CONTEXT MODELING OF WAVELET COEFFICIENTS IN EZW-BASED LOSSLESS IMAGE CODING

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Abstract

The EZW lossless coding framework consists of three stages: (i) a reversible wavelet transform, (ii) an EZW data structure to order the coefficients and (iii) an arithmetic coding using context modeling. In this work, we discuss the various experiments conducted on context modeling of wavelet coefficients for arithmetic coding to optimize the compression efficiency. The context modeling of wavelet coefficients can be classified into two parts: (i) context modeling of significance information and (ii) context modeling of the remaining or residue information. It was observed from our experiments while context modeling of residue helped in achieving considerable compression efficiency, the context modeling of significance information helped only to a modest extent.
Keywords: lossless, image coding, EZW, wavelet, context modeling.

1. INTRODUCTION

Multimedia applications such as HDTV, internet, video conferencing and telemedicine have created interest among researchers on the problem of compressing still images and video. In this paper, we concentrate on lossless image compression using a set partitioning based EZW framework. The EZW based lossless image coding framework consists of three stages: (i) reversible discrete wavelet transform (ii) ordering of wavelet coefficients using the EZW data structure and (iii) context modeling based arithmetic coding. The choice of wavelets plays an important role in decorrelating and compacting the image data and thereby improving the compression efficiency. The EZW data structure orders the wavelet coefficient to exploit the interband correlation. Finally, in the third stage, the arithmetic coding exploits the skewness existing in the distribution of wavelet coefficients using different context models. The research work presented in this paper consists of several experiments to investigate (i) different methods of ordering the wavelet coefficients using the EZW data structure and (ii) different context models that can be used in conjunction with arithmetic coding to exploit the ordering of wavelet coefficients. Context modeling of wavelet coefficients is being recently studied for lossy and lossless image coding applications [5][6].

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2. ABBREVIATIONS AND TERMINOLOGY

The terminology used in this paper is similar to that used by Shapiro [1], and Said *et al* [2] and presented here for convenience. Let W, H represent the width and height of image and let $T = 2^n$ represent the current threshold.

- $S_n(i,j)$: Significance of a coefficient w.r.t T
- $S_n(D(i,j))$: Significance of descendants w.r.t T
- $S_n(L(i,j))$: Significance of grand-descendants w.r.t T
- LIP: *List of Insignificant Pixels*
- LSP: *List of Significant Pixels*
- LIS: *List of Insignificant Sets*
- *Significance Bit:* The bit corresponding to $S_n(i,j)$ needed to update the wavelet coefficients in the decoder.
- *Message Bit:* The bit corresponding to $S_n(D(i,j))$ or $S_n(L(i,j))$ that is needed for ordering information.
- *S Bitmap*: This corresponds to the set of $S_n(i,j)$ values for all coordinates (i,j) in the transformed image with respect to one threshold say 2^n or the n-th bit plane. Then S Bitmap corresponds to $\{S_n(i,j) : 0 \leq i < W, 0 \leq j < H, T = 2^n\}$

Similar definitions hold for SD and SL Bitmaps, corresponding to $S_n(D(i,j))$ and $S_n(L(i,j))$ respectively.

3. THE SET PARTITIONING SCHEME

The outline of the set partitioning algorithm is given below.
The Algorithm:
Begin

- Perform the reversible discrete wavelet transform.
- Split the coefficients into:

- significance map containing the first few bit planes of the wavelet coefficients.
- residue map containing the remaining bits of the wavelet coefficients

- Encode significance map (i) without grouping or (ii) with grouping and context model based arithmetic coding.
- Encode residue map using context model based arithmetic coding.

End

The set partitioning scheme consists of two passes namely the sorting and the refinement pass. The sorting pass sorts the coefficients in LIP and moves them to LSP when they become significant. In other words, the wavelet coefficients remain in LIP till the most significant bit (MSB) that is set to 1 is encountered in the sorting pass. The refinement pass outputs the remaining bits following the first significant bit,namely the first bit that is set to 1 in the wavelet coefficients. The algorithm is iterated by decrementing the threshold (threshold usually being a power of two) each time and terminated when the minimum threshold is reached. The significance map consisting of first few bit planes are encoded as the significance and message bits. The significance bit is used to update the wavelet coefficient in the decoder and the message bits are used for ordering significance information. In the sorting pass, the S, SD and SL bitmaps are used to compute the significance information. The S, SD and SL bitmaps are computed for each threshold using recursive functions 'Create_SMaps', 'Create_SDMaps' and 'Create_SLMaps'. These functions can be found in appendix section of [8].

4. ORDERING OF WAVELET COEFFICIENTS IN THE EZW CODING

As discussed in the previous section, the wavelet coefficients are split into significance and residue map. The significance information consists of the significance and message bits. The sorting and refinement passes in the set partitioning algorithm produces significance and message bits to allow proper decoding of symbols. The significance bits in the sorting pass are encoded as either groups of 2×2 or as individual bits. Similarly the message bits namely the $S_n(D(i,j))$ and $S_n(L(i,j))$ can also be encoded as a 2×2 group or individual bits.

- In *Algorithm A*, the sorting pass outputs $S_n(i,j)$, $S_n(D(i,j))$ and $S_n(L(i,j))$ as individual bits depending on the coordinate (i,j) in LIS as Type A or Type B.
- In *Algorithm B*, the sorting pass outputs $S_n(i,j)$, $S_n(D(i,j))$ and $S_n(L(i,j))$ information as symbols formed from a 2×2 grouping of the bits from S, SD and SL bitmaps.

In Algorithm B, the data packet to be encoded has the following form:

- A symbol formed by a 2×2 grouping of $S_n(D(i,j))$.
- Depending on the bit that is set to 1 in $S_n(D(i,j))$, a symbol is formed by a 2×2 grouping of $S_n(i,j)$. Hence a maximum of four $S_n(i,j)$ symbols needs to be coded.
- A symbol formed by a 2×2 grouping of $S_n(L(i,j))$ if the grandchildren exist.

The details of each algorithm can be found in [8]. The grouping of significance information in algorithm B allows us to exploit up to m-th order entropy, m being {1, 2, 3, 4}. Better context models can be formed to encode the symbols rather than individual bits and hence the context models can exploit the correlation between the symbols formed by $S_n(i,j)$ and $S_n(D(i,j))$. In the following sections, the experiments conducted on different context modeling techniques to encode significance and residue values are discussed.

5. CONTEXT MODELING OF SIGNIFICANCE INFORMATION

In [2], Said and Pearlman propose the set partitioning approach in which the significance bits information is encoded as a tree node, with root and 4 children as it's elements. This is performed for each bit plane of the wavelet coefficients. Separate models for the tree root and descendants are proposed to encode the significance information of the wavelet coefficients and its descendants formed as a 2×2 set of coordinates.

In our work, initially, a single context model was employed to code the significance information. This is equivalent to an adaptive arithmetic coding without exploiting any context information or conditioning on the source symbols. Then 16 models were used to encode the significance information of $S_n(i,j)$, $S_n(D(i,j))$ and $S_n(L(i,j))$ together. These were very few context models to really exploit the skewness. Using too many models can cause a phenomenon called context dilution. Hence appropriate number of models have to be used to exploit the skewness. Also, there is an initial model cost to encode the symbols.

In our experiments, it was observed that one common set of context models for the significance information cannot exploit the correlation between the significance bits and the message bits namely $S_n(i,j)$ and $S_n(D(i,j))$ (or $S_n(L(i,j))$). Hence a set of 16 different models each for $S_n(D(i,j))$ and $S_n(L(i,j))$ and a set of 4 models for $S_n(i,j)$ were used. Initially, the contexts were a polynomial function of the types of the coordinates (Type A, B, or C). Type C is our modification to the set partitioning algorithm to convey the message that the coordinates have already been processed as Type A and Type B and does not need any further processing. Then contexts based on the previously encoded symbols formed by 2×2 block were used. Both were unsuccessful attempts to code the significance information efficiently. Finally, after extensive experiments with the context models, it was found that the only correlation in the wavelet coefficients is between two successive thresholds $T = 2^n$ and $T = 2^{n-1}$ This is due to the fact that once a coordinate becomes significant with respect to a threshold, that coordinate is significant with respect to all lower thresholds. It was observed that this context modeling improved the compression efficiency to a small extent only. The symbol encoded formed from the significance information in a 2×2 group with respect to previous threshold was stored as a context information in the LIS list structure.

6. CONTEXT MODELING OF RESIDUE

The last step in the two algorithms A and B is context modeling of the residue. The residue coefficients are formed after the set partitioning based EZW coding has been performed up to a last threshold (say 2^L, usually L=3). The range of residue is between ($-2^L - 1$ to $2^L - 1$) As mentioned in [7], since the significance of the coefficient was found by checking whether a coefficient is $\geq$ to the current threshold, there could be a conflict in decoding the symbol zero. This problem was avoided by mapping the coefficients that are equal to current threshold to an escape symbol and mapping the symbols back to 2^L. A major contribution to the storage cost is from the residue since the entropy content is high. A vast number of experiments were conducted to find an optimal coding of the residue coefficients. One experiment was to adapt the CALIC (Context based Adaptive Lossless Image Coding) encoding to exploit the edges and the texture patterns in the residue, if there are any. A brief discussion of the actual CALIC encoding and the experiments on adapting it to the residue coefficients will be explained in a later section. Another experiment was to improve the context modeling proposed in [3].

6.1. Context Modeling based on Set Partitioning Approach

In [3], the residue values (ranging from -2^3 to 2^3) are categorized into bins or buckets. A model number to encode the current category number is computed based on the values of the previously scanned neighboring categories. More specifically, let $C(i,j)$ be the category value of the current residue value and $M(i,j)$ the model number of the current pixel which needs to be computed. Let

$$C_{ave}(i,j) = \{C(i,j-1) + C(i-1,j-1) + C(i-1,j) + C(i,j+1)\}/4 \quad (1)$$

and

$$C_{par}(i,j) = C(\frac{i}{2},\frac{j}{2}) \quad (2)$$

Then,

$$M(i,j) = Min(4, C_{ave}(i,j) + 5 * Min(4, C_{par}(i,j)) \quad (3)$$

After encoding the category values, actual value within a category is encoded using a single fixed uniform model.

Our experiments based on the set partitioning approach are as follows. As mentioned earlier, the significance with respect to previous thresholds were used as contexts to encode the symbols formed in the current threshold. This is due to the reason that there exists a correlation in the significance computation between successive thresholds. Some experiments were conducted to encode the residue coefficients using the last threshold significance computed in the sorting and refinement passes of the set partitioning approach. The intuitive reasoning to such an approach is as follows: Since there is an initial cost involved in updating the cumulative frequencies of the symbols in adaptive arithmetic coding it was expected that this could be reduced using the correlation existing between the significance of the last threshold and the residue coefficients. It is known that the distribution of each subband is not the same. Hence a different modeling technique after the first 2 to 3 resolutions was experimented similar to that followed in other multiresolution techniques like Hierarchical INTerpolation (HINT) technique. This did not help in improving the compression efficiency either and is due to the fact that initial model cost of the arithmetic coding for the new model seem to offset the gains that can be exploited by varying distributions. After extensive experiments it was observed that the compression efficiency did improve when the siblings of the parents namely the S, SE and SE parents were also included in the computation of the context model. Hence, the average of the four parents was calculated instead of the parent category alone.

$$C_{par}(i,j) = \frac{1}{4}\{C(\frac{i}{2},\frac{j}{2}) + C(\frac{i}{2},\frac{j}{2}+1) + C(\frac{i}{2}+1,\frac{j}{2}) + C(\frac{i}{2}+1,\frac{j}{2}+1)\} \quad (4)$$

6.2. Context Modeling based on CALIC

Instead of modeling the residue based on set partitioning approach, one can encode the residue coefficients based on CALIC style of modeling and coding but with significant modifications to the CALIC algorithm. If CALIC style of encoding is used then categorization is not necessary as used in SPIHT. In CALIC [4], a gradient-adjusted prediction is employed. A compound context is formed using the error energy term and a texture pattern. The prediction errors are updated and encoded using the context formed. The details of the CALIC algorithm can be found in [4].

Different predictors based on CALIC style of coding were carried out in the experiments that we conducted. Since the predictors did not help in the residue, only context modeling based arithmetic coding was performed on the residue coefficients (unlike the categorization in SPIHT). It was also observed that bias cancellation did not help in improving the compression efficiency to a large extent. On the other hand, bimodal operation of CALIC helps in encoding the residue. Since the range of intensity values is small in the residue map, there is a potential for neighborhood to possess similar intensity values. This fact is exploited in the residue coefficients Again, since there is a correlation in the residue coefficients in an absolute or magnitude sense, a remapping function denoted by $r(x)$ is

Table 1: Compression Efficiencies of S+P and (5,3) filters with Last Threshold = 32

	Compression Efficiency					
	S+P (Type C)			(5,3)		
Image	M-1	M-2	M-3	M-1	M-2	M-3
Challenger	43.34	43.24	42.25	43.79	43.60	42.65
Coral	38.22	38.09	37.08	38.32	38.10	37.12
F16	57.03	56.94	56.21	58.37	58.25	57.41
House	44.85	44.63	44.39	45.96	45.65	45.48
Planets	78.41	78.29	77.99	78.83	78.60	78.26
LAX	37.49	37.48	36.52	39.18	39.17	38.44
Lenna	34.51	34.24	33.38	35.31	34.97	34.45
Man	63.09	63.05	62.54	61.23	61.15	60.67
Shuttle	49.35	49.27	48.12	50.82	50.67	49.89
Sphere	46.28	46.19	45.67	45.16	45.10	44.37

used to map the coefficients to positive values.

$$r(x) = \begin{cases} 2x & \text{if } x \geq 0 \\ 2x+1 & \text{if} x < 0 \end{cases}$$

The above remapping was performed just before the actual arithmetic encoding of the residue coefficients. The contexts were formed using the neighboring coefficients and average of the four parent coefficients similar to the coding of categories in SPIHT approach. The tail truncation used in CALIC also helps in improving the compression efficiency. It needs to be emphasized that by fine tuning the last threshold of the sorting pass and the maximum frequency in arithmetic coding, a modest gain in compression efficiency can be achieved.

7. SUMMARY

In this paper, various context models were studied for efficient encoding of wavelet coefficients in an EZW-based lossless image coding framework. Context models from two state of the art lossless coders (namely SPIHT and CALIC) were used to study on the improvement in compression efficiency. The results of compression efficiency obtained without and with grouping of significance information and context modeling of the residues using CALIC modeling for two different last thresholds namely 32 and 8 are shown in Table 1, and Table 2. It was observed that context modeling of significance information in the sorting and refinement passes involves the significance of previous threshold. It was also observed that context modeling of residue coefficients based on CALIC coding improved the compression efficiency only to a modest extent.

8. REFERENCES

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Table 2: Compression Efficiencies of S+P and (5,3) filters with Last Threshold = 8

	Compression Efficiency					
	S+P (Type C)			(5,3)		
Image	M-1	M-2	M-3	M-1	M-2	M-3
Challenger	41.21	40.66	40.73	42.17	41.15	41.54
Coral	36.03	35.33	35.57	36.62	35.28	35.99
F16	56.06	55.56	55.47	57.74	57.01	56.89
House	43.26	42.16	43.07	44.67	43.18	44.40
Planets	77.46	76.88	77.11	78.02	77.10	77.55
LAX	33.95	33.61	33.80	36.28	35.90	36.24
Lenna	32.04	31.15	31.85	33.42	32.18	33.27
Man	62.74	62.43	62.03	60.66	60.10	60.00
Shuttle	47.57	47.11	46.95	49.82	49.09	49.28
Sphere	44.68	44.10	44.10	43.60	42.68	42.73

M-1: Inidividual Bit Encoding of Significance SPIHT Modeling & Encoding of Residue.
M-2: 2×2 Grouping & Context Modeling of Significance SPIHT Modeling & Encoding of Residue.
M-3: 2×2 Grouping & Context Modeling of Significance CALIC Modeling & Encoding of Residue.

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