CHANNEL ESTIMATION FOR DS-CDMA WITH APERIODIC SPREADING CODES

Kemin Li[1] *and Hui Liu*[2]

1. Department of EE, University of Virginia, Charlottesville, VA 22903
2. Department of EE, University of Washington, Seattle, WA 98195

ABSTRACT

For high performance CDMA communications, multiuser detection is often required to suppress the multiple access interference (MAI). Most multiuser detectors rely on accurate channel information to recover the multiuser digital signals. This paper studies the blind channel estimation problem for DS-CDMA systems using aperiodic spreading codes. The Maximum Likelihood (ML) estimator is formulated for channel estimation. We first convert the multiuser parameter estimation problem into a set of single user optimization problems via alternating optimization, and then determine the channel parameters for each user using an iterative algorithm derived. It is shown by computer simulation that this iterative algorithm can reach global maxima almost always under medium SNR values.

Keywords: CDMA, Channel estimation

1. INTRODUCTION

Most multiuser detectors [6, 4, 8, 9] premise on the knowledge of communication channels, i.e. multipath channel coefficients, phases and carrier offsets, which usually need to be estimated in practice. Abundant research has been conducted on estimating the channel parameters for DS-CDMA systems where periodic spreading codes are used.

In north America IS-95 standard, very long pseudo-random spreading codes which span many symbol periods are used to reduce the interference from adjacent cells. We term the pseudo-random spreading codes as aperiodic codes in the sequel. Most channel estimators for aperiodic CDMA systems exploit the statistics of received signals of individual users after despreading [3, 7]. Data efficient parametric multiuser channel estimation is much involved, partly due to the lack of structure information in the received data. The contribution of this paper is the formulation of the Maximum Likelihood channel estimator for aperiodic CDMA systems, and more importantly, an efficient algorithm to solve the multidimension optimization problem efficiently.

The rest of the paper is organized as follows. In Section 2 the discrete time data model of CDMA systems is presented. Section 3 presents the Maximum Likelihood estimator and a reduced complexity algorithm which solves the multidimensional optimization iteratively. The performance of the proposed algorithm is examined through computer simulations. The paper is then concluded in Section 5.

2. DATA FORMULATION

We consider a quasi-synchronous CDMA system where signals from P users are synchronized within L chip durations[1]. L is maximum delay that accounts for all timing ambiguities and multipath delays. The channel effect will cause minor inter-symbol interference (ISI) in $(L-1)$ out of M samples within each symbol period T_s. In a quasi-synchronous CDMA system, we assume $L \ll M$, therefore a majority part of received chip rate samples are unaffected by ISI [5]. According to [2], the ISI-free chip-rate received samples within one symbol period can be expressed in vector form as

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[1] Such is assumed for notational simplicity. The method presented here can be applied to asynchronous CDMA systems as well.

$$
\begin{aligned}
\mathbf{y}(k) &= \begin{bmatrix} y(kM+L-1) \\ y(kM+L) \\ \vdots \\ y(kM+M-1) \end{bmatrix} \\
&= \sum_{i=1}^{P} s_i(k) \begin{bmatrix} w_{i,k}(L-1) \\ w_{i,k}(L) \\ \vdots \\ w_{i,k}(M-1) \end{bmatrix} + \begin{bmatrix} v(kM+L-1) \\ v(kM+L) \\ \vdots \\ v(kM+M-1) \end{bmatrix} \\
&\stackrel{\text{def}}{=} \sum_{i=1}^{P} s_i(k)\mathbf{w}_i(k) + \mathbf{v}(k),
\end{aligned} \tag{1}
$$

where $\mathbf{w}_i(k)$ is the ith user's signature waveform for the kth symbol, $s_i(k)$ is the ith user's information bearing sequence, and $\mathbf{v}(k)$ the additive noise. Let K=M-L+1, it is easy to verify that $\mathbf{w}_i(k)$ can be decomposed as follows,

$$
\begin{aligned}
&\mathbf{w}_i(k) = \\
&\begin{bmatrix} c_{i,k}(L-1) & \cdots & c_{i,k}(0) \\ c_{i,k}(L) & \cdots & c_{i,k}(1) \\ \vdots & \ddots & \vdots \\ c_{i,k}(M-1) & \cdots & c_{i,k}(K-1) \end{bmatrix} \begin{bmatrix} h_i(0) \\ \vdots \\ h_i(L-1) \end{bmatrix} \\
&= \mathbf{C}_i(k)\mathbf{h}_i
\end{aligned}
$$

Here, $\{c_{i,k}(m)\}$ are the ith user's spreading codes for the kth symbol, and $h_i(l)$ is the ith user's channel response. Note that $\mathbf{w}_i(k)$ is time varying because of the aperiodic spreading codes employed. If the spreading codes are periodic, *i.e.*, $\mathbf{w}_i(k) \equiv \mathbf{w}_i$, the channel parameters can be estimated efficiently using a subspace algorithm described in [1, 2]. The problem addressed here is the estimation of the channel parameters, $\{\mathbf{h}_i\}_{i=1}^{P}$, given only the received data samples when the spreading codes are aperiodic.

3. ALGORITHM DEVELOPMENT

In this section, we first formulate the Maximum Likelihood channel estimator and then present an iterative solution. We derive our algorithm based on the alternating optimization idea in [10]. We show that after reducing the multiuser optimization problem to a set of single-user problems, the remaining problems bear certain structure that allows one to tackle them efficiently.

3.1. The Maximum Likelihood Estimator

Rewrite Equation (1) as

$$
\begin{aligned}
\mathbf{y}(k) &= \sum_{i=1}^{P} s_i(k)\mathbf{w}_i(k) + \mathbf{v}(k) \\
&= \underbrace{[\mathbf{w}_1(k)\ \cdots\ \mathbf{w}_P(k)]}_{\mathbf{W}(k)} \begin{bmatrix} s_1(k) \\ \vdots \\ s_P(k) \end{bmatrix} + \mathbf{v}(k) \\
&= \mathbf{W}(k)\mathbf{s}(k) + \mathbf{v}(k).
\end{aligned} \tag{2}
$$

Under the AWGN assumption and treating the transmitted digital sequence as deterministic unknowns, the ML channel estimator with N received data vectors are given by

$$
\{\hat{\mathbf{h}}_i\}_{i=1}^{P} = \arg\min_{\{\mathbf{h}_i\}} \sum_{k=1}^{N} \left\| \mathbf{y}(k) - \mathbf{W}(k)\mathbf{s}(k) \right\|^2. \tag{3}
$$

It can be shown that Equation (3) is equivalent to

$$
\{\hat{\mathbf{h}}_i\} = \arg\max_{\{\mathbf{h}_i\}} \sum_{k=1}^{N} \left\| \mathbf{P}_{\mathbf{W}(k)}\mathbf{y}(k) \right\|^2, \tag{4}
$$

where $\mathbf{P}_{\mathbf{W}(k)}$ is the projection matrix onto the subspace spanned by column vectors of $\mathbf{W}(k)$:

$$
\mathbf{P}_{\mathbf{W}(k)} = \mathbf{W}(k)\Big(\mathbf{W}^H(k)\mathbf{W}(k)\Big)^{-1}\mathbf{W}(k)^H. \tag{5}
$$

3.2. Step one: converting multiuser problem to single-user ones

We solve the problem in (4) in two steps. In the first step, we adopt the alternating maximization methodology which quintessentially resorts to P L-dimensional maximization. Conceptually at each iteration a maximization is performed with respect to channel of one user while all the other parameters are hold fixed. That is,

$$
\hat{\mathbf{h}}_i^{(n)} = \arg\max_{\mathbf{h}_i} \sum_{k=1}^{N} \left| \mathbf{P}_{[\bar{\mathbf{W}}_i^{(n)}(k), \mathbf{w}_i^{(n)}(k)]}\mathbf{y}(k) \right|^2, \tag{6}
$$

where $\mathbf{W}_i^{(n)}(k)$ is

$$
[\mathbf{w}_1^{(n)}(k), \cdots, \mathbf{w}_{i-1}^{(n)}(k), \mathbf{w}_{i+1}^{(n-1)}(k), \cdots, \mathbf{w}_P^{(n-1)}(k)], \tag{7}
$$

$\mathbf{w}_i^{(n)}(k) = \mathbf{C}_i(k)\mathbf{h}_i^{(n)}$, and superscript $^{(n)}$ stands for the nth iteration.

By alternating maximization, we equivalently restrict our optimization in the channel vector space for the ith user while use the estimated channel vectors of all other users to form $\bar{\mathbf{W}}_i^{(n)}(k)$ in Equation (6). This procedure is repeated for all users.

Using a well-known result on the projection matrix of partitioned matrices, we have

$$\mathbf{P}_{[\bar{\mathbf{W}}_i^{(n)}(k),\ \mathbf{w}_i^{(n)}(k)]} = \mathbf{P}_{\bar{\mathbf{W}}_i^{(n)}(k)} + \mathbf{Q}_i^{(n)}(k) \tag{8}$$

where

$$\mathbf{Q}_i^{(n)}(k) = \left(\mathbf{I} - \mathbf{P}_{\bar{\mathbf{W}}_i^{(n)}(k)}\right)\mathbf{w}_i^{(n)}(k). \tag{9}$$

Note $\bar{\mathbf{W}}_i^{(n)}(k)$ is known from previous iteration.

Plugging Equation (8) into (6), and notice that the first summand in (8) is not depended on $\mathbf{h}_i$, we can therefore drop it in the final expression. Consequently we arrive at the following single user problems

$$\hat{\mathbf{h}}_i^{(n)} = \arg\max_{\mathbf{h}_i} \sum_{k=1}^{N} \|\mathbf{Q}_i^{(n)}(k)\mathbf{y}(k)\|^2, \quad i = 1, \cdots, P. \tag{10}$$

3.3. Step two: solution to single-user problems

Equation (10) is nontrivial to solve given its current form. Before proceeding to its solution, we shall express it in a more revealing form. First we introduce some notations to simplify our derivation. Define

$$\mathbf{F}_i(k) = (\mathbf{I} - \mathbf{P}_{\bar{\mathbf{W}}_i(k)}). \tag{11}$$

The single user cost function in (10) becomes

$$\begin{aligned} \sum_{k=1}^{N} \left|\mathbf{Q}_i^{(n)}(k)\mathbf{y}(k)\right|^2 &= \sum_{k=1}^{N} \mathbf{y}(k)^H \mathbf{Q}_i^{(n)}(k)\mathbf{y}(k) \\ = \quad & \sum_{k=1}^{N} \mathbf{y}(k)^H \mathbf{F}_i(k)\mathbf{w}_i(k) \\ & \times\left(\mathbf{w}_i^H(k)\mathbf{F}_i^H(k)\mathbf{F}_i(k)\mathbf{w}_i(k)\right)^{-1} \\ & \times\mathbf{w}_i^H(k)\mathbf{F}_i^H(k)\mathbf{y}(k) \end{aligned} \tag{12}$$

Upon defining

$$\begin{aligned} \mathbf{A}_i(k) &= \mathbf{C}_i^H(k)\mathbf{F}_i^H(k)\mathbf{y}(k)\mathbf{y}^H(k)\mathbf{F}_i(k)\mathbf{C}_i(k), \\ \mathbf{B}_i(k) &= \mathbf{C}_i^H(k)\mathbf{F}_i^H(k)\mathbf{F}_i(k)\mathbf{C}_i(k), \end{aligned} \tag{13}$$

it is easy to verify that the parameter estimation problem in Equation (12) becomes

$$\hat{\mathbf{h}}_i = \arg\max_{\mathbf{h}_i} \sum_{k=1}^{N} \frac{\mathbf{h}_i^H \mathbf{A}_i(k)\mathbf{h}_i}{\mathbf{h}_i^H \mathbf{B}_i(k)\mathbf{h}_i}. \tag{14}$$

Maximization of this sum of quotients is not straightforward due to the variation of spreading codes. No closed-form solution seems to be possible. Next we will derive a fast iterative solution.

Rewrite (14) as

$$\hat{\mathbf{h}}_i = \arg\max_{\mathbf{h}_i} \sum_{k=1}^{N} \frac{\alpha_k \mathbf{h}_i^H \mathbf{A}_i(k)\mathbf{h}_i}{\alpha_k \mathbf{h}_i^H \mathbf{B}_i(k)\mathbf{h}_i}, \tag{15}$$

subject to constraint $\alpha_i \mathbf{h}_i^H \mathbf{B}_i(k)\mathbf{h}_i = 1$. Note the introduction of an additional parameter, α_k, does not change the value of the cost function. However it does allow us to express the cost function in a more convenient form. Using Lagrange multiplier, the unconstrained cost function becomes

$$J = \sum_{k=1}^{N} \alpha_k \mathbf{h}_i^H \mathbf{A}_i(k)\mathbf{h}_i - \sum_{k=1}^{N} \lambda_k(\alpha_k \mathbf{h}_i^H \mathbf{B}_i(k)\mathbf{h}_i - 1). \tag{16}$$

Taking derivative of J with respect to $\mathbf{h}_i, \alpha_k, \lambda_k$, and set these derivatives to 0, we reach the following set of equations,

$$\sum_{k=1}^{N} \alpha_k \mathbf{A}_i(k)\mathbf{h}_i - \sum_{k=1}^{N} \lambda_k \alpha_k \mathbf{B}_i(k)\mathbf{h}_i = 0 \tag{17}$$

$$\alpha_k \mathbf{h}_i^H \mathbf{B}_i(k)\mathbf{h}_i - 1 = 0, \qquad k = 1, \cdots, N \tag{18}$$

$$\mathbf{h}_i^H \mathbf{A}_i(k)\mathbf{h}_i = \alpha_i \mathbf{h}_i^H \mathbf{B}_i(k)\mathbf{h}_i,\ k = 1, \cdots, N \tag{19}$$

These equations suggest that we can find the equilibrium point iteratively. With previously calculated values of λ_k and α_k, $\mathbf{h}_i$ is obtained as the generalized eigenvector corresponding to the largest generalized eigenvalue of matrix pair $(\mathbf{A}, \mathbf{B})$. New values of α_k and λ_k are then calculated from Equation (18) and (19) based on the channel estimate previously computed. After $\mathbf{h}_i$ or the cost function converges, we arrive at the final $\mathbf{h}_i$ estimate.

We summarize the multiuser parameter estimation procedure as follows:

1. Initialize all channel estimates.
2. For $i = 1 : P$,
   (a) Calculate the single user cost function for user i, construct Equation (14) based on Equations (6) and (8).
   (b) Solve the single-user channel estimation problem by computing the solutions of Equations (17-19) iteratively.
3. Repeat Step 2 until convergence or certain number of iterations is reached.

4. SIMULATION RESULTS

In this section, we provide some computer simulation results to illustrate the performance of the proposed algorithm. In all examples FIR channels of length 3 is assumed. We run 200 Monte-Carlo trials and the mean squared error (MSE) of channel estimates is used as the performance measure.

In this example, we simulate a 5-user DS-CDMA system with spreading factor of 16. Random binary sequence are used as the spreading codes. We vary the SNR from 10dB to 20dB, the MSE of first user's channel estimates are plotted in Figure 1. Also plotted is the *deterministic* CR bound[2] for channel estimates. Almost all the simulations converge to global maxima after about 6-12 iterations. The performance of proposed algorithm comes very close to the CR bound. This indicates a superior convergence property of the proposed algorithm. Note that channels estimates were obtained from only 10 received data symbols (vectors).

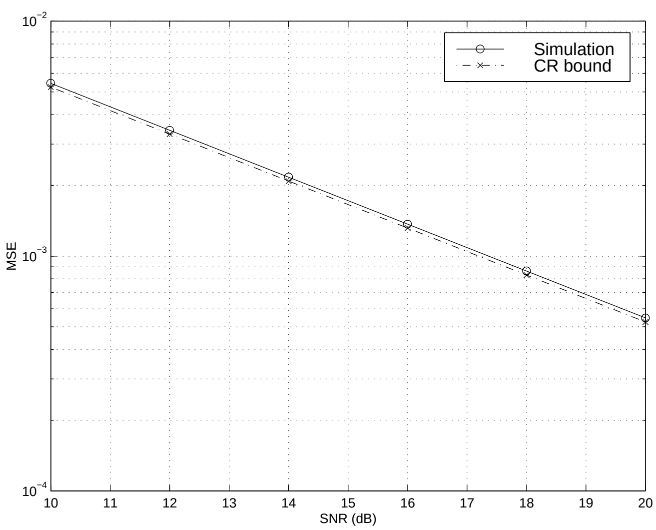

Figure 1: MSE versus SNR

5. CONCLUSIONS

In this paper, we studied the problem of blind channel estimation for CDMA systems using aperiodic spreading codes. The multiuser Maximum Likelihood parameter estimation problem is solved using an iterative method with relatively low complexity. The proposed estimator is very data efficient with excellent convergence property. More analyses on convergence and statistical properties of the proposed algorithm remain to be done.

[2]The results are not included due to space limitation.

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