

DIRECT JOINT SOURCE LOCALIZATION AND PROPAGATION SPEED ESTIMATION

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ABSTRACT

This paper describes two new techniques for the joint estimation of source location and propagation speed using measured time difference of arrival (TDOA) for a sensor array. Previous methods for source location either assumed the array consisted of widely separated subarrays, or used an iterative procedure that required a good initial estimate. The first method directly estimates the source location and propagation speed by converting the solution of a system of nonlinear equations to an overdetermined system of linear equations with two supplemental variables. The second method provides improved estimates by using the solution of the first method as initial condition for further iteration. The Cramér-Rao Bound (CRB) on the joint estimation is derived, and simulations show the new methods compare favorably to the bound.

Keywords: source localization, propagation speed estimation, seismic remote sensing, joint estimation

1. INTRODUCTION AND PROBLEM FORMULATION

Consider finding the location of a source with a N passive sensors. In this situation, all that can be found by the sensors is the time difference of arrival (TDOA) between sensors. It is desirable to find the location of the source using the knowledge of the sensor locations and the measured TDOAs. This is a common problem that has application to seismic remote sensing, battlefield atmospheric acoustics, and underwater acoustic sensing. This can also be shown to be equivalent to determining the location of a single sensor receiving synchronized signals from remote transmitters [3], as in the Global Positioning System (GPS).

Because of the broad applicability of this problem, many researchers have addressed the problem in detail over the last 30 years. However, in most cases it has

been assumed that speed of propagation is known. This is quite reasonable for RF applications, and perhaps for acoustic environments if the temperature and wind speed is known. In some cases, such as seismic remote sensing, this is an unrealistic assumption.

The issue of estimating source position with unknown propagation has been addressed in the past (it is hinted at in [5]). There an iterative approach is used. Since the problem is not convex, a good initial estimate is required for convergence. In this paper a direct solution to the problem is proposed.

Consider a single source received by N passive sensors. Then $\|\mathbf{x}_s - \mathbf{x}_i\| = ct_i$, where $\mathbf{x}_s$ is a vector of the known Cartesian coordinates of the source, $\mathbf{x}_i$ is a vector of the known Cartesian coordinates of sensor i , t_i is the time of arrival at sensor i , c is the speed of propagation, and $\|\cdot\|$ is the Euclidean norm (i.e. distance).

Only the TDOA can be measured, leading to the equations

$$\|\mathbf{x}_s - \mathbf{x}_i\| - \|\mathbf{x}_s - \mathbf{x}_1\| = c(t_i - t_1), \quad i = 2, \dots, N, \quad (1)$$

where sensor 1 has been chosen as the reference sensor (without loss of generality).

If c is known, this forms a nonlinear (and nonconvex) set of equations with four unknowns: $\mathbf{x}_s$ and t_1 . If more than four sensors are available, the problem is overdetermined. If c is also unknown, the problem is underdetermined for four or fewer sensors, and overdetermined for more than five sensors.

This very general problem formulation includes many special cases some of which are of great practical interest. These include arrays constrained to a line or a plane, near field or far field cases, more sensors than the number of unknown parameters, as well as the key feature of this paper, which is known versus unknown speed of propagation.

Consider the special case where the source and all the sensors are on a line. If the source is inside the array, we can determine the location with two or more sensors and known speed or three (or more) sensors

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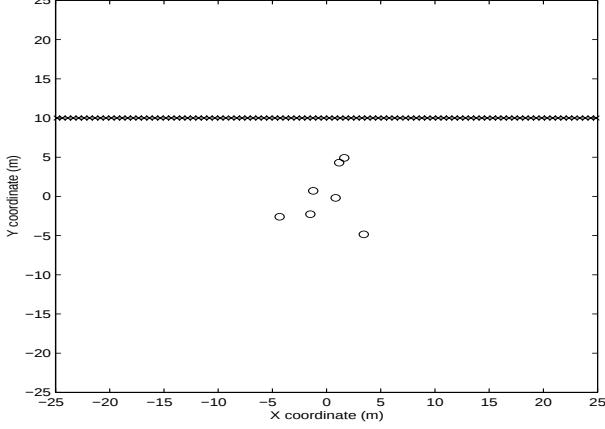


Figure 1: Sensor geometry and source positions for the experiments. The sensors are located at the circles and the source positions at the x's.

with unknown speed. On the other hand, if the source is outside the array the source range is profoundly unobservable, while the speed of propagation can be estimated from as few as two sensors.

Next, consider the same linear array with the source not constrained on the line. For known velocity and source in the far field, we can estimate the bearing with two or more sensors, while in the near field case three or more sensors are required to obtain an estimate of source range and bearing. With unknown velocity the far field bearing is not observable, while for the near field case range and bearing are technically observable and practical application is usually limited to cases near broadside.

The methods described in this paper are applicable to cases where the source is not constrained to the array subspace, but the example is a scenario where the array dimension is equal to the dimension of source location uncertainty. This case permits accurate joint estimation of velocity and source location.

2. CRAMÉR-RAO BOUNDS

This section extends the results of [1] to the case where the propagation speed is unknown. Let an object be located at position $\mathbf{x}_s$, at a distance of r_s from the origin in the direction given by the unit vector $\mathbf{b}_s$ ($\mathbf{x}_s = r_s \mathbf{b}_s$). Define the distance from the source to the i -th sensor as

$$D_i \equiv \|\mathbf{x}_s - \mathbf{x}_i\|.$$

Define the unit vector from the source to sensor i as

$$\mathbf{b}_i \equiv \frac{\mathbf{x}_s - \mathbf{x}_i}{D_i}.$$

The range difference between sensors i and 1 and

the vector of range differences are defined by

$$d_{1i} \equiv D_1 - D_i \text{ and } \mathbf{d} \equiv [d_{12}, \dots, d_{1N}]^T.$$

The time delay from the source to sensor is given by $\tau_i = D_i/c$, where c is the propagation speed. The time delay vectors are given by $\mathbf{t} = (1/c)\mathbf{d}$. Denote the estimated time delays as t_{1i}^o to form the vector $\mathbf{t}^o$. If $\mathbf{t}^o$ is estimated using a Maximum Likelihood (ML) estimator, it can be modeled as the sum of $\mathbf{t}$ and a zero-mean Gaussian distributed random variable, with its covariance given by the inverse of the Fisher information matrix.

For unbiased estimators with small measurement errors, the variance of the estimators is bounded below by the Cramér-Rao bound (CRB). For estimating parameter Θ drawn from a real Gaussian density with mean $\mu(\Theta)$ and covariance Σ , then

$$\text{var}(\hat{\Theta}) \geq \mathbf{J}^{-1} = \left(\frac{\partial \mu^T}{\partial \Theta} \Sigma^{-1} \frac{\partial \mu}{\partial \Theta^T} \right)^{-1},$$

where $\mathbf{J}$ is the Fisher information matrix. For source localization with unknown speed of propagation,

$$\Theta = [\mathbf{x}_s^T \ c]^T \text{ and } \mu(\Theta) = \mathbf{t}(\mathbf{x}_s, c) = \frac{1}{c} \mathbf{d}(\mathbf{x}_s).$$

Define the differencing matrix $\mathbf{Z} \equiv [\mathbf{1} \ -\mathbf{I}]$, where $\mathbf{1}$ is a column vector of ones and $\mathbf{I}$ is and $(n-1) \times (n-1)$ identity matrix. Then $\mathbf{t} = (1/c)\mathbf{Z}\mathbf{D}$, with $\mathbf{D} = [D_1, \dots, D_N]^T$, so

$$\frac{\partial \mu}{\partial \Theta^T} = \left[\frac{\partial \frac{1}{c} \mathbf{Z}\mathbf{D}}{\partial \mathbf{x}_s^T} \quad \frac{\partial \frac{1}{c} \mathbf{Z}\mathbf{D}}{\partial c} \right] = \frac{1}{c} \mathbf{Z} [\mathbf{B} \quad -\frac{1}{c} \mathbf{D}],$$

where $\mathbf{B} = [\mathbf{b}_1, \dots, \mathbf{b}_N]^T$, because

$$\frac{\partial D_i}{\partial \mathbf{x}_s^T} = \frac{(\mathbf{x}_s - \mathbf{x}_i)^T}{D_i} = \mathbf{b}_i^T.$$

Thus, the variance bound of the joint estimator is given by

$$\mathbf{J}^{-1} = c^2 \begin{bmatrix} \mathbf{B}^T \\ -\frac{1}{c} \mathbf{D}^T \end{bmatrix} \mathbf{Z}^T \Sigma_t^{-1} \mathbf{Z} \begin{bmatrix} \mathbf{B} & -\frac{1}{c} \mathbf{D} \end{bmatrix}^{-1}.$$

The Fisher information matrix may also be used to find estimator bounds on position, bearing, and speed of propagation individually. Defining the projection matrices by

$$\mathbf{P}_{\mathbf{x}_s} \equiv \frac{\mathbf{x}_s \mathbf{x}_s^T}{r_s^2} \text{ and } \mathbf{P}_{\mathbf{x}_s}^\perp \equiv \mathbf{I} - \mathbf{P}_{\mathbf{x}_s},$$

the following bounds can be derived

$$\begin{aligned} \text{var}(\hat{r}_s) &\geq \begin{bmatrix} \mathbf{P}_{\mathbf{x}_s} \\ \mathbf{0} \end{bmatrix}^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{P}_{\mathbf{x}_s} \\ \mathbf{0} \end{bmatrix}, \\ \text{var}(\hat{\mathbf{b}}_s) &\geq \begin{bmatrix} \mathbf{P}_{\mathbf{x}_s}^\perp \\ \mathbf{0} \end{bmatrix}^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{P}_{\mathbf{x}_s}^\perp \\ \mathbf{0} \end{bmatrix}, \\ \text{var}(\hat{c}) &\geq \begin{bmatrix} \mathbf{0} \\ \mathbf{1}^T \end{bmatrix}^T \mathbf{J}^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{1}^T \end{bmatrix}. \end{aligned}$$

3. DIRECT SOLUTION

By introducing two supplemental variables, the solution to (1) can be obtained from an overdetermined system of linear equations. This does not give the Maximum Likelihood (ML) estimate (which can only be obtained by an iterative approach) but the equations' least-squares solution yields an excellent approximation for seven or more sensors in a three-dimensional scenario (six or more sensors in a two dimensional scenario).

Using vector notation, let $\mathbf{x}_s = (x, y, z)$ and $\mathbf{x}_i = (x_i, y_i, z_i)$. Without loss of generality, we choose $i = 1$ as the reference sensor for the differential time delays.

Now define the translated and normalized position location vector variable and the two new supplemental variables by

$$\begin{aligned} \mathbf{x}'_s &\equiv [x_1, x_2, x_3]^T = \frac{\mathbf{x}_s - \mathbf{x}_1}{c\|\mathbf{x}_s - \mathbf{x}_1\|}, \\ x_4 &\equiv \frac{1}{2c\|\mathbf{x}_s - \mathbf{x}_1\|}, \\ x_5 &\equiv \frac{c}{2\|\mathbf{x}_s - \mathbf{x}_1\|}. \end{aligned} \quad (2)$$

Thus, as described in [4], (1) can be written as a linear set of equations $\mathbf{A}\mathbf{x} = \mathbf{b}$, where

$$\mathbf{A} = \begin{bmatrix} -(\mathbf{x}_2 - \mathbf{x}_1) & \|\mathbf{x}_2 - \mathbf{x}_1\|^2 & -(t_2 - t_1)^2 \\ \vdots & \vdots & \vdots \\ -(\mathbf{x}_N - \mathbf{x}_1) & \|\mathbf{x}_N - \mathbf{x}_1\|^2 & -(t_N - t_1)^2 \end{bmatrix}, \quad (3)$$

$\mathbf{x} = [\mathbf{x}'_s, x_4, x_5]^T$, and $\mathbf{b} = [t_2 - t_1, \dots, t_N - t_1]^T$.

We note in the three-dimensional problem, the first column on the r.h.s. of (3) is a sub-matrix of dimension $(N - 1) \times 3$ and thus $\mathbf{A}$ is a $(N - 1) \times 5$ matrix, $\mathbf{x}$ is a 5×1 vector, and $\mathbf{b}$ is a $(N - 1) \times 1$ vector. In the two-dimensional problem, the first column on the r.h.s. of (3) is a sub-matrix of dimension $(N - 1) \times 2$ and thus $\mathbf{A}$ is a $(N - 1) \times 4$ matrix, $\mathbf{x}$ is a 4×1 vector, and $\mathbf{b}$ is an $(N - 1) \times 1$ vector.

If $\{\hat{\mathbf{x}}'_s, \hat{x}_4, \hat{x}_5\}$ are the least-squares solution of $\mathbf{A}\mathbf{x} = \mathbf{b}$, then the desired source location and the velocity are given by

$$\hat{\mathbf{x}}_s = \frac{\hat{\mathbf{x}}'_s}{2\hat{x}_4} + \mathbf{x}_1, \quad \hat{c} = \sqrt{\frac{\hat{x}_5}{\hat{x}_4}}. \quad (4)$$

If the source and sensors are known to be in two-dimensional space, then all the above results are valid except the minimum number of sensors can be reduced by one.

The method used to derive (4) can also be used to derive a method that estimates source location when the speed of propagation is known. This requires the introduction of only a single supplemental variable. Then

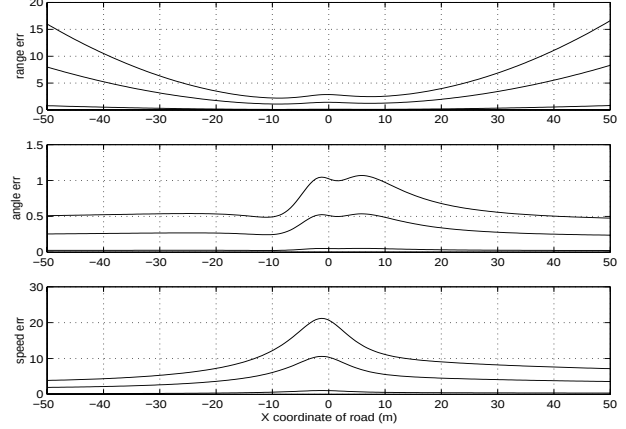


Figure 2: Cramér-Rao bound on range, azimuth angle of arrival, and propagation speed. Range is in m, angle in degrees, and propagation speed in m/s.

the source localization estimate can be derived by solving a set of linear equations. This method has the advantage over the algorithm in [2] of not requiring the solution of a quadratic equation.

4. FOUR STEP METHOD

Although the above method produces a good initial estimate of the source position and speed of propagation when nothing about them is known a priori, the estimate of propagation speed is not particularly good. The direct solution can be used as an initial estimate for an iterative method. However, since existing iterative methods require a good initial estimate of speed of propagation, they may not converge. A new iterative approach is found based on the assumption that source location estimate is better than the initial speed estimate. Ignoring the initial speed estimate and treating the source location estimates as if they were correct, the least-squares estimate of c is then given by

$$\hat{c} = \frac{\mathbf{t}^{oT} \mathbf{d}(\mathbf{x}_s)}{\mathbf{t}^{oT} \mathbf{t}^o}. \quad (5)$$

This suggests the following iterative method may be used to jointly estimate source position and speed of propagation given no a priori knowledge of either:

1. Find $\hat{\mathbf{x}}_s$ using (4).
2. Find $\hat{c}$ using (5).
3. Use any method to estimate source location for known speed of propagation, using $\hat{c}$.
4. Repeat steps 2-4 as many times as desired.

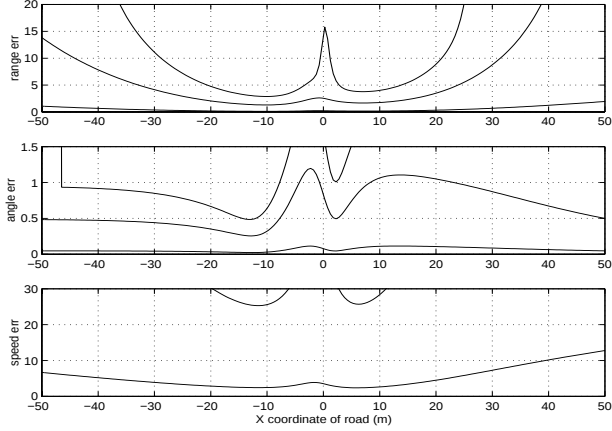


Figure 3: RMS error using direct method. Range is in m, azimuth angle in degrees, and propagation speed in m/s.

5. SIMULATIONS

The performance of the proposed methods is compared to the CRB using computer simulations. Seven sensors are randomly distributed in three dimensions (the X and Y coordinates are shown in Figure 1; the altitudes are small compared to X and Y). The coordinate system is chosen such that the array centroid is at the origin. The range and bearing are defined with respect to the centroid. The performance is investigated at 200 points along a path that passes near the array, as shown in Figure 1. The true speed of propagation is 343 m/s.

The CRB can be determined once the time delay estimate variance is specified. It is assumed for this experiments that each sensor's error is indendent and identically distributed. Thus, $\Sigma = \sigma^2 \mathbf{I}$, where σ^2 is the variance of the time delay estimation errors. In these simulations, the time delay estimates are assumed to be uniformly distributed on $[-\Delta, \Delta]$. Three values of Δ are simulated, 0.005, 0.05, and 0.1 m. The CRBs on range, azimuth angle, and propogation speed estimation for this geometry are shown in Figure 2. The standard deviation, not the variance, of the bounds is shown in the plots.

Monte Carlo simulations were performed using the two methods described in this paper. At each source location, 100 trials were performed, and the RMS errors on range, azimuth angle, and propagation speed were computed. The results using the direct method are shown in Figure 3. For small errors, the method approaches the CRB, but it diverges significantly for large errors. The direct method can provide an excellent starting point for the Four Step Method. The RMS errors using five iterations is shown in Figure 4. In general, five iterations is quite sufficient, while excellent performance with only two iterations is commonly

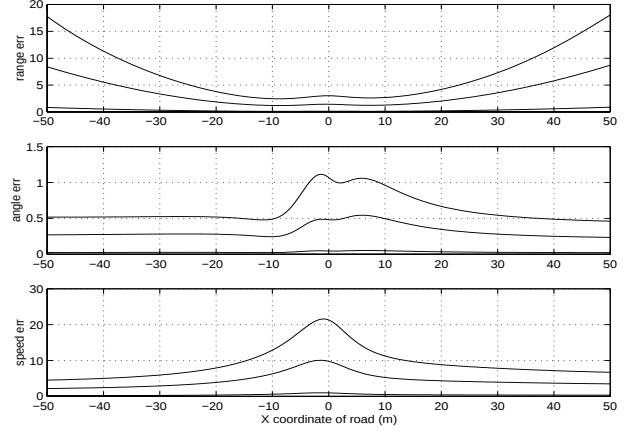


Figure 4: RMS error using direct method followed iterations with known speed. Range is in m, azimuth angle in degrees, and propagation speed in m/s.

encountered. The known speed solution was performed using the known speed analogous to the unknown speed direct method. By comparing Figure 2-4, we note the results of the Four Step method approach those of the CRB even for larger errors.

6. REFERENCES

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