Unless an initial carrier offset acquisition strategy is used, the carrier offset estimation algorithm in [3] has a limit working range: the offset should be less that 1/2 of the carrier spacing. In this work, we present a new and computationally efficient algorithm for carrier offset correction. It combines the conditional maximum likelihood approach and the low-rank property of the narrowband sub-channels. The algorithm also takes advantage of available multiple frames of received data to refine the carrier offset estimation. Using the proposed estimation algorithm, we are able to decode the information bits with minimum BER.

2. PROBLEM FORMULATION

Let us denote the kth frame of baseband digital OFDM signal, represented in vector space, as,

$$\mathbf{s}(k) = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \cdots & \mathbf{w}_P \end{bmatrix} \begin{bmatrix} b_1(k) \\ b_2(k) \\ \vdots \\ b_P(k) \end{bmatrix} = \mathbf{W}_P\, \mathbf{b}(k)\,,$$

where $\mathbf{b}(k)$ is a vector of information bits (in our simulation, we used BPSK bits, but the algorithm for estimating carrier offset also allows $\mathbf{b}(k)$ to be other complex-valued information vector). Columns of $\mathbf{W}_P$ are simply length N IDFT vectors, i. e.
$\mathbf{w}_i = \left[\begin{array}{cccc} 1 & e^{j\,2\,\pi\,\frac{i-1}{N}} & \ldots & e^{j\,2\,\pi\,\frac{i-1}{N}(N-1)} \end{array} \right]^T$.
In the above simplified notation, we assume that the symbol interval of the original data stream prior to OFDM is normalized to unity, so that the spacing between adjacent sub-carriers is $1/N$. In a more general notation, one can replace the normalized version of inter-carrier spacing by $1/(N\,T_b)$, where T_b is the bit interval of the original data stream. Note that columns of $\mathbf{W}_P$ are orthogonal, i. e., $\mathbf{w}_m^H \mathbf{w}_n$=N $\delta_{m,n}$. Under ideal Gaussian channel, a bank of matched filters $\mathbf{W}_P^H$ followed by a decision maker will enable us to optimally recover information bits. In practice, only a sub-set of N orthogonal FDM channels are used for information transmission, hence $P < N$ holds. The difference $N \Leftrightarrow P$ also allows us to add prefix of length Ng to each frame during the data transmission for combating any inter-symbol-interference (ISI) due to the channel.

When channel introduces a carrier offset f_d, the received baseband OFDM signal (the kth frame) proir to demultiplexing using $\mathbf{W}_P^H$ can be modelled as,

$$\mathbf{r}(k) = \underbrace{\mathbf{E}(f_d)\,\mathbf{W}_P}_{\mathbf{W_f}}\;\underbrace{\mathbf{H}\,\mathbf{b}(k)}_{\boldsymbol{\eta}(k)} + \mathbf{n}(k)\,, \qquad (1)$$

where $\mathbf{E}(f_d) = diag\left\{1,\, e^{j\,2\,\pi\,f_d},\, \cdots,\, e^{j\,2\,\pi\,f_d\,(N-1)}\right\}$ is a diagonal matrix attributed to the carrier offset, which can be caused either by the Doppler effect or by the linear phase response of the channel. The diagonal matrix $\mathbf{H} = diag\{\,H(1),\, H(2),\, \cdots,\, H(P)\,\}$ reflects the amplitude response of the channel. $\mathbf{n}(k)$ is a vector of complex white Gaussian noise, $\mathbf{n}(k) \sim \mathcal{CN}(\mathbf{0},\, 2\,\sigma^2\mathbf{I})$.

It is reasonable to assume that the diagonal matrix $\mathbf{H}$ is positive definite, hence, information vector $\mathbf{b}(k)$ can be decoded from the vector $\boldsymbol{\eta}(k)$. In this work, we assume that the channel response associated matrices $\mathbf{H}$ and $\mathbf{W_f}$ are all unknown but deterministic. Given a transmitted information vector $\mathbf{b}(k)$, we can write the conditional probability density function (pdf) of data in (1) as,

$$F\,(\,\mathbf{r}(k)\,|\,\mathbf{b}(k)\,) = \frac{1}{(2\,\pi\,\sigma^2)^N}\exp\left\{\Leftrightarrow \frac{\|\,\mathbf{r}(k) \Leftrightarrow \mathbf{W_f}\,\boldsymbol{\eta}(k)\,\|^2}{2\,\sigma^2}\right\} \qquad (2)$$

We adopt a conditional maximum likelihood approach to estimate the carrier offset f_d, and then use the estimated $\mathbf{W_f}$ to decode the information vector $\mathbf{b}(k)$. In doing so, we treat $\boldsymbol{\eta}(k)$ in equation (1) as a linearly-entered deterministic unknown parameter vector.

3. PROPOSED ESTIMATION ALGORITHM

In proposing our carrier offset correction algorithm, we notice that the carrier offset f_d is contained in the $N \times P$ matrix $\mathbf{W_f} = \left[\begin{array}{cccc} \mathbf{e}_1 & \mathbf{e}_2 & \cdots & \mathbf{e}_P \end{array}\right]$, with $\mathbf{e}_i = \left[\begin{array}{cccc} 1 & e^{j\,2\,\pi\,(\frac{i-1}{N}+f_d)} & \ldots & e^{j\,2\,\pi(\frac{i-1}{N}+f_d)\,(N-1)} \end{array}\right]^T$.
Finding the conditional maximum likelihood estimates of the carrier offset f_d and the information vector $\mathbf{b}(k)$ is equivalent to the optimization problem of minimizing the exponential term in (2) over the parameter space,

$$\arg\max_{f_d,\boldsymbol{\eta}(k)} F\,(\mathbf{r}(k)\,|\,\mathbf{b}(k)) \Leftrightarrow \arg\min_{f_d,\boldsymbol{\eta}(k)} \left\|\mathbf{r}(k) \Leftrightarrow \mathbf{W_f}\,\boldsymbol{\eta}(k)\right\|^2$$

where the non-linearly entered parameter f_d is contained in the matrix $\mathbf{W_f}$ and $\boldsymbol{\eta}(k)$ is a linearly entered parameter vector.

This non-linear separable estimation problem can be solved in two stages [7] by first finding the linear parameter $\boldsymbol{\eta}(k)$ assuming a fixed f_d and then plugging the LS solution of $\boldsymbol{\eta}(k)$ back into the cost function to get the non-linear parameter f_d as follows,

$$\hat{f}_d = \arg\max_{f_d}\left\{\left\|\mathbf{P}_{\mathbf{W_f}}\,\mathbf{r}(k)\right\|^2\right\}\,, \qquad (3)$$

$$\hat{\mathbf{b}}(k) = \mathrm{sgn}\left\{\,\hat{\mathbf{W}}_{\mathbf{f}}^{\,H}\,\mathbf{r}(k)\,\right\}\,, \qquad (4)$$

where $\mathbf{P}_{\mathbf{W_f}} = \mathbf{W_f}\,\mathbf{W_f}^H$ is simply the orthogonal projection matrix (ignoring a scale factor 1/N) associate with the subspace $< \mathbf{W_f} >$.

We notice that each columns of $\mathbf{W_f}$ are parameterized by the only non-linear parameter f_d. The presence of non-zero f_d in the matrix $\mathbf{W_f}$ does not change the orthogonality among its own columns, i.e., $\mathbf{W_f}^H\,\mathbf{W_f} = N\,\mathbf{I}_{p\times p}$, even though it does destroy the orthogonality between $\mathbf{W}_P$ and $\mathbf{W_f}$. Therefore, we have the decomposed form for projection matrix $\mathbf{P}_{\mathbf{W_f}} = \sum_{i=1}^{P}\mathbf{P}_{\mathbf{e}_i}$. The above observations lead us to propose a computationally efficient estimation scheme for f_d. The proposed scheme uses the following meaningful formula to replace the one in equation (3).

$$\left\|\mathbf{P}_{\mathbf{W_f}}\,\mathbf{r}(k)\right\|^2 = \mathbf{r}^H(k)\,\mathbf{P}_{\mathbf{W_f}}\,\mathbf{r}(k) = \sum_{i=1}^{P}\mathbf{r}^H(k)\,\mathbf{P}_{\mathbf{e}_i}\,\mathbf{r}(k) = \frac{1}{N}\sum_{i=1}^{P}|\,\mathbf{e}_i^H\,\mathbf{r}(k)\,|^2\,, \qquad (5)$$

where $\frac{1}{N}\,|\,\mathbf{e}_i^H\,\mathbf{r}(k)\,|^2 = \frac{1}{N}\left|\sum_{n=0}^{N-1} r(n)e^{-j\,2\,\pi\,(\frac{i-1}{N}+f_d)\,n}\right|^2$

$= \frac{1}{N} \left| \sum_{n=0}^{N-1} r_i(n) e^{-j\,2\,\pi\, f_d\, n} \right|^2 = P_{\mathbf{r}_i}(f_d)$, which is simply the periodogram of the demodulated version of the OFDM data $\mathbf{r}(k)$ using the ith sub-carrier.

Therefore, the carrier offset estimator $\hat{f}_d$ in equation (3) can be simplified as the following one,

$$\hat{f}_d = \arg\max_{f_d} \sum_{i=1}^{P} P_{\mathbf{r}_i}(f_d)\,, \qquad (6)$$

where the periodogram of $r_i(n) = r(n)\, e^{-j\,2\,\pi\, \frac{i-1}{N}\, n}$ can be easily calculated using the computationally efficient FFT algorithm. That is, $P_{\mathbf{r}_i}(f) = \frac{1}{N} \mid R_i(f) \mid^2$, where $R_i(f)$ is the Fourier transform of $\mathbf{r}_i$. In order to get the estimate of f_d upto the accuracy of its Cramer-Rao bound (CRB), one need to use zero-padding in getting the Fourier transform of the vector $\mathbf{r}_i$ using the FFT.

Note that in communications applications, we always have more than just one block of the OFDM data $\mathbf{r}(k)$ available. Suppose we have K blocks of baseband OFDM data $\{\,\mathbf{r}(k)\,\}_{k=1}^{K}$ available. We then can improve our estimate of f_d by taking advantage of multiple *independent* data vectors. The reason for doing so is that the coditional pdf for the K blocks of data $\{\,\mathbf{r}(k)\,\}_{k=1}^{K}$ equals the product of each individual conditional pdfs, i. e.

$$F(\mathbf{r}(1), \mathbf{r}(2), \cdots, \mathbf{r}(K) \mid \mathbf{b}(1), \mathbf{b}(2), \cdots, \mathbf{b}(K)\,) = \Pi_{k=1}^{K} F(\,\mathbf{r}(k) | \mathbf{b}(k)\,).$$

Therefore, the conditional maximum likelihood approach suggests us to sum up all K individual cost functions in estimating the carrier frequency f_d. One can then obtain a better estimate of f_d as follows when more than just one block of OFDM data is available,

$$\begin{aligned} \hat{f}_d &= \arg\max_{f_d} \left\{ \sum_{k=1}^{K} \left\| \mathbf{P}_{\mathbf{W_f}}\, \mathbf{r}(k) \right\|^2 \right\} \\ &= \arg\max_{f_d} \left\{ \sum_{k=1}^{K} \sum_{i=1}^{P} \mid \mathbf{e}_i^H\, \mathbf{r}(k) \mid^2 \right\} \qquad (7) \\ &= \arg\max_{f_d} \left\{ \sum_{k=1}^{K} \sum_{i=1}^{P} P_{\mathbf{r}_{i(k)}}(f_d) \right\}. \end{aligned}$$

In the simulation examples, we demonstrated the effect of using the refined estimate of the carrier offset f_d of (7) on improving the estimation accuracy and the BER of the OFDM systems.

4. SIMULATION EXAMPLES

We provide simulation results of using the proposed carrier offset correction scheme prior to OFDM data demultiplexing at the receiver. In figure 1, we simulated the effect of carrier offset on the BER performance of a baseband OFDM system with N=32, and P=20. The carrier offset f_d changes from zero to the sub-carrier spacing $0.6/N$ with a increment of $0.1/N$. One can see from the figure 1 that even a very small amount of f_d can cause severe degradation in system performance.

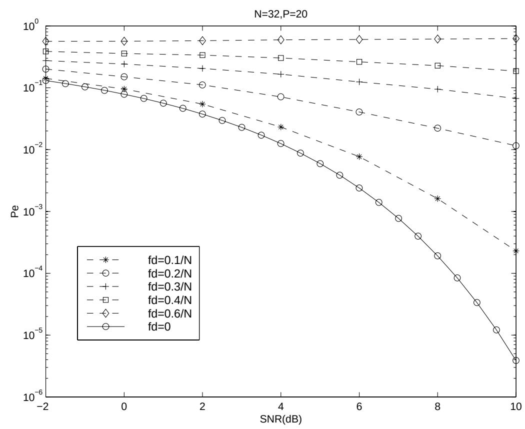


Figure 1: Effect of the carrier offset on the BER performance of a FDMA system.

In figure 2, we shown the estimation accuracy of the proposed carrier offset estimator in comparison with the Cramer-Rao bound (CRB). In this simulation, we used a OFDM system with $N = 32$, $P = 20$, $f_d = 1/N$. In this figure, we plotted the mean squared error (MSE) of $\hat{f}_d$ versus SNR for various values of the number of data blocks, K, used in estimating f_d using (7). In getting the $MSE(\hat{f}_d)$, we used $K_B = 200$ independent trials during the simulation. From the simulation results, one can see that the proposed estimator provides a very good estimation accuracy above the SNR threshold of 0dB when only 50 blocks of OFDM data is used.

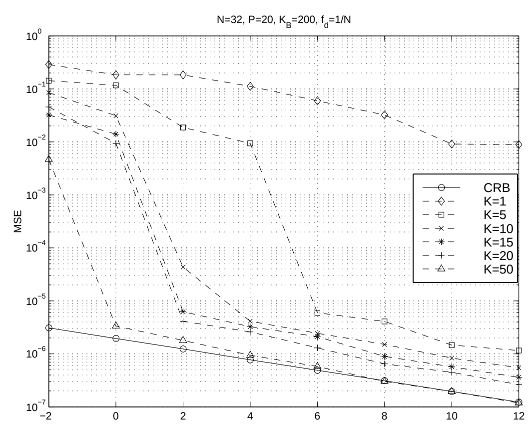
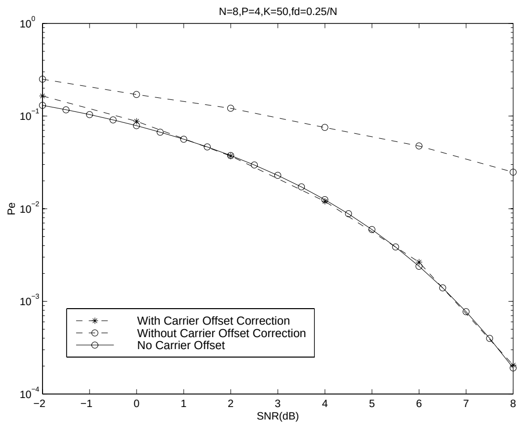
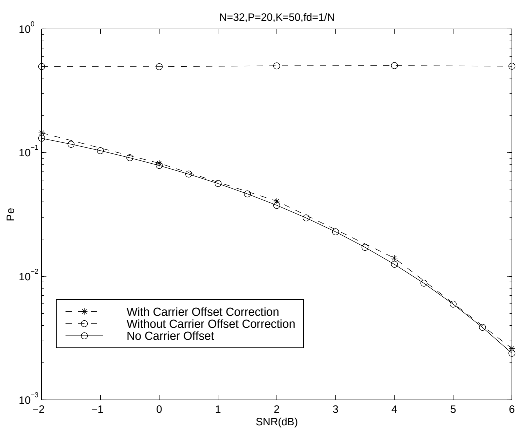


Figure 2: Estimation accuracy of the proposed carrier offset estimator in comparison with the CRB.

In figure 3, we shown the BER performance after carrier offset correction using the proposed algorithm. After carrier offset estimation and correction, we were able to bring the BER back down to the minimum BER of an ideal OFDM system with no carrier offset.



(a). $N = 8$, $P = 4$, $f_d = 0.25/N$.



(b). $N = 32$, $P = 20$, $f_d = 1/N$.

Figure 3: BER performance using the proposed algorithm to estimate and correct the carrier offset. Parameters used are: (a). $N = 8$, $P = 4$, $f_d = 0.25/N$; (b). $N = 32$, $P = 20$, $f_d = 1/N$. Dashed line with circles shows the BER of the system without carrier offset correction. Dashed line with asterisks shows the BER of the system after the carrier offset correction using the proposed algorithm. Solid line with circles shows the BER of an ideal OFDM-BPSK system with no carrier offset.

5. CONCLUSIONS

In this work, we proposed an efficient scheme for carrier offset correction in the OFDM systems. The proposed carrier offset estimator is based on the conditional maximum likelihood function and it relies on the low-rank nature of the problem ($P < N$). The algorithm formulates the objective function, which is simply the summation of all the periodograms of demodulated (with all used sub-carriers) OFDM data. Computationally efficient FFT is used for obtaining the estimate. Refined estimation accuracy and improved BER performance can be achieved by incorporating multiple blocks of available OFDM data. Simulation results demonstrated that satisfactory performance can be achieved with only 50 blocks of OFDM data.

6. REFERENCES

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