A NEW APPROACH FOR SYMBOL FRAME SYNCHRONIZATION AND CARRIER FREQUENCY ESTIMATION IN OFDM COMMUNICATIONS

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ABSTRACT

This work considers the problem of jointly estimating symbol frame boundaries and carrier frequency offsets for orthogonal frequency division multiplexed (OFDM) communications in frequency selective fading environments. Orthogonality between the modulated and virtual carriers over an interference free window of the received signal is used to develop an algorithm for estimating the carrier frequency offset and detecting the beginning of a symbol frame. By using a cyclic prefix to remove interference from neighboring frames, the the method is applicable in the presence of dispersive channels. The main contribution of this work is the joint estimation of the frequency offset and the frame boundary.

1. INTRODUCTION

In orthogonal frequency division multiplexing (OFDM) systems, a set of orthogonal subcarriers are modulated during each transmission. It is the orthogonality between the subcarriers that assures the prefect reconstruction of the symbol stream at the receiver. Even a small difference between the oscillators in the transmitter and receiver causes a loss of orthogonality between subcarriers resulting in intercarrier interference (ICI). Therefore, the carrier frequency offset must be estimated very accurately and compensated for at the receiver before demodulation.

Before demodulation, the OFDM transmitted signal consists of a sequence of long (relative to the sample rate) frames. The receiver must decide when the start of a frame occurs. A dispersive channel complicates this decision by causing interference from consecutive frames (interframe interference-IFI). In OFDM, a common mechanism for coping with IFI is to insert a cyclic prefix at the beginning of the frame. Then the IFI is confined to the prefix region. Deleting samples of the received signal corresponding to the prefix removes IFI. However, knowing which samples to delete requires frame timing information. The problem we address in this paper is the joint estimation of the carrier frequency offset and frame timing in the presence of a dispersive channel. The methods that have been proposed for this purpose in the literature can be classified into two groups. The first group of estimators estimate the frame timing and frequency offset using pilots or training signals [1, 2, 3, 4, 5, 6]. These methods decrease bandwidth efficiency since the pilots must occupy part of the useful bandwidth.

Several investigators recognized that pilots were not necessary since the OFDM signal already contains rich structural information which is sufficient to estimate timing and frequency parameters [7, 8, 9, 10, 11]. A new approach was taken in [12]. It falls into the second category of estimators which exploit OFDM signal structure. In [12] orthogonality between the modulated and unmodulated "virtual carriers" is exploited. If the transmit and receive oscillators are perfectly synchronized, the modulated carriers in the received signal and virtual carriers are orthogonal. The degree to which these two sets of subcarriers are orthogonal then is a measure of how far out of synchronization the receiver oscillator is. A criterion is established which has a unique minimum at the correct frequency offset. Closed form and adaptive methods are then suggested to minimize the criterion. The work in [12] addresses the problem of carrier frequency offset estimation assuming that the frame timing is already known. In practice, the frequency offset and the timing must be estimated jointly. We build upon the work in [12] in this paper and extend the work to jointly estimate both frame timing and carrier frequency offset.

2. PROBLEM FORMULATION

In this section, we illustrate how frame timing error, carrier frequency offset, and the fading channel destroy orthogonality between the virtual and the modulated subcarriers contained in the transmitted signal. This investigation will guide the choice of an optimization criterion and an algorithm for detecting frame boundaries and estimating the carrier frequency offset. In the following $\mathbf{A}^T$ and $\mathbf{A}^*$ denote the transpose and complex conjugate transpose of a matrix $\mathbf{A}$.

Define the $P \times 1$ OFDM symbol vector by

$$\mathbf{s}_k = [s_k(1), \cdots, s_k(P)]^T,$$

the $N \times 1$ prefix-free frame signal vector by

$$\mathbf{x}_k = [x_k(1), \cdots, x_k(N)]^T$$

and let $\mathbf{W}$ be the $N \times N$ unitary DFT matrix ($\mathbf{W}^*\mathbf{W} = \mathbf{W}\mathbf{W}^* = \mathbf{I}$). The $(k,\ell)^{th}$ element of $\mathbf{W}$ is $\mathbf{W}_{k,\ell} = e^{j\frac{2\pi(k-1)(\ell-1)}{N}}$ for $k, \ell = 1, \cdots, N$. Let the P columns of the $N \times P$ matrix $\mathbf{W}_p$ be the columns of $\mathbf{W}$ corresponding to the modulated subcarriers and collect the remaining $N-P$ columns into $\mathbf{W}_\perp$. The columns of $\mathbf{W}_p$ contain N samples of the P modulated subcarriers while $\mathbf{W}_\perp$ contain the virtual carriers. Because $\mathbf{W}$ is unitary, the virtual carriers are orthogonal to the modulated carriers

$$\mathbf{W}_\perp^* \mathbf{W}_p = 0. \tag{1}$$

This is the key relationship that we exploit to obtain synchronization at the receiver.

Using the notation from above, the modulated OFDM signal in the k^{th} frame can be writen as

$$\mathbf{x}_k = \mathbf{W}_p \mathbf{s}_k.$$

Assume that the OFDM signal passes through a frequency selective fading channel with finite impulse response $h(n)$ of length L. To avoid interference between consecutive OFDM frames, a cyclic prefix can be inserted at the beginning of the OFDM frame prior to transmission. If frame timing information is available at the receiver, the samples of the received signal corresponding to the cyclic prefix can be deleted. After removing these samples, the received signal can be writen as

$$\mathbf{y}_k = \mathbf{W}_p \mathbf{H}_p \mathbf{s}_k \tag{2}$$

where $\mathbf{H}_p$ is a $P \times P$ diagonal matrix containing the samples of the N-point DFT of $h(n)$ corresponding to the P modulated subcarriers. The mathematical form of (2) arises from the use of a cyclic prefix rather than an arbitrary one. Deleting the cyclic prefix converts the linear convolution of the channel into a circular one and the effect of the channel can be writen as a complex scaling of the symbols as in (2). This is elaborated on in [13]. Note that (2) represents the ideal case where the received signal is not corrupted by additive noise and frame timing is available. Under these conditions the received OFDM signal is orthogonal to the virtual subcarriers

$$\mathbf{W}_\perp^* \mathbf{y}_k = \underbrace{\mathbf{W}_\perp^* \mathbf{W}_p}_{\mathbf{0}} \mathbf{H}_p \mathbf{s}_k = 0$$

Next, consider the situation where there is a frequency offset, ϕ between the transmitter and receiver oscillators. Then the received signal becomes

$$\mathbf{y}_k = \mathbf{E}(\phi) \mathbf{W}_p \mathbf{H}_p \mathbf{s}_k \tag{3}$$

where $\mathbf{E}(\phi) = \text{diag}\left(1, e^{j\phi}, \cdots, e^{j\phi(N-1)}\right)$. If the offset ϕ were known, then the received OFDM signal is orthogonal to a modulated version of the virtual subcarriers $\mathbf{E}(\phi)\mathbf{W}_\perp$ as follows

$$\begin{aligned} \mathbf{W}_\perp^* \mathbf{E}^*(\phi) \mathbf{y}_k &= \mathbf{W}_\perp^* \mathbf{E}^*(\phi) \mathbf{E}(\phi) \mathbf{W}_p \mathbf{H}_p \mathbf{s}_k \\ &= \mathbf{W}_\perp^* \mathbf{W}_p \mathbf{H}_p \mathbf{s}_k = 0 \end{aligned}$$

where we used the fact that $\mathbf{E}^*(\phi)\mathbf{E}(\phi) = \mathbf{I}$. In this equation, if ϕ is not the correct frequency offset, then $\mathbf{y}_k$ is not orthogonal to $\mathbf{E}(\phi)\mathbf{W}_\perp$. This leads to the idea that ϕ can be estimated by minimizing the cost function

$$J(\phi) = \|\mathbf{W}_\perp^* \mathbf{E}^*(\phi) \mathbf{y}_k\|$$

which is precisely the approach taken in [12].

It is important to note that in order to exploit orthogonality, we had to know the frame boundries. In particular, the elements of $\mathbf{y}_k$ are consist of N samples of the received signal which are not corrupted by interference from adjacent OFDM frames. The cyclic prefix was inserted to avoid this interference. However, we had to know which samples corresponded to the cyclic prefix in order to delete them. A general expression for an N sample window of the received signal at time n is more complicated than (3) owing to the fact that it may contain contributions from the k^{th} and the $(k-1)^{th}$ frames. A general expression for $\mathbf{y}(n)$ is derived in [13]. The point is that $\mathbf{y}(n)$ will only be orthogonal to $\mathbf{E}(\phi)\mathbf{W}_\perp$ under two conditions. First, ϕ must equal the correct frequency offset. Second, $\mathbf{y}(n)$ must be an interframe interference (IFI) free N sample window of the received signal. Both of these conditions are required for othogonality. This discussion suggests a two step approach to estimating the frame boundry n_o and the offset ϕ. First, hypothesize that $n = n_o$ and estimate ϕ. Denote this estimate by $\hat{\phi}$. Then evaluate the criterion

$$J(n, \hat{\phi}) = \|\mathbf{W}_\perp^* \mathbf{E}^*(\hat{\phi}) \mathbf{y}(n)\|.$$

If $J(n, \hat{\phi}) = 0$ then we know that $\mathbf{y}(n)$ is an interference free N-sample window of data. Hence, we can decide $n = n_o$ is the start of the frame and we also have $\hat{\phi}$ an estimate of the frequency offset. Basically, we have suggested computing a frequency offset estimate using a sliding N-sample window on the data. When $J(n, \hat{\phi})$ is equal to zero, we obtain synchronization. Note that only a finite number of steps is necessary since we know that $1 \leq n_o \leq N+L$. Of course, this method relies on the availability of an accurate estimator for ϕ which is computationally efficient since the estimates must be recomputed. In the next section we briefly review the method proposed in [12] which gives very high resolution estimates of ϕ but can be computationally demanding. We suggest a modification to this method which can be implemented more efficiently.

3. CARRIER OFFSET ESTIMATION

In this section, we review a method for high resolution carrier frequency offset estimation that was originally presented in [12]. We begin by assuming that the receiver is frame synchronized to the transmitter but not carrier synchronized. Let $\mathbf{y}$ denote an interference free N-sample window of data. From the discussion in the previous section, we know that the criterion $\|\mathbf{W}_\perp^* \mathbf{E}^*(\phi)\mathbf{y}\|$ will be minimized when ϕ is equal to the carrier offset. This is the approach taken in [12]. Let $z = e^{j\phi}$ and $\mathbf{E}(z) = \text{diag}(1, z, z^2, \cdots, z^{(N-1)})$. Then the criterion

$$P(z) = \|\mathbf{W}_\perp^* \mathbf{E}^*(z)\mathbf{y}\|^2 = \mathbf{y}^* \mathbf{E}(z) \mathbf{W}_\perp \mathbf{W}_\perp^* \mathbf{E}^*(z) \mathbf{y} \tag{4}$$

is an order $2N-1$ polynomial in z. Upon computing the roots of $P(z)$, the carrier frequency offset can be determined by the unique root which lies on the unit circle $z = e^{j\phi}$. Rooting may be avoided by a fine grained search of the cost function on the unit circle. As reported in [12], this procedure has excellent performance giving mean square errors close to the Cramer-Rao bound. Next, we show how to obtain closed form estimates of ϕ which avoid polynomial rooting or searching.

We begin by rewriting the cost function as follows

$$P(z) = \mathbf{y}^* \mathbf{E}(z) \mathbf{W}_\perp \mathbf{W}_\perp^* \mathbf{E}^*(z) \mathbf{y} = \mathbf{z}^T(\phi) \mathbf{G} \bar{\mathbf{z}}(\phi) \tag{5}$$

where $\mathbf{z}(\phi) = [1, z, z^2, \cdots, z^{(N-1)}]^T, z = e^{j\phi}$,

$$\mathbf{G} = (\mathbf{W}_\perp \mathbf{W}_\perp^*) \odot \left(\bar{\mathbf{y}} \mathbf{y}^T\right), \tag{6}$$

$\odot$ denotes element-wise multiplication, and the overbar denotes element-wise complex conjugation. Next we relax the constraint that $\mathbf{z}(\phi)$ be structured and simply compute the minimizing solution to

$$\hat{\mathbf{z}} = \arg\min_{\|\mathbf{z}\|=1} \mathbf{z}^T \mathbf{G} \bar{\mathbf{z}} \tag{7}$$

The solution is given by the eigenvector of $\mathbf{G}$ corresponding to the smallest eigenvalue. Note that the smallest eigenvalue is zero (at least when there is no noise) since $P(e^{j\phi}) = 0$. With one data vector, the rank of $\mathbf{G}$ in (6) is $N-P$. Hence, there are P linearly independent vectors that minimize (7). We can build the rank of $\mathbf{G}$ to $\min(N-1, K(N-P))$ by

$$\mathbf{G} = (\mathbf{W}_\perp \mathbf{W}_\perp^*) \odot \mathbf{R}_y \tag{8}$$

where $\mathbf{R}_y = \sum_{k=1}^{K} \bar{\mathbf{y}}_k \mathbf{y}_k^T$ is rank K. Hence, in practice we will simply use K frames so that $K(N-P) > N-1$. Then $\mathbf{G}$ is rank deficient by one and $\mathbf{z}$ in (7) is unique. Generally, this requirement on K is not too demanding.

The next step is to fit a structured $\mathbf{z}(\phi)$ to the estimate $\hat{\mathbf{z}}$ by minimizing the least squares criterion

$$\mathcal{Q}_0(\phi) = \|\hat{\mathbf{z}} - \mathbf{z}(\phi)\alpha\|^2$$

After eliminating α we have

$$\mathcal{Q}_0(\phi) = \left\| \mathbf{P}^{\perp}_{\bar{\mathbf{z}}(\phi)} \bar{\hat{\mathbf{z}}} \right\|^2$$

where $\mathbf{P}^{\perp}_{\bar{\mathbf{z}}(\phi)} = \mathbf{I} - \frac{1}{N}\bar{\mathbf{z}}(\phi)\mathbf{z}^T(\phi)$ is an orthogonal projection matrix. Note that $\mathbf{z}^T(\phi)\bar{\mathbf{z}}(\phi) = N$ because of the structure of $\mathbf{z}(\phi)$. Following an approach used repeatedly in array processing with uniform linear arrays [14], we reparameterize the problem in terms of a first order polynomial $b(z) = z - b$ which has a root at $z = b = e^{j\phi}$. Define the 2×1 vector $\mathbf{b} = [-b, 1]^T$ which contains the coefficients of $b(z)$. Because of the special structure of $\mathbf{z}(\phi)$, the vector $\mathbf{b}$ completely characterizes the orthogonal compliment of the space spanned by $\bar{\mathbf{z}}(\phi)$. To see this, define the $N \times (N-1)$ Toeplitz matrix $\mathbf{B}(b)$ by

$$\mathbf{B}(b) = \begin{bmatrix} -b & & & \\ 1 & -b & & \\ & 1 & \ddots & \\ & & \ddots & -b \\ & & & 1 \end{bmatrix}$$

Now we have the important relationship

$$\mathbf{B}^*(b)\bar{\mathbf{z}}(\phi) = 0 \qquad \Longrightarrow \qquad \mathbf{P}^{\perp}_{\bar{\mathbf{z}}(\phi)} = \mathbf{P}_{\mathbf{B}(b)}$$

which leads to the new criterion

$$\mathcal{Q}_1(b) = \left\| \mathbf{P}_{\mathbf{B}(b)} \bar{\hat{\mathbf{z}}} \right\|^2 = \text{trace}\left(\mathbf{B}(b)\left(\mathbf{B}^*(b)\mathbf{B}(b)\right)^{-1}\mathbf{B}^*(b)\mathbf{R} \right)$$

where $\mathbf{R} = \bar{\hat{\mathbf{z}}}\hat{\mathbf{z}}^T$. Before proceeding, we make one more simplifying step. Let $\mathbf{A} = (\mathbf{B}^*\mathbf{B})^{-1}$, then we have the following identity

$$\mathcal{Q}_1(b) = \text{trace}\left(\mathbf{B}(b)\mathbf{A}\mathbf{B}^*(b)\mathbf{R}\right) = \mathbf{b}^T\mathbf{Q}\bar{\mathbf{b}}$$

where

$$q_{m,i} = \sum_{n=1}^{N-1}\sum_{j=1}^{N-1} r_{n+m-1,j+i-1}a_{j,n} \tag{9}$$

and $q_{i,j}, r_{i,j}, a_{i,j}$ are the $(i,j)^{th}$ elements of $\mathbf{Q}, \mathbf{R}, \mathbf{A}$ respectively. This transformation reduces the complexity of estimating b provided $\mathbf{A}$ is known since $\mathbf{Q}$ is a 2×2 matrix. To minimize $\mathcal{Q}_1(b)$ we will follow the approach proposed in [15] which is non-iterative and produces consistent estimates of b. It involves two steps. In the first step, we assume that $\mathbf{A} = (\mathbf{B}^*\mathbf{B})^{-1} = \mathbf{I}$. This simplifies (9) considerably. In particular, $q_{m,i}$ becomes

$$q_{m,i} = \sum_{\ell=1}^{N-1} \bar{\hat{z}}_{\ell+m-1}\hat{z}_{\ell+i-1}, \qquad i, m = 1, 2 \tag{10}$$

Hence, in the first step, $q_{1,1}$ $q_{2,2}$ are just estimates of the zero lag autocorrelation of the elements of $\bar{\hat{\mathbf{z}}}$ while $q_{1,2} = \bar{q}_{2,1}$ is an estimate of the one lag autocorrelation. The estimate $\hat{b}_1$ is determined by the eigenvector of the 2×2 matrix $\mathbf{Q}$ corresponding to the smallest eigenvalue, σ_{min}. This is a simple computation.

The second step uses values computed during the first step to further improve the estimate. First, $\mathbf{A}$ is estimated using $\hat{b}_1$ by $\mathbf{A} = \left(\mathbf{B}^*(\hat{b}_1)\mathbf{B}(\hat{b}_1)\right)$ and $\mathbf{R}$ is replaced by $\mathbf{R} - \sigma_{min}\mathbf{I}$ for consistency. Then $\mathbf{Q}$ is recomputed according to (9) and a new estimate $\hat{b}_2$ is obtained from the eigendecomposition of $\mathbf{Q}$. While this re-estimation process could be continued, it was shown in [15] that the estimate $\hat{b}_2$ achieves its asymptotic variance. Hence, only two steps are necessary. In practice, we have found that the estimates obtained from the first step ($\mathbf{A} = \mathbf{I}$) are very good. While the second step does improve the estimates it may also be omitted during initial carrier acquisition.

Initialize: $n = 1$
While $J(n, \hat{\phi}) > T$
1. Update $\mathbf{G}_n$ using $y(n)$
2. Compute smallest eigenvalue λ_{min} of $\mathbf{G}$ and corresponding eigenvector $\hat{\mathbf{z}}$
3. Compute $\mathbf{Q} = \left[\begin{smallmatrix} q_{1,1} & q_{1,2} \\ q_{2,1} & q_{2,2} \end{smallmatrix}\right]$ using $\hat{\mathbf{z}}$ and (10)
4. Compute $\hat{b}_n$ using EVD of $\mathbf{Q}$
5. $\hat{\phi}_n = \angle\hat{b}_n$ where $\hat{\mathbf{b}}_n = |\hat{\mathbf{b}}_n|e^{j\angle\hat{\mathbf{b}}_n}$
6. $n = n + 1$

End

Table 1: Frame Timing and Carrier Synchronization Algorith

4. ALGORITHM SUMMARY

The above procedure may be incorporated into a joint frame time and frequency offset estimator as suggested above in a two step fashion. First, n is hypothesized to be the start of a frame and $\hat{\phi}$ is computed by the above procedure. Then $J(n, \hat{\phi})$ is evaluated. If it is zero (or smaller than a threshold T), then the hypothesis $n = n_o$ is correct, otherwise increment n and repeat the procedure. We summarize the algorithm in Table 1.

A few comments about the algorithm in Table 1 are in order. The influence of the new data can be easily incorporated into $\mathbf{G}_n$ in step 1. Note the relationship between $\mathbf{G}_{n-1}$ and $\mathbf{G}_n$.

$$\mathbf{G}_{n-1} = \left[\begin{array}{c|c} \alpha & \boldsymbol{\beta}^* \\ \hline \boldsymbol{\beta} & \mathbf{A} \end{array}\right] \qquad \mathbf{G}_n = \left[\begin{array}{c|c} \mathbf{A} & \boldsymbol{\gamma} \\ \hline \boldsymbol{\gamma}^* & \xi \end{array}\right]$$

Here, the $N-1 \times N-1$ matrix $\mathbf{A}$ is a submatrix of both $\mathbf{G}_{n-1}$ and $\mathbf{G}_n$. The new data $y(n)$ only effects the last column and row of $\mathbf{G}_n$. And, since $\mathbf{G}_n$ has Hermitian symmetry, only the N elements of $\boldsymbol{\gamma}$ and ξ have to be computed. Hence, $\mathbf{G}_n$ can be updated very efficiently.

The eigen computation in Step 2 of Table 1 is the only significant computation of our algorithm. Step 2 requires computing the eigenvector $\hat{\mathbf{z}}$ of $\mathbf{G}_n$ corresponding to the smallest eigenvalue λ_{min}. Since $\mathbf{G}_n$ is nearly the same as $\mathbf{G}_{n-1}$ we would expect that λ_{min} and $\hat{\mathbf{z}}$ could be updated efficiently rather than recomputed from scratch. However, we leave this subject as an item for future research. For now, we suggest that λ_{min} and $\hat{\mathbf{z}}$ can be updated from one time to the next using one or two steps of inverse iteration [16]. This is a procedure for computing the smallest eigenvalue and eigenvector of a matrix which is computationally efficient relative a full eigendecomposition. With reasonable signal to noise ratios, the difference between the smallest eigenvalue λ_{min} and the next-to-smallest eigenvalue is large and inverse iteration converges very quickly.

5. SIMULATIONS

In this section, we consider the performance of our algorithm on simulated OFDM data. For the simulations, the symbols were randomly selected from a 16-QAM constellation. The total number of carriers was set to $N = 16$ with $P = 12$ modulated carriers

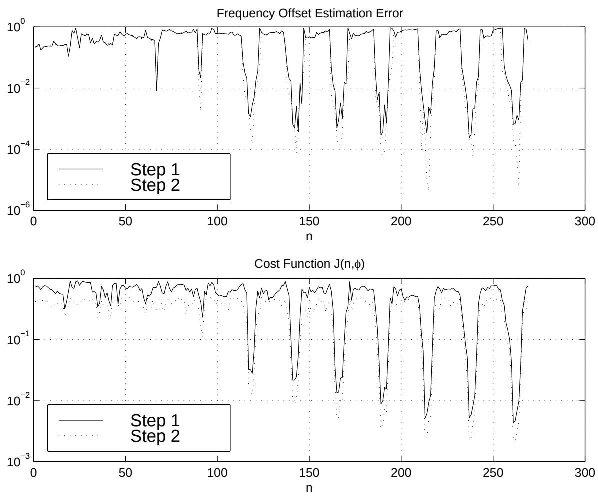

Figure 1: Synchronization performance for channel with length $L = 8$.

and $N - P = 4$ virtual carriers. An $L = 8$ point cyclic prefix was used so that the total frame length was $N + L = 24$ samples. An OFDM discrete-time time-domain signal was generated using an N-point IFFT of the symbols. Then, the cyclic prefix was added at the beginning of each frame before the entire signal was convolved with the channel and corrupted by additive white Gaussian noise with variance σ_n^2. The signal to noise ratio (SNR) is given by $SNR = 10\log_{10}(\sigma_y^2/\sigma_n^2)$ where σ_y^2, the sample signal variance, was computed numerically during the simulation. The SNR was set to 30 dB. To simulate the effect of timing uncertainty a random number of zeros was inserted at the beginning of of the transmitted signal. The effect of a carrier offset was synthesized by multiplying the received signal by $e^{j\phi n}$ where the frequency offset ϕ was set to 0.4444.

Figure 1 shows the performance of our algorithm for a channel with length 8 samples. The channel was generated as a random complex vector. The channel coefficients $h(n)$ are -0.0454 $-0.0227i$, 0.1699 $+0.0373i$, 0.2712 $-0.2692i$, -0.4230 $-0.1739i$, -0.3712 $+0.1179i$, 0.4350 $+0.1857i$, 0.0625 $-0.3672i$, -0.2132 $-0.2290i$.

In the figure, the upper plot shows the error $\phi - \hat{\phi}$ between the estimate and the true offset versus n. The lower plot shows the cost function $J(n, \hat{\phi})$ versus n. The frame boundaries can be detected by locating the nulls in the cost function as discussed in Section 3. Note that when the cost function $J(n, \hat{\phi})$ is small (lower subfigure), which corresponds to frame synchronization, the error in the frequency estimate is also very small (upper subfigure). It was varified through simulations that when when the channel is shorter than the cyclic prefix, the null in $J(n, \hat{\phi})$ simply gets wider. When this is the case, any point in the null interval may be taken as the beginning of a frame.

In Figure 1, the solid line shows the estimation error for $\hat{\phi}$ and the resulting cost function $J(n, \hat{\phi})$ using only the first step of the above algorithm. The dotted line corresponds to using the full algorithm (both step one and step two). As mentioned previously, the first step (solid line) yields very good estimates of the carrier offset during the in-sync nulls. The second step (dotted line) does offer some improvement. It is expected that for low SNR's the second step will be needed to obtain accurate estimates of ϕ.

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