QUANTIZATION NOISE ANALYSIS OF WAVE DIGITAL AND LOSSLESS DIGITAL INTEGRATOR ALLPASS/LATTICE FILTERS

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ABSTRACT

Quantization noise levels of two low-sensitive allpass filter structures, namely wave digital circulator filters (WDCF) and lossless digital integrator filters (LDIF), are compared. Allpass filters are of interest for design of lowpass and bandpass lattice filters. The results show, primarily, that second-order LDIFs have lower total quantization noise gains than corresponding WCDFs for any pole configuration within the right half-circle of the z plane. The benefit of using ladder LDIFs rather than cascaded first and second order sections is also demonstrated.

1. INTRODUCTION

Digital lattice filters[1] consist of two parallel connected allpass filters [1]. They can be designed as odd order elliptic, Butterworth, or Chebyshev lowpass filters. A highpass function can also be obtained in the same structure by using the power complementary output. In the same way, two times odd order bandpass and bandstop filters can be obtained. Thus, these filters are useful, among other things, for bandsplitting. Lattice filters have low sensitivity in the passband, which is often very beneficial, since the number of bits of the multiplier coefficients can be reduced without significant deterioration of the magnitude function [2].

The allpass filters are normally realized using wave digital (WD) filter techniques [3]. Most commonly used is the wave digital circulator filter (WDCF), which consists of cascade connected allpass adaptors. The advantage of this type of structure is that it has only one multiplier in the critical loop, provided that three-port adaptors are employed [4]. Thus, maximal speed can be achieved in a hardware implementation [2].

Another way of realizing the allpass filters are via lossless digital integrator (LDI) structures. This was first introduced in [5]–[7], but the filters had, unfortunately, two multipliers in the critical loop. In [8] this was circumvented by changing the locations of the multipliers, thus reducing the critical loop to one multiplier only. The structure was further modified in [9], by interchanging delayed and non-delayed integrators, in order to eliminate overflow limit cycles [10]. The structure in [9] is considered in this paper.

WD and LDI lattice filters (both with one multiplier in the critical loop) were compared in [8], where properties such as response to coefficient quantization, noise gain, and sensitivity were investigated. A well known fact is that the noise gain of a digital filter is of great importance, as it affects the signal to noise ratio (SNR) at the filter output. The noise gain problem has been investigated during several decades. A significant result of [8] was that, for certain test filters, the total noise gain at the output of the LDIF was *much lower* than for the WDCF.

In this paper, analytic noise gain expressions are established, for filter orders one and two. This allows more general conclusions to be drawn as compared to numerical analyses of test filters.

2. MAIN RESULT

Signal quantization is most commonly made directly after each multiplier. Zero input granularity limit cycles then often arise, however, for filter orders higher than one. By placing the quantizer after the adder following a multiplier, the risk for

[1]Not to be confused with the Gray–Markel lattices.

this undesirable phenomenon can be reduced, if magnitude truncation is used [10].

A quantization introduces an error which is in general modeled as a white noise source with variance σ_e^2 [10]. The noise variance at the filter output resulting from only one noise source is

$$\sigma_e^2 \int_{-1/2}^{1/2} |H_i(e^{j2\pi\nu})|^2 d\nu, \qquad (1)$$

where $H_i(z)$ is the transfer function from noise source i to the output and ν is the normalized frequency. The various quantization noise sources are, following common practice, assumed independent of each other and can thus be analyzed by superposition. The *total noise gain* (TNG), G, is defined as

$$G = \sum_{i=1}^{N} \int_{-1/2}^{1/2} |H_i(e^{j2\pi\nu})|^2 d\nu, \qquad (2)$$

where N is the number of noise sources. Thus, the total noise variance at the output is $G\sigma_e^2$.

When using the wave digital circulator technique for realization of lattice filters, the allpass filters are realized as a chain of first and second order sections. The former consist of two-port and the latter of three-port adaptors. When using LDI building blocks, the same principle can be followed (although this is, as we shall see, not the best available approach). Therefore, first and second order LDI and WD allpass filters are analyzed in this section, which allows analytical noise gain expressions to be derived.

2.1. First Order Filters

The filter structures compared are depicted in Figure 1. The WDCF is, in fact, just a two-port adaptor. Plus and minus signs in the WDCF, i.e., usage of a delay with and without a sign inverter, correspond to usage of a capacitor and an inductor, respectively, in the prototype network. Let us label these two variants WDCF–C and WDCF–L. In the following, where "±" occurs, the plus sign corresponds to WDCF–C. Since the filter order is one, no granularity limit cycles can occur if magnitude truncation is used. Therefore, quantization is made directly after the multiplier, as indicated by the noise source $e(n)$. With $\alpha_1 = 1 - a_1$, both structures realize the same allpass transfer function, $(1-a_1z)/(z-a_1)$, from u to y. The transfer functions from the noise sources to the output are $z/(z-a_1)$ and $(z\pm1)/(z-a_1)$, for

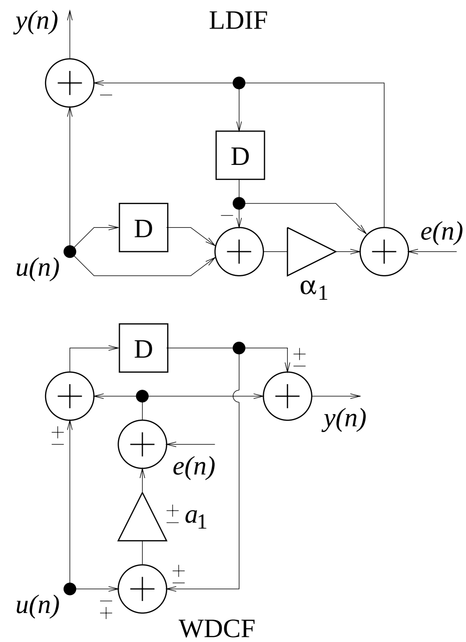

Figure 1: First order allpass filter structures.

the LDIF and the WDCF respectively, yielding the following TNGs:

$$G_{\mathrm{LDI}} = \frac{1}{1-a_1^2}, \quad G_{\mathrm{WD}} = \frac{2}{1 \pm a_1}. \qquad (3)$$

The functions are depicted in Figure 2. The preferable structure, depending on the pole location, is obvious.

2.2. Second Order Filters

The structures compared are depicted in Figure 3. Quantization is in this case made after the adder following a multiplier. The WDCF is a parallel three-port adaptor. (A series adaptor can be used as well, however, at the expense of an extra quantizer.) The two structures realize the same transfer function from input to output if $\gamma_1 = \alpha_1$ and $\gamma_3 = (4-2\alpha_1-\alpha_2)/2$. It is assumed that the poles are complex-conjugated, $z = re^{\pm j\theta}$. The coefficients can then be expressed as $\alpha_1 = 1-r^2$ and $\alpha_2 = 1 + r^2 - r\cos\theta$. Evaluating (2), using residues, yields

$$\begin{aligned} G_{\mathrm{LDI}} &= \frac{2}{(1-r^2)(1+2r\cos\theta + r^2)}, \\ G_{\mathrm{WD}} &= \frac{2}{1-r^2}, \end{aligned} \qquad (4)$$

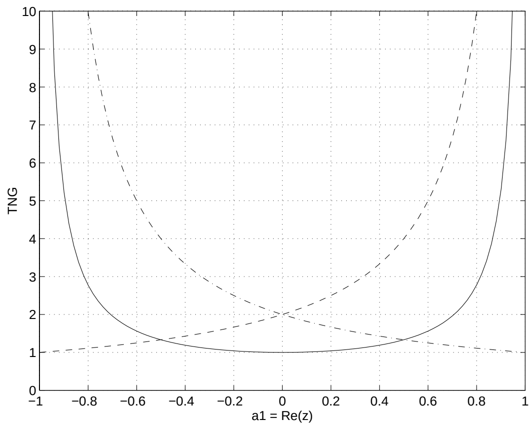

Figure 2: Noise gains for first order LDIF (solid), WDCF–C (dashed), and WDCF–L (dash-dotted).

giving the following relative noise gain:

$$\frac{G_{\text{LDI}}}{G_{\text{WD}}} = \frac{1}{1 + 2r\cos\theta + r^2}. \tag{5}$$

A plot of this relation is shown in Figure 4. The result tends to agree with the comparison LDIF vs. WDCF–C for the first order case. For any pole location within the right half-circle of the z plane—and also for some in the left half-circle—the LDIF has a lower noise gain than the WDCF. However, unlike the first order case, using delays with sign inversion does not alter the noise gain, due to structural symmetry. The LDI structure therefore has a clear advantage in this case.

2.3. LDI Filters of Higher Order

While it is possible to realize higher order LDI allpass filters as a chain of cascaded first and second order sections, a generally better approach is to extend the LDI structure by additional interconnected integrators. This is known as an *LDI ladder network*; see Figure 5. This approach can be used for any filter order without increasing beyond one the number of multipliers in the critical loop.

Since analytical expressions are now much more difficult to obtain than for orders one and two, we in this paper restrict the analysis to numerical evaluations. The allpass filters considered are both of order three with poles at $z = \{0.32, 0.19 \pm j0.86\}$ (AP1) and $\{0.80, 0.90 \pm j0.32\}$ (AP2), respectively. Naturally, WDCF–L is used for the first order WDCF allpass sections. The following total noise gains are obtained.

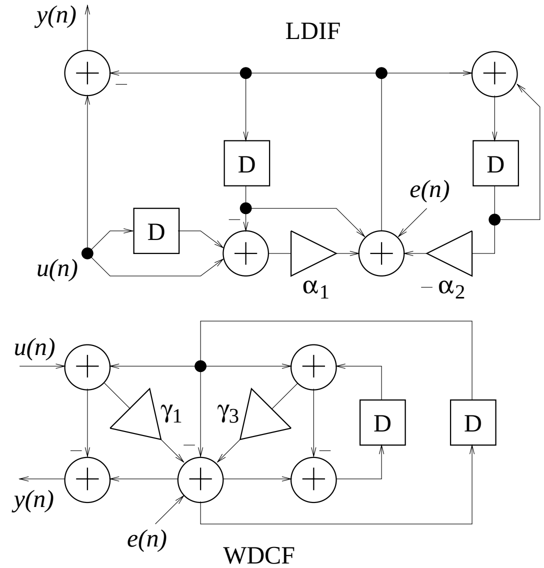

Figure 3: Second order allpass filter structures.

	AP1	AP2
WDCF:	2.55	6.06
Cascaded LDIF:	1.30	2.25
Ladder LDIF:	0.38	0.80

The benefit of using the ladder approach is obvious, as is the benefit of using LDI rather than WD realization of a filter of this type.

3. CONCLUSION

In this paper, lossless digital integrator and wave digital circulator realizations of allpass filters were analyzed with respect to quantization noise. Suitably designed, two allpass filters can be added to form a lattice lowpass or bandpass filter.

For first order filters, it was shown that WDCFs realizations are preferable for pole locations in $0.5 < |z| < 1$, while LDIFs are preferable for $|z| < 0.5$.

For second order filters, it was shown that for all pole locations in the right half-circle of the z plane (and for certain pole location in the left half-circle as well), the LDI structure yields a lower level of quantization noise at the output than the WD structure.

It was further indicated—numerically—that for most (if not all) allpass filters of orders higher than two, a ladder structure is preferable to a

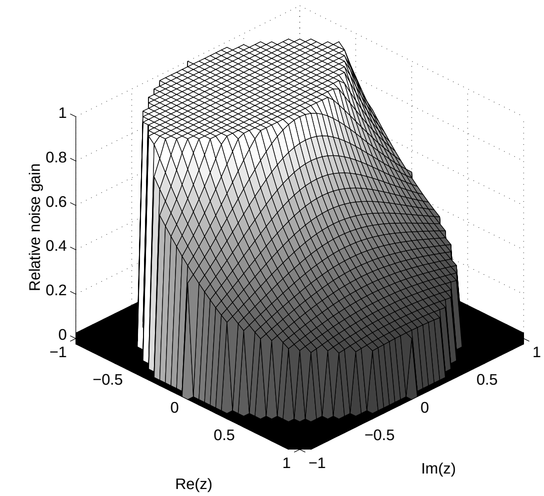

Figure 4: Relative noise gain, $G_{\mathrm{LDI}}/G_{\mathrm{WD}}$, for second order allpass filters.

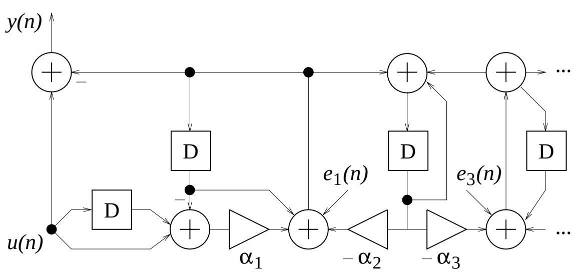

Figure 5: LDI ladder allpass filter of arbitrary order.

structure of cascaded first and second order sections.

The results in this paper have striking similarities to those presented in [9]. In [9] it is found that overflow limit cycles will not occur in the LDI allpass structure in Figure 5 if the pairwise sums of the coefficients are less than two, i.e., $\alpha_1 + \alpha_2 < 2$, $\alpha_3 + \alpha_4 < 2$, etc. This is shown in [9] to correspond mainly to pole locations within the right half-circle, i.e., where the second order LDIF has good noise properties. These two results indicate that allpass/lattice LDIFs in many cases are strong contenders to wave digital allpass/lattice filters.

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