

# A BLIND PROJECTION RECEIVER FOR CODED CDMA SYSTEMS

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## ABSTRACT

This paper presents a blind adaptive CDMA receiver that requires no knowledge of the spreading codes, the delays, and the energy of the received signals associated with the interfering users. Our receiver is based on linear interference cancellation and adaptive interference signal subspace tracking. It has error control coding embedded in the detector structure and employs “branch processing” to detect the bit stream. Simulation results demonstrating that the receiver suffers negligible performance loss over systems with complete knowledge of the interfering users are presented in the paper.

## 1. INTRODUCTION

The purpose of this paper is to present a blind CDMA receiver based on the Projection Receiver (PR) principle [1][2], a method which cancels interference through orthogonal projection to the interfering signal subspaces. Our method offers the following features: (1) It employs “blind” algorithms that do not require knowledge of spreading codes, signal energies and time delays for all but one user of interest. (2) It is suitable for synchronous and asynchronous communication systems. (3) It incorporates error control coding into the receiver.

Blind multiuser receivers fit in the downlink CDMA systems where accurate information from other active users is hard to obtain, or inappropriate to know. Honig, Madhow and Verdu presented a minimum-output-energy (MOE) blind receiver in [3]. For a processing gain of  $N$ , the complexity of their system is  $O(N^2)$  arithmetical operations per sample for recursive least-squares adaptation and  $O(N)$  operations for least-mean-square adaptation. A blind decorrelating receiver and a blind minimum-mean-square-error (MMSE) receiver were developed by Wang and Poor [4]. They estimated the signal subspace parameters with complexity  $O(NK)$  operations, where  $K$  denotes the number of users in the channel. Our method, also having complexity  $O(NK)$ , only estimates the interfering signal subspaces. Unlike the two receivers cited above, our method combines a forward error control (FEC) decoder and signal detector in the receiver. When concatenated with FEC decoders, the receivers of [3] and [4] can generate soft metrics only on a per symbol basis, while our receiver can generate soft metrics either for symbols by “symbol processing” or for branch calculation. We assume that a branch equals  $n$  symbols, and that the code rate equals  $1/n$ . This work is also

a significant improvement over our own previous work [6]. The method in [6] operates on symbol metrics, and has performance capabilities similar to the method in [4]. In this work, by introducing “branch processing” we obtain theoretically provable and significant performance improvement over the method in [4] in asynchronous systems. Furthermore, the subspace estimation procedure is more robust than the method in [6] due to the use of delayed decision feedback of the output of the FEC decoder.

## 2. THE SYSTEM MODEL OF CDMA

We first study an antipodal  $K$ -user CDMA system operating in a synchronous additive white Gaussian noise (AWGN) channel. The results are extended to the asynchronous case in Section 5. The length  $N$  of the spreading code is assumed to be identical to the processing gain in this paper. Let  $\mathbf{c}_j$  represent  $N$ -element vector containing the spreading code of the  $j$ th user. The signal at the receiver input can be expressed in the form

$$\mathbf{r}[i] = \sum_{j=1}^K \sqrt{E_j} d_j[i] \mathbf{c}_j + \mathbf{n}[i], \quad (1)$$

where  $\mathbf{r}[i]$  is an  $N$ -dimensional vector containing the received signal over one symbol period,  $E_j$  is the received energy of the  $j$ th user,  $d_j[i]$  denotes the data of  $j$ th user at symbol index  $i$  that are independent and equally likely to be  $-1$  or  $+1$ , and  $\mathbf{n}[i]$  represents a  $N$ -dimensional white Gaussian noise process with covariance matrix given by  $\sigma^2 \mathbf{I}$ .

Let  $\mathbf{C} = [\mathbf{c}_1, \mathbf{c}_2, \dots, \mathbf{c}_K]$  be an  $(N \times K)$ -element matrix whose columns are the spreading codes of the  $K$  users in the network,  $\mathbf{A} = \text{diag}[\sqrt{E_1}, \sqrt{E_2}, \dots, \sqrt{E_K}]$ , and  $\mathbf{d}[i] = [d_1[i], d_2[i], \dots, d_K[i]]^T$ , where  $(\cdot)^T$  denotes the transpose of  $(\cdot)$ . The input-output relationship in (1) can now be expressed in matrix form as

$$\mathbf{r}[i] = \mathbf{C} \mathbf{A} \mathbf{d}[i] + \mathbf{n}[i]. \quad (2)$$

Without loss of generality, we assume that the spreading code of the desired user is  $\mathbf{c}_1$ . Define  $\mathbf{C}_u$  as the matrix for the rest of the spreading codes, *i.e.*,  $\mathbf{C}_u = [\mathbf{c}_2, \dots, \mathbf{c}_K]$ . Similarly, let  $\mathbf{A}_u = \text{diag}[\sqrt{E_2}, \dots, \sqrt{E_K}]$ , and  $\mathbf{d}_u[i] = [d_2[i], \dots, d_K[i]]$ . Then,

$$\mathbf{y}[i] = \mathbf{C}_u \mathbf{A}_u \mathbf{d}_u[i] + \mathbf{n}[i]. \quad (3)$$

is the interference component from the other users and the channel noise in the received signal  $\mathbf{r}[i]$ .

### 3. THE PROJECTION RECEIVER

The projection receiver (PR) is a near-far resistant multiuser receiver that cancels the interference without explicitly detecting the transmitted data of the interfering users [1][2]. As shown in Figure 1, the PR utilizes a metric generator to collect *soft* metric information for a forward error control (FEC) decoder. For a synchronous CDMA system,

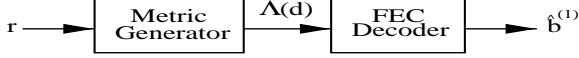


Figure 1: The projection receiver.

the metric generator computes

$$\Lambda(\hat{d}_1[i]) = \left\| \mathbf{M}(\mathbf{r}[i] - \mathbf{c}_1 \sqrt{E_1} \hat{d}_1[i]) \right\|^2, \quad (4)$$

where

$$\mathbf{M} = \mathbf{I} - \mathbf{C}_u (\mathbf{C}_u^T \mathbf{C}_u)^{-1} \mathbf{C}_u^T \quad (5)$$

for each hypothesis  $\hat{d}_1[i]$  in the set  $\{+1, -1\}$ . These metrics are then used by the FEC decoder to estimate the bit sequence of the desired user. The operator  $\mathbf{C}_u (\mathbf{C}_u^T \mathbf{C}_u)^{-1} \mathbf{C}_u^T$  projects its input vector onto the subspace of the interfering signals. Consequently, the matrix  $\mathbf{M}$  is the projection matrix onto the null space of  $\mathbf{C}_u$ .

### 4. A BLIND SUBSPACE TRACKING PROJECTION RECEIVER

The projection receiver requires knowledge of the spreading codes of the interfering users in order to determine the projection matrix  $\mathbf{M}$ . We now show that knowledge of the interfering spreading codes is not necessary since the subspace generated by the interfering signals can be estimated from the received signal.

Let  $\mathbf{Q}$  be an orthonormal  $(N \times (K-1))$ -element matrix and let  $\mathbf{C}_u = \mathbf{Q}\mathbf{V}$ , where  $\mathbf{V}$  is an upper triangular matrix containing  $((K-1) \times (K-1))$  elements. If the column rank of  $\mathbf{C}_u$  is equal to  $K-1$ , the upper triangular matrix  $\mathbf{V}$  will be nonsingular. Using the relationship  $\mathbf{C}_u = \mathbf{Q}\mathbf{V}$  in (5), we can show that  $\mathbf{M}$  satisfies the relationship

$$\mathbf{M} = \mathbf{I} - \mathbf{Q}\mathbf{Q}^T. \quad (6)$$

This result implies that the metrics in (4) can be calculated without knowledge of the spreading codes of the other users if an orthogonal basis set for the column vectors of  $\mathbf{C}_u$  can be evaluated. We can adaptively track the basis vectors using a subspace tracking algorithm [5].

#### 4.1. Adaptive Subspace Tracking

We assume that  $E \{ \mathbf{d}[i] \mathbf{d}^T[i] \} = \mathbf{I}$ , where  $E \{ \cdot \}$  denotes the statistical expectation of  $\{ \cdot \}$ . It follows from (3) that the correlation matrix of the observation vector is given by

$$\mathbf{R} = E \{ \mathbf{y}[i] \mathbf{y}^T[i] \} = \mathbf{C}_u \mathbf{A}_u^2 \mathbf{C}_u^T + \sigma^2 \mathbf{I}. \quad (7)$$

Let  $\mathbf{R} = \mathbf{U}\Sigma\mathbf{U}^T$ , where  $\Sigma = \text{diag}(\lambda_1, \dots, \lambda_N)$  is a diagonal matrix containing the eigenvalues of  $\mathbf{R}$  and  $\mathbf{U} = [\mathbf{u}_1, \dots, \mathbf{u}_N]$  is a unitary matrix whose columns define the

eigenvectors of  $\mathbf{R}$ . Let  $r = K-1$  denote the rank of  $\mathbf{C}_u \mathbf{A}_u$ . The eigenvalues are arranged in descending order such that

$$\lambda_1 \geq \dots \geq \lambda_r > \lambda_{r+1} = \dots = \lambda_N = \sigma^2. \quad (8)$$

It is straightforward to show that the smallest  $N-r+1$  eigenvalues are equal to the noise variance  $\sigma^2$ . The eigenvectors associated with these eigenvalues are the noise eigenvectors of  $\mathbf{R}$ . Similarly, the eigenvectors associated with the largest  $r$  eigenvalues are the signal eigenvectors of  $\mathbf{R}$ . Let  $\mathbf{U}_s$  be a matrix whose column vectors correspond to the signal eigenvectors, *i.e.*,

$$\mathbf{U}_s = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_{K-1}]. \quad (9)$$

Since  $\mathbf{U}_s$  and  $\mathbf{C}_u \mathbf{A}_u$  have the same column span, and since  $\mathbf{A}_u$  is a diagonal matrix,  $\mathbf{U}_s$  and  $\mathbf{C}_u$  have the same column span. In other words, any  $\mathbf{c}[i]$  for  $i = 1, 2, \dots, K-1$  can be represented by a linear combination of the column vectors of  $\mathbf{U}_s$ .

The optimal least-squares estimate of an input vector  $\mathbf{y}[i]$  in the signal subspace is given by [5]

$$\hat{\mathbf{y}}[i] = \mathbf{U}_s \mathbf{U}_s^T \mathbf{y}[i]. \quad (10)$$

We can now develop an iterative algorithm to estimate the signal subspace that attempts to minimize the cost function as

$$J(\mathbf{W}) = E \left\{ \left\| \mathbf{y}[i] - \mathbf{W}\mathbf{W}^T \mathbf{y}[i] \right\|^2 \right\}. \quad (11)$$

where  $\mathbf{W}$  is a  $(N \times (K-1))$ -element matrix of orthonormal column vectors. In this work we employ the recursive least-squares algorithm described in [5] to adaptively track the subspaces associated with the residue vector  $\mathbf{y}[i]$  online. This algorithm is given in Table 1. The parameter  $\beta$  in the table is a positive constant that is close to but smaller than one. We note that the time index  $i$  has been added to  $\mathbf{W}$  to indicate that the calculations are made during each symbol intervals. Yang [5] has proved that the cost function  $J(\mathbf{W})$  achieves a global minimum whenever  $\mathbf{W} = \mathbf{U}_s \mathbf{T}$ , where  $\mathbf{T}$  is an arbitrary unitary matrix. Although  $\mathbf{W}$  is not a unique solution for the minimization problem, the outer product  $\mathbf{W}\mathbf{W}^T$  is unique and is equal to the signal subspace projection matrix  $\mathbf{U}_s \mathbf{U}_s^T$ .

With an orthonormal basis  $\mathbf{W}[i]$  for the interference signal space available at the  $i$ th symbol interval, the metric in (4) can be written as

$$\Lambda(\hat{d}_1[i]) = \left\| (\mathbf{I} - \mathbf{W}[i] \mathbf{W}^T[i]) (\mathbf{r}[i] - \mathbf{c}_1 \sqrt{E_1} \hat{d}_1[i]) \right\|^2, \quad (12)$$

and therefore, we can implement the projection receiver using the estimated subspaces without any knowledge of the spreading waveforms employed by users other than the one of interest.

Recall that the input to the subspace tracker is not the received signal  $\mathbf{r}[i]$  but the residual signal  $\mathbf{y}[i]$ . An estimate of the component  $\mathbf{c}_1 \sqrt{E_1} \hat{d}_1[i]$  of the desired user must be removed from  $\mathbf{r}[i]$  before subspace tracking is attempted. Since the FEC decoder estimates the bits with an inherent delay of  $t$  bit intervals, we can use a delayed estimate of the residual signal obtained as

$$\hat{\mathbf{y}}[i-t] = \mathbf{r}[i-t] - \mathbf{c}_1 \sqrt{E_1} \hat{d}_1[i-t] \quad (13)$$

Table 1: The recursive least-square algorithm for subspace tracking.

<b>Initialization</b>	
$\mathbf{P}[i] =$	$\mathbf{I}_{(K-1) \times (K-1)}$
<b>Main Iteration</b>	
$\mathbf{z}_i[i] =$	$\mathbf{W}^T[i-1]\mathbf{y}[i]$
$\mathbf{h}[i] =$	$\mathbf{P}[i-1]\mathbf{z}_i[i]$
$\mathbf{g}[i] =$	$\frac{\mathbf{h}[i]}{\beta + \mathbf{z}_i^T[i]\mathbf{h}[i]}$
$\mathbf{P}[i] =$	$\frac{1}{\beta}(\mathbf{P}[i-1] - \mathbf{g}[i]\mathbf{h}^T[i])$
$\mathbf{e}[i] =$	$\mathbf{y}[i] - \mathbf{W}^T[i-1]\mathbf{z}_i[i]$
$\mathbf{W}[i] =$	$\mathbf{W}[i-1] + \mathbf{e}[i]\mathbf{g}^T[i]$

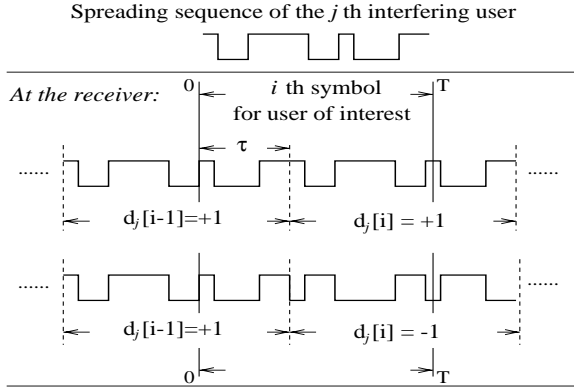


Figure 2: Two distinct possibilities for the spreading sequence coming from the  $j$ th user, as seen by a synchronous receiver in asynchronous transmission.

as the input to the subspace tracker. This procedure assumes that the statistics of the signals changes slowly over the  $t$  bit intervals.

## 5. AN ASYNCHRONOUS CDMA RECEIVER

If the signal timing associated with the user of interest is known, and the other users transmit asynchronously, we can use the blind adaptive receiver described in the previous section by treating each interfering user of an asynchronous CDMA system as two virtual synchronous users. Figure 2 displays the relative timing associated with the spreading codes of the  $j$ th user when the signal from that user arrives with a delay of  $\tau$  seconds relative to the user of interest. For a fixed value of  $\tau$ , the interference from the  $j$ th user in the  $i$ th symbol interval may take one of two possible forms, depending on whether the two symbols  $d_j[i-1]$  and  $d_j[i]$  have the same or different signs. Thus, in an asynchronous environment, one interfering user can be equivalently viewed as two synchronous users that employ different spreading codes. Without any modification we could use the blind, adaptive receiver described in the previous section for asynchronous CDMA systems. However, this receiver for asynchronous CDMA systems only allows the maximum number of user to be half the spreading gain because it operates on symbol metrics. This drawback can be easily mitigated in our blind receiver by operating on branch metrics. Without loss of generality, let us assume

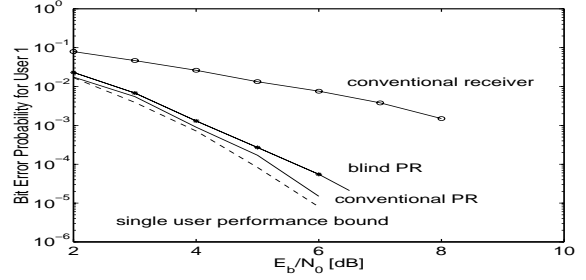


Figure 3: Synchronous CDMA, gold codes ( $N = 15$ ): Bit error probability for different receivers with  $K = 8$  users.

that the code rate of the forward control coding is  $1/n$  such that a branch includes  $n$  symbols. Instead of observing one symbol at a time in Figure 2, we may process one branch at a time. Each asynchronous user contributes  $n + 1$  independent forms of sequences during such a branch period. Consequently, the interfering signal subspaces will be of size  $nN \times nN$  with rank  $(n + 1)(K - 1)$ . Thus, the receiver operating on branch metrics allows more users in a CDMA system than the receiver operating on symbol metrics. Furthermore, the receiver operating on branch metrics has better bit-error-probability performance than receivers operating on symbol metrics. A proof of this statement for  $n = 2$  case is given in the Appendix.

## 6. EXPERIMENTAL RESULTS

Experiments for finding the bit error rates for the desired user were first performed in a synchronous CDMA system with spreading gain  $N = 15$  for different signal-to-noise ratios. All the systems used a forward error control code defined by a four-state convolutional code with generator

matrix  $\mathbf{g} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ . A set of *Gold* codes were used as

the spreading codes. We found the probability of bit errors in each experiment by counting up to 100 errors and dividing the number of errors by the number of bits transmitted before they occurred. Figure 3 compares the performance of the blind adaptive projection receiver with the performance of the original projection receiver with complete knowledge of the spreading codes of all users and a conventional single-user receiver (a single matched filter followed by a Viterbi decoder) when eight users share the CDMA channel. The signal-to-interference-ratio (*SIR*) was set to be  $-10$  dB, i.e., the signal energy of each interfering user appears 10 dB stronger than the desired signal energy at the receiver end. The forgetting factor  $\beta$  was chosen to be 0.9999. We can see from the results that the performances of the blind and the conventional projection receivers are not much different. Both the projection receivers perform significantly better than the conventional single-user receiver and reasonably close to the single user performance bound.

Another set of experiments were performed in a four-user asynchronous CDMA environment to compare the performance of the conventional single-user receiver, the receiver suggested in [3], the original PR and the blind PR. We again used *Gold* codes as the spreading codes and set the signal-to-interference-ratio to be  $-10$  dB. The delays between the desired user and the interfering users were 4,

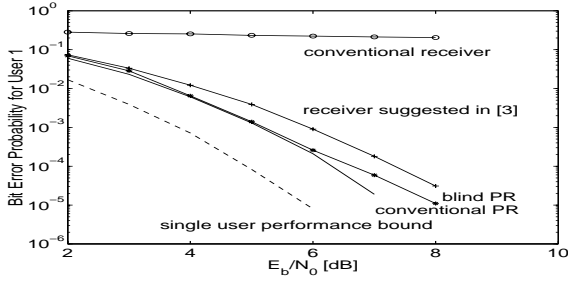


Figure 4: Asynchronous CDMA, gold codes ( $N = 15$ ): Bit error probability for different receivers with  $K = 4$  users.

7 and 11 chip lengths, respectively. The blind projection receiver operated on branch metrics with  $n = 2$ . We implemented the receiver in [3] using the RLS algorithm as well as error control coding. Figure 4 shows that the blind projection receiver has relatively small performance loss over the projection receiver that requires knowledge of the interfering users' spreading codes as well as time delays in an asynchronous CDMA system. Our receiver also outperforms the receiver suggested in [3].

## 7. CONCLUDING REMARKS

A blind adaptive subspace tracking projection receiver was presented in this paper. This receiver does not require knowledge of the interfering users' spreading codes, but performs close to the method that requires complete knowledge of every user's spreading code. Our system can be used in asynchronous CDMA systems without modification of the algorithm. Simulation results show that our receiver has performance capabilities that are comparable to those of multiuser detectors employing perfect knowledge of all the users' spreading codes. Our present research on this area is focused on *asynchronous* CDMA detection in *multipath* and *fading* channels as well as acquisition issues.

## 8. ACKNOWLEDGMENT

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## A. APPENDIX

The bit-error-probability based on PR symbol metrics is given by [2]

$$P_e = Q \left( \sqrt{\frac{4|\mathbf{M}\mathbf{c}_1|^2}{2N_0}} \right), \quad (14)$$

where  $N_0 = 2\sigma^2$ . Compared with the single-user bound, the performance loss of the PR in decibels is given by

$$L_{PR} = 10 \log_{10} \left( \frac{4|\mathbf{M}\mathbf{c}_1|^2}{4|\mathbf{c}_1|^2} \right) = 10 \log_{10} \left( \frac{\mathbf{c}_1^T \mathbf{M} \mathbf{c}_1}{\mathbf{c}_1^T \mathbf{c}_1} \right), \quad (15)$$

where the second equation follows the fact that  $\mathbf{M}$  is idempotent, *i.e.*,  $\mathbf{M}^T \mathbf{M} = \mathbf{M}$ . Note that a projection receiver operating on symbol basis has identical performance as the decorrelating receiver [7]. For  $n = 2$ , let us define  $\mathbf{C}_{u2}$  as the  $2N \times 3(K - 1)$  matrix containing all the independent

sequences of the interfering users in a branch period, and the projection matrix  $\mathbf{M}_2$  as

$$\mathbf{M}_2 = \mathbf{I}_2 - \mathbf{C}_{u2} (\mathbf{C}_{u2}^T \mathbf{C}_{u2})^{-1} \mathbf{C}_{u2}^T, \quad (16)$$

where  $\mathbf{M}_2$  and the identity matrix  $\mathbf{I}_2$  both contain  $2N \times 2N$  elements. In order to prove that the receiver operating on branch metrics has better performance than the one operating on symbol metrics, we need to show that

$$\frac{|\mathbf{M}\mathbf{c}_1|}{|\mathbf{c}_1|} = \frac{\left\| \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix} \begin{bmatrix} \mathbf{c}_1 \\ \pm \mathbf{c}_1 \end{bmatrix} \right\|}{\left\| \begin{bmatrix} \mathbf{c}_1 \\ \pm \mathbf{c}_1 \end{bmatrix} \right\|} < \frac{\left\| \mathbf{M}_2 \begin{bmatrix} \mathbf{c}_1 \\ \pm \mathbf{c}_1 \end{bmatrix} \right\|}{\left\| \begin{bmatrix} \mathbf{c}_1 \\ \pm \mathbf{c}_1 \end{bmatrix} \right\|}. \quad (17)$$

Let  $\mathbf{C}_{eqv} = \begin{bmatrix} \mathbf{C}_u & \mathbf{C}_u \\ \mathbf{C}_u & -\mathbf{C}_u \end{bmatrix}$ . Then

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix} = \mathbf{I}_2 - \mathbf{C}_{eqv} (\mathbf{C}_{eqv}^T \mathbf{C}_{eqv})^{-1} \mathbf{C}_{eqv}^T. \quad (18)$$

The columns of the matrix  $\mathbf{C}_{eqv}$  spans a  $2N$ -dimensional space and  $\mathbf{C}_{eqv}$  has rank  $4(K - 1)$ .  $\mathbf{C}_{u2}$  also spans a  $2N$  space, but has rank  $3(K - 1)$ . Careful consideration of the structures of  $\mathbf{C}_{u2}$  and  $\mathbf{C}_{eqv}$  will show that the column span of  $\mathbf{C}_{u2}$  is a subset of the column span of  $\mathbf{C}_{eqv}$ . This implies that the nullspace associated with  $\mathbf{C}_{eqv}$  is a subset of the nullspace associated with  $\mathbf{C}_{u2}$ . Since  $\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix}$  and  $\mathbf{M}_2$  are projection matrices into these null spaces, it follows that the magnitude of the projection is larger with  $\mathbf{M}_2$  than with  $\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix}$ . This proves the inequality of (17). The result for  $n = 2$  can be easily extended to arbitrarily values of  $n$ .

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