SOURCE DETECTION AND LOCALIZATION USING A MULTI-MODE DETECTOR: A BAYESIAN APPROACH

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ABSTRACT

This paper considers a class of detection/localization problems in which the detector offers multiple operating modes. The modes differ in their detection performance and geographical coverage: "focused" modes offer higher detection performance but less coverage area than "broad search" modes. It is assumed that a signal source is to be detected and localized using a sequence of tests, each possibly employing a different mode. The goal is to determine a strategy for mode selection in the sequence of tests that will yield optimal payoff in terms of a pre-established criterion. A mathematical model capturing the key characteristics of this situation is proposed and used to develop optimal mode selection strategies.

1. INTRODUCTION

Recent advances in microprocessor and other technologies have greatly enhanced the capabilities and practicality of tunable, deployable, and configurable sensors in a wide variety of applications. Building upon pioneering work that considered optimal measurement map selection in estimating the state of certain classes of stochastic dynamical systems [1, 2, 7], research explicitly addressing the development of strategies to effectively control the configuration of a sensor system in order to estimate parameters of the process being measured has begun to emerge over the past few years [4, 5, 9, 10].

This paper considers a situation in which a *detector* is configurable in such a way as to provide multiple modes of operation that differ in their detection performance and geographical coverage. The development that follows focuses on the case of a detector with two operating modes: a "broad search" mode that provides wide coverage and a "focused" mode that provides better detection performance but covers less area. This is indeed the case in the actual application that motivated this research, in which an aircraft is to search for a target using long-range and short-range radar systems that cannot be operated simultaneously. At any given moment, the pilot can test the entire area of interest with the long-range radar with the expense of either low probability of detection or many false alarms; alternatively a small sub-region can be examined with higher probability of detection and lower false alarm rate, but limited time on station generally prevents the entire region from being searched in this mode. The ideas formulated in this paper generalize to detectors having more than two modes in a straightforward way – provided, of course, that the modes are satisfactorily modeled as described in the following sections.

2. MATHEMATICAL FORMULATION

A useful mathematical description of the situation introduced in the previous section must account for differences in the detector's operating modes, both in detection performance and geographical coverage. In the case of only two operating modes, as in the motivating application, an idealized model for the geographical coverage is obtained by considering the entire region of interest S to be partitioned into N disjoint "cells" $C_1, ..., C_N$. Operating in the broad search mode (*Mode A*), the detector tests for the presence of a signal source in S. In the focused mode (*Mode B*), however, the test may be applied to exactly one cell C_n.

To account for difference in detector performance in the two operating modes, detector performance is modeled as arising from the problem of detecting of a known signal in white gaussian noise of known variance. This model provides a well understood solution (i.e., the matched filter) in each test, admits several straightforward generalizations, and allows detection performance in *Mode B* to be distinguished from that in *Mode A* by simply raising the signal-to-noise ratio (SNR). More specifically, in each mode of operation the detector encounters a problem of the form

$$H_0 \quad : \quad \mathbf{X} = \mathbf{N} \qquad (1)$$

$$H_1 \quad : \quad \mathbf{X} = S + \mathbf{N} \qquad (2)$$

where S is a known signal M-vector with energy $||S||^2 = 1$ and $\mathbf{N}$ is a zero-mean white gaussian M-vector having known variance σ^2; i.e., $\mathbf{N} \sim \mathcal{N}[\mathbf{0}, \sigma^2 \mathbb{I}]$ where $\mathbb{I}$ is the $n \times n$ identity matrix. Since $||S||$ is fixed, the SNR (and hence the performance of the detector) in each mode can be adjusted by varying σ^2.

Assuming at most one signal source is present, denote by H_1 and H_0 the events that the signal source is, respectively, present in and absent from S. For $n = 1, ..., N$, denote by $H_{1,n}$ the event that the signal source is present in cell n and by $H_{0,n}$ the event that the signal source is absent from cell n. With these definitions, $H_1 = \cup_{n=1}^N H_{1,n}$ and $H_0 = \cap_{n=1}^N H_{0,n}$. Regardless of whether it is operating in *Mode A* or *Mode B*, the system yields both a detection decision ($\rightarrow H_0$ or $\rightarrow H_1$) and a localization decision ($\rightarrow H_{1,i}$ for some $i = 1, ..., N$ or $\rightarrow H_0$).

Recall that the optimal solution, in terms of minimal probability of error, to a detector problem of the form (1) is a test on the inner product $S^T \mathbf{X}$ where the detection threshold is a function of the *a priori* probability that a signal is present [6, 8]. The probabilities of detection and false alarm for each test are given by error functions of the detection thresholds. In particular, the tests applied in both operating modes will be of this form, but their detection thresholds and probabilities of detection and false alarm will all be different (even when *Mode B* is applied to distinct cells) because of their dependence on $\Pr(H_1)$ and $\Pr(H_{1,n})$, $n = 1, ..., N$.

Generally, the overall goal of the detection system will be to both detect the signal source and localize it (i.e., identify which cell it is in) – though these two subgoals may not be of equal importance. This is modeled by considering a payoff function consisting of the convex sum of two terms

$$J = \alpha U + \beta V \qquad \alpha, \beta \geq 0,\ \alpha + \beta = 1 \tag{3}$$

where U represents the probability that the correct detection decision (i.e., between H_0 and H_1) is attained and V represents the probability that the correct cell is identified. A precise mathematical formulation of these two terms is given in the following section.

Beginning with prior probabilities $\Pr(H_{1,n})$ for $n = 1, ..., N$, the detection system is faced with the problem of selecting a sequence of operating modes and cells (whenever *Mode B* is selected) that will yield the best value of the payoff function (3) after each test.[1]

[1] This may not be the same sequence that yields the best value of the payoff function after a fixed number K of tests is completed, possibly a more realistic criterion in some applications.

3. AN ATTENTIVE DETECTION FILTER

To address the problem described above, the detection/localization system can perform an iterative sequence:

1. Determine whether to operate in *Mode A* or *Mode B* and, if *Mode B* is chosen, which cell to test.
2. Perform the test and decide whether the signal source is present and, if so, which cell it is in.
3. Update $\Pr(H_{1,n})$, $n = 1, ..., N$, for use in step 1 of the next test.

The following subsections describe these three steps in detail. The natural order of development is to treat the steps in reverse order.

3.1. Update of the priors

At the outset of this step a test has been performed, either in *Mode A* on all of S or in *Mode B* on a specific cell n. The *a posteriori* probabilities of H_1 and $H_{1,n}$ for $n = 1, ..., N$ given the outcome of the test can be evaluated by Bayes' rule. Denoting $p_i = \Pr(H_i)$ and $p_{i,n} = \Pr(H_{i,n})$, $i = 0, 1$, the posterior probabilities are computed on a case-by-case basis in terms of the detectors' probabilities of detection $P_{d,A}$, $P_{d,B}^{(n)}$ and false alarm $P_{f,A}$, $P_{f,B}^{(n)}$ as follows.

- *Mode A*, detect:

$$\begin{aligned} \Pr(H_1 | \rightarrow H_1) &= \sum_n \Pr(H_{1,n} | \rightarrow H_1) \\ \Pr(H_{1,n} | \rightarrow H_1) &= \frac{P_{d,A} p_{1,n}}{P_{f,A} p_0 + P_{d,A} p_1} \end{aligned}$$

- *Mode A*, no detect:

$$\begin{aligned} \Pr(H_1 | \rightarrow H_0) &= \sum_n \Pr(H_{1,n} | \rightarrow H_0) \\ \Pr(H_{1,n} | \rightarrow H_0) &= \frac{(1 - P_{d,A}) p_{1,n}}{(1 - P_{f,A}) p_0 + (1 - P_{d,A}) p_1} \end{aligned}$$

- *Mode B* on cell n, detect:

$$\Pr(H_1 | \rightarrow H_{1,n}) = \sum_k \Pr(H_{1,k} | \rightarrow H_{1,n})$$

$$\Pr(H_{1,k} | \rightarrow H_{1,n}) = \begin{cases} \dfrac{P_{f,B}^{(n)} p_{1,k}}{P_{d,B}^{(n)} p_{1,n} + P_{f,B}^{(n)} (1 - p_{1,n})} & k \neq n \\ \dfrac{P_{d,B}^{(n)} p_{1,n}}{P_{d,B}^{(n)} p_{1,n} + P_{f,B}^{(n)} (1 - p_{1,n})} & k = n \end{cases}$$

- *Mode B* on cell n, no detect:

$$\Pr(H_1| \rightarrow H_{0,n}) = \sum_k \Pr(H_{1,k}| \rightarrow H_{0,n})$$

$$\Pr(H_{1,k}| \rightarrow H_{0,n}) = \begin{cases} \frac{(1-P_{f,B}^{(n)})p_{1,k}}{(1-P_{d,B}^{(n)})p_{1,n}+(1-P_{f,B}^{(n)})(1-p_{1,n})} & k \neq n \\ \frac{(1-P_{d,B}^{(n)})p_{1,n}}{(1-P_{d,B}^{(n)})p_{1,n}+(1-P_{f,B}^{(n)})(1-p_{1,n})} & k = n \end{cases}$$

3.2. Test and decision

At this point, a mode has been selected and, in the case of *Mode B*), a cell has also been selected. The rule for making the selection will be developed in the next subsection; the purpose of this subsection is to arrive at a rule for generating a decision using the outcome of the test selected. For notational convenience, the decision will be represented here by an ordered pair (d, ℓ). The detection component d takes on the value 0 or 1 to reflect the decision as to whether the signal source is present in S. The localization component ℓ assumes values $1, ..., N$ denoting the cell in which the source is located; $\ell = 0$ is reserved for the special case in which $d = 0$.

The decision criterion will be the *a posteriori* probabilities given in the previous subsection. Specifically, d will be assigned the value 1 if the posterior probability of H_1 following the measurement is larger than that of H_0 and assigned the value 0 otherwise. Similarly, ℓ is given the value 0 if $\Pr(H_0) > \Pr(H_1)$; otherwise, ℓ takes the index of the most probable $H_{1,n}$ (in posterior probability).

3.3. Mode selection

At the outset of Step 1, the mode selection step in the algorithm, the system must select among $N+1$ possible tests $T_0, ..., T_N$ providing $2N+2$ possible outcomes $o_n \in \{0, 1\}$, $n = 0, ..., N$. Each possible outcome requires evaluation of $N + 1$ posterior probabilities in order to determine the system decision (d, ℓ).

As suggested in section 2, mode selection is based on a payoff function $J(T_n)$ formed as a convex sum of two terms $J(T_n) = \alpha U(T_n) + \beta V(T_n)$. These terms are defined explicitly in terms of *a posteriori* probabilities:

$$\begin{aligned} U(T_n) &= \Pr(H_0|o_n = 0)\Pr(o_n = 0) \\ &\quad + \Pr(H_1|o_n = 1)\Pr(o_n = 1) \end{aligned} \tag{4}$$

$$\begin{aligned} V(T_n) &= \Pr(H_{1,\ell}|o_n = 0)\Pr(o_n = 0) \\ &\quad + \Pr(H_{1,\ell}|o_n = 1)\Pr(o_n = 1) \end{aligned} \tag{5}$$

where ℓ in (5) assumes the index the cell chosen if o_n is the conditioning value. In both of these expressions, $\Pr(o_n = 0) = \{(1 - P_{d,A})p_1 + (1 - P_{f,A})p_0\}$ and $\Pr(o_n = 1) = \{P_{d,A}p_1 + P_{f,A}p_0\}$ for the *Mode A* test; $\Pr(o_n = 0) = \{(1 - P_{d,B}^{(n)})p_{1,n} + (1 - P_{f,B}^{(n)})p_{0,n}\}$ and $\Pr(o_n = 1) = \{P_{d,B}^{(n)}p_{1,n} + P_{f,B}^{(n)}p_{0,n}\}$ for the *Mode B* tests. The other factors in (4) are the *a posteriori* probabilities of H_0 and H_1 while those in (5) are the maximal posterior probabilities among $\{H_0, H_{1,1}, ..., H_{1,N}\}$ under each of the two test outcomes. In particular, all of the numbers needed to evaluate J for each of the $N+1$ candidate tests are available as *a priori* probabilities, *a posteriori* probabilities, or detector performance data. Evaluation of $2(N + 1)^2$ posterior probabilities *before mode selection* to provide an exhaustive list of the $N + 1$ possible post-test values of J is the crucial step in mode selection. The test T_n is simply selected to yield the maximum value of $J(T_n)$.

3.4. Summary of the approach

To summarize, the system first uses the *a priori* probabilities to compute probabilities of detection and false alarm for the optimal Bayesian detectors for *Mode A* and *Mode B*. Using these, *a posteriori* probabilities are computed under each of the $2N + 2$ possible test outcomes. These are used to evaluate J for each of the candidate modes, and the one offering the maximal value of J is selected. At this point, the selected test is actually run and the outcome is used to determine the system output (d, ℓ) and also to decide which set of previously computed *a posteriori* probabilities will be adopted as the *a priori* probabilities at the start of the next iteration.

4. ATTENTIVE DETECTION EXAMPLE

Figure 1 shows the behavior of the two-mode detection/localization system operating in a five-cell (i.e., $N = 5$) scenario. The test signal and noise vectors are of length $M = 10$ and the SNRs in the two modes are established by the parameters $\sigma_A^2 = 2$ and $\sigma_B^2 = 0.67$. The initial prior probabilities are $\Pr(H_{1,1}) = .0839$, $\Pr(H_{1,2}) = .1482$, $\Pr(H_{1,3}) = .0479$, $\Pr(H_{1,4}) = .1910$, $\Pr(H_{1,5}) = .0289$. The posterior probabilities of the first test, which are used as the prior probabilities in the second test, appear in the first column of the grid – and so forth. In this example, a signal source is actually present in cell 4. The system chooses *Mode A* for the initial test (indicated by light shading of all cells in the first column), does not detect (per the annotation beneath the column), and decides for H_0 (indicated by the lack of a dark-shaded cell). *Mode B* on cell 4 is cho-

	Test 1 (no detect)	Test 2 (detect)	Test 3 (detect)	Test 4 (detect)
Cell 5	.0124	.0106	.0054	.0006
Cell 4	.0820	.2198	.5991	.9535
Cell 3	.0206	.0175	.0090	.0010
Cell 2	.0636	.0541	.0278	.0032
Cell 1	.0360	.0306	.0157	.0018

Figure 1: An example illustrating the behavior of the two-mode detector in a five-cell scenario. A detailed description of this figure is given in the text.

sen in the following three tests with detects in all three tests. Following test 2, the system still decides in favor of H_0 (note the high initial prior of 0.5 on H_0). After tests 3 and 4, however, H_1 is chosen and cell 4 is selected (dark shading) because the posterior probability of $H_{1,4}$ is the largest of any cell.

5. DISCUSSION AND CONCLUSIONS

This paper has developed a Bayesian approach for optimal management of a switchable-mode detection system. Although attention was focused on the two-mode case, the principles employed should extend directly to cases involving more modes.

The example in section 4 was terminated when posterior probability of some cell (or H_0) exceeded a threshold, with correct decisions on both signal presence and location. In other trials, the system occasionally did not attain this termination condition in a reasonable number of iterations, suggesting the need for convergence analysis of the iterative algorithm and attention to development of appropriate termination criteria.

As noted, one case of particular interest is where it is known *a priori* that a fixed number of iterations will be used. It would appear to be possible, in principle, to evaluate the propagation of the hypothesis probabilities exhaustively to the last iteration — considering every possible combination of possible tests and outcomes — *before actually performing any of the tests.* This would allow selection of an optimal sensing strategy at the expense of a substantial computational burden. Efficient algorithms for attaining near-optimal sensing strategies in such cases would be of interest.

This work is part of the authors' ongoing research into attentive sensing and biologically motivated approaches to sensor management.

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7. REFERENCES

[1] M. Athans, "On the Determination of Optimal Costly Measurement Strategies for Linear Stochastic Systems," *Automatica,* vol. 8, pp. 397-412, 1972.

[2] J.S. Baras and A. Bensoussan, "Optimal Sensor Scheduling in Nonlinear Filtering of Diffusion Processes." *SIAM Journal of Control and Optimization,* vol. 27, no. 4, pp. 786-813, July 1989.

[3] J.J. Clark and A.L. Yuille, *Data Fusion for Sensory Information Processing Systems.* Kluwer Academic Publishers, 1990.

[4] D. Cochran and R. Martin, "Nonlinear Filtering Models of Attentive Vision," Proceedings of the IEEE International Conference on Circuits and Systems, Atlanta, May 1996.

[5] J. Evans and V. Krishnamurthy, "Optimal Sensor Scheduling for Hidden Markov Models," *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, Seattle, May 1998.

[6] C.W. Helstrom, *Elements of Signal Detection and Estimation.* Prentice-Hall, 1995.

[7] Y. Oshman, "Optimal Sensor Selection Strategy for Discrete-Time State Estimators." *IEEE Transactions on Aerospace and Electronic Systems,* vol. 30, no. 2, April 1994.

[8] L.L. Scharf, *Statistical Signal Processing.* Addison Wesley, 1991.

[9] D. Sinno and D. Cochran, "Estimation with Configurable and Constrained Sensor Systems," *Signal Processing* (to appear).

[10] D. Sinno and D. Cochran, "Dynamic Estimation with Selectable Linear Measurements," *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing*, Seattle, May 1998.