MULTIUSER BLIND IDENTIFICATION USING A LINEAR PARAMETERIZATION OF THE CHANNEL MATRIX AND SECOND-ORDER STATISTICS *

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ABSTRACT

We observe that the channel matrix in the standard multiuser, multichannel (MIMO) digital communications model is linear in the channel coefficients. Also, recent work incorporating "basis functions" suggests that the multipath channel itself is in a subspace formed by delayed versions of the transmission pulse. Hence the channel matrix is linear in the coefficients of this subspace. We propose two algorithms based on the sample covariance matrix of the received signal (i.e., second-order statistics) that take advantage of this linear parameterization: a new identification algorithm that estimates the outer product of the model coefficients via multiplication by a predetermined matrix, and a multiuser version of the previously presented "subspace method" that employs knowledge of the transmission pulse. While both methods are superior to the original non-parameterized subspace method in terms of computation and performance, the new method requires less computation and in some cases outperforms the other.

1. INTRODUCTION

We consider a digital communications system employing multiple antennas to receive a fixed, known number of linearly modulated information sequences transmitted over different multipath channels. The multipath channels are assumed to be unchanging in time over a small number of symbols. Furthermore the channels are assumed to be made up of a small number (< 10) of propagation paths incurring delays uniformly distributed between 0 and the time delay spread, and that the delay spread is a non-negligible fraction of the symbol interval (large enough to not be considered "flat"). With this very specific type of channel model, each multipath channel (between a given user and antenna) is well approximated by a small number of delayed versions of the transmission pulse, the delays fixed and uniformly spaced across the duration of the multipath time delay spread. This model has been shown to be appropriate [6] for certain TDMA cellular standards used in practice, namely IS-136.

In the standard decomposition of the received data matrix into channel matrix times information symbol matrix plus noise $\mathbf{X} = \mathbf{HS} + \mathbf{N}$, the channel matrix $\mathbf{H}$ is linear in the elements of the channel vector. Since the channel vector is in the span of a known basis, there is a corresponding set of known basis *matrices* for the channel matrix $\mathbf{H}$, i.e. $\mathbf{H}$ is linearly parameterized. This new way of thinking about the channel matrix has lead us to a new algorithm and to the reformulation of an existing algorithm.

A classic method for blind channel identification is the "subspace" method of [4], which is based on the noise subspace of the received data space-time correlation matrix. This method has the advantageous ability to provide the exact channel when there is little or no noise, even for a relatively small number of symbols. However, the original subspace method doesn't use any knowledge of the basis functions. Also the method suffers when the channel matrix becomes ill-conditioned due to the simultaneous decay of the channel impulse responses, which is exactly what happens when a common transmission pulse is used.

The subspace method was modified to take advantage of the knowledge of the transmission pulse in [2]. In our paper, using the linear parameterization of the channel matrix, we derive essentially the same algorithm. However we take advantage of oversampling in a way not addressed in [2]. In particular, with P times oversampling, the P "virtual" channel responses for a given antenna, although in different subspaces formed by the P polyphase components of the delayed transmission pulses, have the same coefficients. While this leads to slightly less computation (since in the end we estimate P times fewer parameters), our initial experience does not indicate much of a performance enhancement.

We also present a new algorithm based on the expected form of $\mathbf{XX}^H$ ($(.)^H$ represents conjugate transpose). The outer-product matrix $\mathbf{HH}^H$ is in the span of the outer-products of the basis matrices of $\mathbf{H}$, where the coefficients are products of channel coefficients. Hence by simply solving a linear system, we can construct a matrix whose dominant eigenvector is made up of the coefficients of the channel in the known basis set.

The subspace method has been extended to multiple users in [1] and briefly outlined in [5]. For multiple users, independent linear combinations of the different user's channels are estimated. Inverting these channel estimates yields (approximately) instantaneous mixtures of the different user's channels that can be separated via various techniques, not addressed in this paper. Note that we are solving the problem of finding the unknown channel coefficients assuming an FIR model; we do not here address recovery of the information symbols (equalization).

In this paper, we first review the assumed communications system model which results in a linear relationship between the information symbols and the received data. We then develop the linear parameterization of the channel matrix, and present the two aforementioned algorithms. Finally, we provide numerical simulation results and discussion.

2. SYSTEM DESCRIPTION

We consider a communications system in which there are d users transmitting independent white information sequences $s^{(m)}[n], m = 1, ..., d$. This work assumes the channel duration is known to be approximately L symbol intervals, based on the known delay spread and pulse shape. The signals are

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received at an antenna array of M sensors and digitized at P times the baud rate, so the length L snapshot vector for the i-th antenna, p-th polyphase component, at time index n is

$$\mathbf{x}_{ip}[n] = \sum_{m=1}^{d} \mathbf{H}_{ip}^{(m)} \mathbf{s}^{(m)}[n] + \boldsymbol{\eta}_{ip}[n]$$

This vector is made up of additive filtered user sequences $\mathbf{s}^{(m)}[n] = [s^{(m)}[n], \ldots, s^{(m)}[n-(2L-2)]]^T$, and a noise term. See [3] for a more detailed description of the system.

Representing the channel coefficients for the i-th antenna, p-th polyphase component, and m-th user in the $L \times 1$ vector $h_{ip}^{(m)}[k]$ we have the filtering matrix of the form $\mathbf{H}_{ip}^{(m)} = T_L(\mathbf{h}_{ip}^{(m)})$ where

$$T_L(\mathbf{h}) = \begin{pmatrix} h[0] & \cdots & h[L-1] & 0 & \cdots & 0 \\ 0 & h[0] & \cdots & h[L-1] & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & h[0] & \cdots & h[L-1] \end{pmatrix}.$$

All of the channel snapshots for a given time index n are stacked on top of each other to form an $LMP \times 1$ vector

$$\mathbf{x}[n] = [\mathbf{x}_{1,1}[n]^T, \mathbf{x}_{1,2}[n]^T, ..., \mathbf{x}_{MP}[n]^T]^T$$

with a similar definition for $\boldsymbol{\eta}[n]$. This "super"-snapshot vector $\mathbf{x}[n]$ can be written as the product of the "channel matrix" with stacked snapshot vector $\mathbf{s}[n] = [\mathbf{s}^{(1)}[n]^T, \ldots, \mathbf{s}^{(d)}[n]^T]^T$ plus noise

$$\mathbf{x}[n] = \mathbf{H}\mathbf{s}[n] + \boldsymbol{\eta}[n]$$

The channel matrix $\mathbf{H}$ is an $MP \times d$ block matrix composed of $L \times (2L-1)$ blocks $\mathbf{H}_{ip}^{(m)}$, and can be written as

$$\mathbf{H} = \sum_{i=1}^{M} \sum_{p=1}^{P} \sum_{m=1}^{d} [(\mathbf{e}_i^{(M)} \otimes \mathbf{e}_p^{(P)}) \mathbf{e}_m^{(d)T}] \otimes \mathbf{H}_{ip}^{(m)}$$

where $\mathbf{e}_i^{(M)}$ is the i-th length M standard basis vector (a column) consisting of all zeros except a one in the i-th position, $(.)^T$ is the vector transpose, and $\otimes$ is the kronecker product.

3. LINEAR PARAMETERIZATION OF THE CHANNEL MATRIX

In the above derivation, we defined a matrix function of a vector $T_L(\mathbf{h})$, where $\mathbf{h}$ is a length-L channel impulse response vector. Note that $T_L(\mathbf{h})$ is linear. $\mathbf{h}$ may be decomposed in terms of a basis for C^L, for example, the standard basis, as $\mathbf{h} = \sum_{n=0}^{L-1} h(n)\mathbf{e}_{n+1}^{(L)}$. Using the linearity of T_L, $T_L(\mathbf{h}) = \sum_{n=0}^{L-1} h(n) T_L(\mathbf{e}_{n+1}^{(L)})$. From this we see that $T_L(\mathbf{h})$ is in the span of the basis *matrices* $T_L(\mathbf{e}_{n+1})$, $n = 0, 1, ..., L-1$.

The aim of the next two subsections is to detail and simplify the model by using a different basis, of lower dimensionality, for each length L channel vector. This is done by taking advantage of the knowledge of the Nyquist pulse, time-delay spread, and oversampling factor.

3.1. Channel Model

In [6] it is shown that, given a maximum time-delay spread τ_{max}, for which $\frac{\tau_{max}}{T} \approx \frac{1}{2}$, the multipath channel is highly concentrated in a subspace formed from only 3 uniformly delayed Nyquist pulses. This is regardless of the complex gains of each multipath arrival, assuming on the order of 10 dominant multipath rays at uniformly distributed random arrival times between 0 and τ_{max}. Using this fact, we here assume that the channel impulse response for the i-th antenna, m-th user, may be approximated to a high degree of accuracy as

$$h_i^{(m)}(t) = \sum_{n=0}^{N_h-1} h_{ci}^{(m)}[n] p_{rc}(t - nT_s)$$

with $T_s = T/Q$, where Q is an integer, and $N_h T_s \approx \tau_{max}$. The functions $p_{rc}(t - nT_s)$, $n = 0, 1, ..., N_h - 1$ are termed "basis functions" since the channel between the i-th antenna, m-th user is in the span of these linearly independent functions.

The length L vector $\mathbf{h}_{ip}^{(m)}$ consists of symbol-spaced samples of $h_i^{(m)}(t)$, starting at $t = p/T$,

$$h_{ip}^{(m)}[k] = h_i^{(m)}(kT + (p-1)T/P), \quad k = 0, ..., L-1 \quad (1)$$

This vector can be represented as a matrix-vector product between a matrix whose columns are the "basis functions" for this virtual channel, and a smaller vector $\mathbf{h}_{ci}^{(m)}$

$$\mathbf{h}_{ip}^{(m)} = \mathbf{B}_{f_p} \mathbf{h}_{ci}^{(m)}$$

where

$$\mathbf{h}_{ci}^{(m)} = \left[h_{ci}^{(m)}[0], h_{ci}^{(m)}[1], \ldots, h_{ci}^{(m)}[N_h - 1] \right]^T$$

and $\mathbf{B}_{f_p} =$

$$\begin{bmatrix} \tilde{p}(0) & \tilde{p}(-T_s) & \cdots & \tilde{p}(-(N_h-1)T_s) \\ \tilde{p}(T) & \tilde{p}(T - T_s) & \cdots & \tilde{p}(T - (N_h-1)T_s) \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{p}((L-1)T) & \tilde{p}((L-1)T - T_s) & \cdots & . \end{bmatrix}$$

Here we use $\tilde{p}(t)$ to denote an appropriately delayed version of the Nyquist pulse, e.g. $\tilde{p}(t) = p_{rc}(t - (p-1)T/P - LT/2)$, so that its peak is approximately in the middle element of each column.

Define the vector $\mathbf{h}_i^{(m)}$, the channel impulse response vector for the i-th antenna, m-th user, as the stack of all of its polyphase component vectors $\mathbf{h}_{ip}^{(m)}, p = 1, ..., P$: $\mathbf{h}_i^{(m)} = [\mathbf{h}_{i1}^{(m)T}, \ldots, \mathbf{h}_{iP}^{(m)T}]^T$. This vector may be written as

$$\mathbf{h}_i^{(m)} = \left[\mathbf{B}_{f_1}^T, \mathbf{B}_{f_2}^T, \ldots, \mathbf{B}_{f_P}^T \right]^T \mathbf{h}_{ci}^{(m)} \equiv \mathbf{B}_f \mathbf{h}_{ci}^{(m)}$$

The significance of this is that even though the different polyphase components $\mathbf{h}_{ip}^{(m)}, p = 1, ..., P$ of the i-th antenna are in different subspaces defined by the $\mathbf{B}_{f_p}$, the coefficients in each of these subspaces are the same! To our knowledge this has not been observed before, so is a new contribution in this work. It will lead to a slight computational advantage over assuming the different virtual channels of a given antenna are "uncoupled." However, we have experienced and present in [3] that taking advantage of the channel coupling does not significantly improve performance of the channel identification.

3.2. Simplified Linear Model

We now substitute the lower dimensional form $\mathbf{h}_i^{(m)} = \mathbf{B}_f \mathbf{h}_{ci}^{(m)}$ into the linear parameterization of the channel matrix, which will be the basis for the development of the algorithms in the following sections. Define a stacked Toeplitz operator, that operates on length LP vectors:

$$\tilde{T}_L(\mathbf{h}_i^{(m)}) = \left[T_L(\mathbf{h}_{i1}^{(m)})^T, \ldots, T_L(\mathbf{h}_{iP}^{(m)})^T \right]^T$$

As with the smaller T_L, this operator is linear so that

$$\tilde{T}_L(\mathbf{h}_i^{(m)}) = \tilde{T}_L(\mathbf{B}_f \mathbf{h}_{ci}^{(m)}) = \sum_{n=0}^{N_h-1} h_{ci}^{(m)}[n] \tilde{T}_L(\mathbf{b}_n)$$

where $\mathbf{b}_n, n = 0, \ldots, N_h - 1$, are the columns of $\mathbf{B}_f$.

Thus the channel matrix $\mathbf{H}^{(m)}$ of the m-th user can be written as a linear combination of known matrices, with the unknown parameters as the weights:

$$\mathbf{H}^{(m)} = \sum_{i=1}^{M} \sum_{n=0}^{N_h-1} h_{ci}^{(m)}[n] \mathbf{e}_i^{(M)} \otimes \tilde{T}_L(\mathbf{b}_n). \qquad (2)$$

4. ALGORITHM DEVELOPMENT

Since both the algorithms considered in this paper are based on second-order statistics, we begin their common development with a section describing the asymptotic form of the covariance matrix and how it is estimated. The subspace method was chosen for comparison purposes to the new algorithm, since it is well established and well understood.

4.1. Second-Order Statistics

The covariance matrix $\mathbf{R}_{xx}[m]$ of $\mathbf{x}[n]$ has the form

$$\mathbf{R}_{xx}[m] = E\{\mathbf{x}[n]\mathbf{x}^H[n-m]\} = \mathbf{H}\mathbf{R}_{ss}[m]\mathbf{H}^H + \mathbf{R}_{\eta\eta}[m]$$

where $(.)^H$ represents conjugate transpose, and $\mathbf{R}_{ss}[m]$ and $\mathbf{R}_{\eta\eta}[m]$ are the covariance matrices of the signal and noise respectively. Our approach is to estimate $\mathbf{R}_{xx}[0]$ and from it obtain an estimate of the channel based on its expected form. Since the information signals are independent and white (assuming unit variance without loss of generality), $\mathbf{R}_{ss}[0]$ is identity. Note that if different user's information signals are received with different power levels, the differences will manifest themselves as scalars multiplying the estimated channels. The covariance matrix $\mathbf{R}_{xx}[0]$ is estimated using N "super"-snapshot vectors to form the sample covariance matrix:

$$\hat{\mathbf{R}}_{xx}[0] = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{x}[n]\mathbf{x}^H[n] \qquad (3)$$

We assume the channel $\mathbf{H}$ does not change significantly over the interval required to collect N snapshots; this assumption will dictate the value of N in practical circumstances.

In order to separate the noise and signal subspaces, the channel matrix $\mathbf{H}$ must be "tall," i.e., we require $LMP > (2L-1)d$. While a noise subspace is required for the subspace algorithm, it is not yet clear that this is required for the new method presented in this paper, since it forms an outer-product of the coefficients without any eigen-decomposition of $\mathbf{R}_{xx}[0]$.

4.2. The New Method

Assume for simplicity that there is a single user, $d = 1$. Ignoring noise, and assuming $\hat{\mathbf{R}}_{xx}[0]$ takes its asymptotic form, we can use (2) to obtain $\mathbf{R}_{xx}[0] =$

$$\mathbf{H}\mathbf{H}^H = \sum_{i_1=1}^{M} \sum_{i_2=1}^{M} \sum_{n_1=0}^{N_h-1} \sum_{n_2=0}^{N_h-1} h_{ci_1}[n_1] h_{ci_2}^*[n_2] \mathbf{F}_{i_1,i_2}(n_1, n_2)$$

where

$$\mathbf{F}_{i_1,i_2}(n_1, n_2) = (\mathbf{e}_{i_1}^{(M)} \mathbf{e}_{i_2}^{(M)T}) \otimes (\tilde{T}_L(\mathbf{b}_{n_1}) \tilde{T}_L(\mathbf{b}_{n_2})^H).$$

Now consider $\mathbf{H}\mathbf{H}^H$ as a block matrix composed of M^2 blocks, each block of dimension $LP \times LP$. Note that for i_1, i_2 fixed, $\mathbf{F}_{i_1,i_2}(n_1, n_2)$ is mostly zeros with only the i_1, i_2 block non-zero. The i_1, i_2 block of $\mathbf{H}\mathbf{H}^H$ is the $LP \times LP$ matrix

$$\left[\mathbf{H}\mathbf{H}^H\right]_{i_1,i_2} = \sum_{n_1=0}^{N_h-1} \sum_{n_2=0}^{N_h-1} h_{ci_1}[n_1] h_{ci_2}^*[n_2] (\tilde{T}_L(\mathbf{b}_{n_1})) \tilde{T}_L(\mathbf{b}_{n_2}))^H),$$

which is a linear combination of known matrices, with the elements of the outer-product $\mathbf{h}_{ci_1}\mathbf{h}_{ci_2}^H$ as the weights.

Now introduce the *vec* operator which associates with any $m \times n$ matrix the $mn \times 1$ vector formed by stacking its columns. For fixed i_1, i_2, we have

$$\mathbf{B}_o vec(\mathbf{h}_{ci_1}\mathbf{h}_{ci_2}^H) = vec\left(\left[\mathbf{H}\mathbf{H}^H\right]_{i_1,i_2}\right)$$

where $\mathbf{B}_o$ is the "outer-product basis matrix," whose columns are vectorized basis matrices of the blocks of $\mathbf{H}\mathbf{H}^H$. In particular, the columns of $\mathbf{B}_o$ are

$$[\mathbf{B}_o]_k = [\mathbf{B}_o]_{n_2 N_h + n_1} = vec\left(\tilde{T}_L(\mathbf{b}_{n_1}) \tilde{T}_L(\mathbf{b}_{n_2}))^H\right),$$

for $k = 0, ..., N_h^2 - 1$, each of which is $(LP)^2 \times 1$.

The new method solves the system of linear equations

$$\mathbf{B}_o vec(\hat{\mathbf{h}}_{ci_1}\hat{\mathbf{h}}_{ci_2}^H) = vec\left(\left[\hat{\mathbf{R}}_{xx}[0]\right]_{i_1,i_2}\right)$$

for the outer-product of the channel coefficients, for each of the M^2 blocks. Since the matrix $\mathbf{B}_o$ is known in advance based on the knowledge of the basis functions, its pseudo-inverse $\mathbf{B}_o^\dagger$ can be precomputed and used to multiply the vectorized blocks of the covariance matrix estimate $\hat{\mathbf{R}}_{xx}[0]$. Since $\hat{\mathbf{R}}_{xx}[0]$ is hermitian, it is sufficient to compute the outer-product estimates for only its $M(M+1)/2$ blocks on the upper triangle.

Stacking the channel parameters into a vector $\mathbf{h}_c = [\mathbf{h}_{c1}^T, \mathbf{h}_{c2}^T, \ldots, \mathbf{h}_{cM}^T]^T$, the outer-product $\mathbf{h}_c\mathbf{h}_c^H$ is an $M \times M$ block matrix with blocks of size $N_h \times N_h$, with (i_1, i_2) block $\mathbf{h}_{ci_1}\mathbf{h}_{ci_2}^H$. We estimate $\mathbf{h}_c\mathbf{h}_c^H$ by plugging in the estimates of $\mathbf{h}_{ci_1}\mathbf{h}_{ci_2}^H$ for each block $1 \leq i_1, i_2 \leq M$. Then we can find $\mathbf{h}_c$, up to a unit magnitude complex scalar ambiguity, as the largest eigenvector of the resulting $MN_h \times MN_h$ matrix. From the estimated parameter vector $\hat{\mathbf{h}}_c$, we can recover the original channel by using the basis matrix $\mathbf{B}_f$.

When there are multiple users ($d > 1$), it can be shown that each user's channel contributes a rank-1 additive term to the outer-product matrix $\mathbf{h}_c\mathbf{h}_c^H$. In this case, the channels of the different users are determined, up to a d dimensional subspace ambiguity, as the d largest eigenvectors.

4.3. Derivation of Subspace Method using the Linear Parameterization

In this section we present an alternative derivation of an algorithm which is very closely related to that presented in [2]. In the subspace method, the noise eigenvectors of the covariance matrix $\hat{\mathbf{R}}_{xx}[0]$ are found. Under certain restrictions, it can be shown that $\mathbf{v}^H\mathbf{H} = 0$ for any vector $\mathbf{v}$ in the noise subspace. Let $\mathbf{V}$ be a matrix of vectors in the noise subspace. Using the linear parameterization of $\mathbf{H}$ (assuming again that $d = 1$ user), we have

$$\mathbf{V}^H\mathbf{H} = \sum_{i=1}^{M} \sum_{n=0}^{N_h-1} h_{ci}^{(m)}[n] \mathbf{V}^H \left[\mathbf{e}_i^{(M)} \otimes \tilde{T}_L(\mathbf{b}_n)\right] = \mathbf{0}. \qquad (4)$$

This is a linear combination of matrices with the unknown parameters as the weights. Using the *vec* operator, we can write this as

$$\mathbf{B}_v \mathbf{h}_c = 0$$

where $\mathbf{B}_v$ has columns that are the vectorized forms of the matrices in the summation in (4). Specifically, the columns of $\mathbf{B}_v$ are

$$[\mathbf{B}_v]_{(i-1)M+n} = vec\left(\mathbf{V}^H\left[\mathbf{e}_i^{(M)} \otimes \tilde{T}_L(\mathbf{b}_n)\right]\right),$$

for $n = 0, ..., N_h - 1$ and $i = 1, ..., M$. If we use all of the noise eigenvectors, then the noise subspace has dimension $LMP - (2L-1)d$, hence $\mathbf{B}_v$ is $(LMP-(2L-1)d)(2L-1) \times MN_h$. Our subspace method is to use the smallest right singular vector of $\mathbf{B}_v$ as our parameter vector estimate (up to an unknown scalar).

When there are multiple users ($d > 1$), the channels of the different users are determined, up to a d dimensional subspace ambiguity, as the d smallest right singular vectors of $\mathbf{B}_v$.

5. MATLAB SIMULATIONS

In our simulations we used QPSK modulation. The receiver has two antennas ($M = 2$) and employs two times oversampling ($P = 2$). The channel is a static five-ray multipath with a raised cosine pulse of excess bandwidth $\beta = 0.1$, truncated to $L = 7$ symbols. We assume each path arrives at all antennas at the same time. We generated 100 random channels with the 5 arrival times for each user uniformly distributed between 0 and $0.4T$, where T is the symbol interval. For a basis set, we started with $N_h = 3$ and found that the matrix $\mathbf{B}_f$ was very well approximated by its truncated SVD using only the two largest left and right singular vectors. So our basis set had $N_h = 2$, made up of the two largest singular vectors of $\mathbf{B}_f$.

The SNR is estimated as

$$SNR = 10log_{10}\frac{\sum_{m=1}^{M}\|\mathbf{x}_m\|^2}{\sum_{m=1}^{M}\|\mathbf{n}_m\|^2}.$$

where $\mathbf{x}_m$ is the noise-free received vector at antenna m and $\mathbf{n}_m$ is the noise alone at that antenna. The normalized mean square error $NMSE$ is given by

$$NMSE = \frac{\sum_{m=1}^{d}\sum_{i=1}^{M}\sum_{p=1}^{P}\|\hat{\mathbf{h}}_{ip}^{(m)} - \mathbf{h}_{ip}^{(m)}\|^2}{\sum_{m=1}^{d}\sum_{i=1}^{M}\sum_{p=1}^{P}\|\mathbf{h}_{ip}^{(m)}\|^2}$$

In the single user case $d = 1$, we estimate the aforementioned scalar ambiguity by doing a least squares fit of the estimated channel to the actual channel. Similarly, with multiple users $d > 1$, we express the estimated channel for each user as an unknown linear combination of the actual channels of the different users, and estimate the d^2 unknown scalars via least squares. Using this approach the $NMSE$ is a measure of how well the actual channel and the estimated channel span the same d-dimensional space.

We vary both the SNR and the number of symbols used in forming the covariance matrices, and plot the average $NMSE$ for $d = 1$ and $d = 2$ users (Figure 1). 100 trials were performed for each of the 100 channels. We note that each of the curves for the new method starts out better than for the modified subspace method at low SNR, but crosses over and performs worse (on average) at high SNR. This suggests this method can yield improved channel estimates in a noisy environment, while requiring less computation. The original subspace method performs far worse than either of the methods developed in this paper; see [3] for detailed results. Note the modified subspace method took less computation for 2 users due to the much smaller number of noise eigenvectors (2 vs. 15).

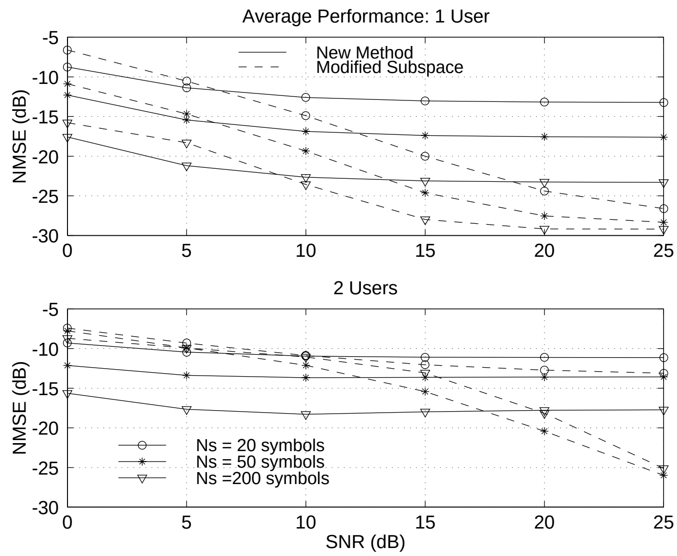

Figure 1. Average performance vs. SNR, 100 randomly generated channels, 100 trials each channel.

6. CONCLUSION

We have presented a computationally attractive blind channel estimation procedure that performs better on average than the subspace algorithm at low SNR. The eigendecomposition of the covariance matrix is the most demanding computational step in the subspace methods, and is avoided by the new method (see [3] for a detailed computational comparison). This method takes advantage of the knowledge of the Nyquist pulse and the structure of the channel matrix.

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