RECURSIVE BLIND CHANNEL IDENTIFICATION AND EQUALIZATION BY ULV DECOMPOSITION

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ABSTRACT

Most eigenstructure-based blind channel identification and equalization algorithms with second-order statistics need SVD or EVD of the correlation matrix of the output signal. In this paper, we show new algorithms based on QR factorization of the output data directly. A recursive algorithm is developed by updating a rank-revealing ULV decomposition. Compared with existing algorithms in the same category, our algorithm is computationally more efficient and numerically (potentially) more robust. The computation in each recursion of the recursive algorithm can be reduced to the order of $O(m^2)$ under some simplifications, where m is the dimension of the received signal vector. Numerical simulations demonstrate the performance of the proposed algorithm.

1. INTRODUCTION

Blind equalization of transmission channels is important in many communication and signal processing applications because only some known statistical properties of the transmitted signals are required. Using cyclostationarity of the channel output, it has been shown that the second-order statistics contain sufficient information for the identification and equalization of finite impulse response channels [1]. Existing second-order statistics-based approaches usually require singular value decomposition (SVD) or eigenvalue decomposition (EVD) of the output correlation matrix [1], [2]. The computation burden of SVD or EVD turns out to be a major obstacle to real time implementation. Although some updating schemes have been proposed that produce an approximate singular value decomposition with computation $O(m^2)$, they can only be used to reduce computation of the first SVD (EVD) of [1] and [2]. The total computation of these two algorithms remains $O(m^3)$.

It is shown in [5] that QR factorization of the received data is in the same spirit of SVD or EVD of the output correlation matrix, thus can be used to identify the channel implicitly based on second-order statistics. Based on it, we will develop a recursive algorithm using rank-revealing ULV decomposition in this paper. The computation of each recursion can be reduced to $O(m^2)$ under some simplifications.

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In section 2, we briefly review the QR factorization approach of [5]. Then in section 3 we present in detail the rank-revealing ULV decomposition to derive a recursive identification algorithm.

2. BLIND CHANNEL IDENTIFICATION AND QR FACTORIZATION

2.1. Problem Formulation

When the channel is time invariant, the received complex baseband signal x can be expressed as

$$x(t) = \sum_{k=-\infty}^{\infty} s_k h(t - kT) + n(t), \tag{1}$$

where s_k denotes the symbol emitted by the digital source at time kT, $h(\cdot)$ the discrete-time channel impulse response, T the symbol interval and $n(\cdot)$ the additive noise.

We assume throughout this paper that i) the input sequence s_k is stationary with zero mean and $E\{s_k s_l^*\} = \delta(k-l)$, ii) the noise n is stationary with zero mean and white with variance σ^2, iii) s & n are uncorrelated, and iv) the channel impulse response $h(\cdot)$ is of finite duration.

In the vector representation, we have

$$\mathbf{x}(iT) = \mathbf{H}\mathbf{s}(iT) + \mathbf{n}(iT), \quad i = 0, 1, \cdots, \tag{2}$$

where $\mathbf{x}(iT)$ and $\mathbf{n}(iT)$ are m-dimensional vectors formed from the m samples of $x(\cdot)$ and $n(\cdot)$ inside the interval $(t_0 + iT, t_0 + iT + m\Delta)$, where $\Delta = T/L$ is the sampling interval and the integer L denotes the number of samples in each symbol interval. Assume $L < m$. $\mathbf{H}$ is the matrix representation of the channel with dimension $m \times d$ as

$$\begin{bmatrix} h(\Delta - K_0 T) & \cdots & h(\Delta - (K_0 + d - 1)T) \\ \vdots & & \vdots \\ h(m\Delta - K_0 T) & \cdots & h(m\Delta - (K_0 + d - 1)T) \end{bmatrix}. \tag{3}$$

We assume $m \geq d$ and that $\mathbf{H}$ has full column rank. The relations between h and $\mathbf{H}$ are discussed in [1]. $\mathbf{s}(iT)$ is a d-dimensional vector consisting of symbols that have "contributions" to the received signal inside the observation interval $(t_0+iT, t_0+iT+m\Delta)$. In other words, at time t_0, the received signal contains information of symbols from K_0 to $(K_0 + d - 1)$.

Let $\{x_i : i = 0, 1, 2, \cdots\}$ denote the sampled data sequence, $x_i = x(t_0 + i\Delta)$, then we have

$$\mathbf{x}(iT) = [\ x_{iL} \quad x_{iL+1} \quad \cdots \quad x_{iL+m-1}\ ]^H \tag{4}$$

where $x_{iL+k} = x(t_0 + iL\Delta + k\Delta) = x(t_0 + iT + k\Delta)$ is the $(iL + k)th$ sampled data.

To simplify the presentation, we ignore the noise for the moment. Let $\mathbf{R_x}(k)$ denote the correlation matrix of the output data vector $\mathbf{x}$.

$$\mathbf{R_x}(k) = E[\mathbf{x}(iT)\mathbf{x}^H((i-k)T)] \tag{5}$$

Since $\mathbf{x}(iT) = \mathbf{Hs}(iT)$, we have

$$\mathbf{R_x}(0) = \mathbf{HH}^H, \quad \mathbf{R_x}(1) = \mathbf{HJH}^H \tag{6}$$

where $\mathbf{J}$ is a $d \times d$ "shifting" matrix

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}.$$

The identifiability of the channel is given by the following theorem.

Theorem 1 [1]: Suppose $\mathbf{H}$ is an $m \times d$ complex matrix of full column rank. Then $\mathbf{H}$ is uniquely determined up to a constant by $\mathbf{R_x}(0)$ and $\mathbf{R_x}(1)$.

2.2. QR Factorization of Output Data Matrix

The output data block matrix is formed by stacking the sampled data vector

$$\mathbf{X}_{N,m}(0) = [\ \mathbf{x}(0) \quad \mathbf{x}(T) \quad \cdots \quad \mathbf{x}((N-1)T)\ ]^H \tag{7}$$

The first subscript denotes the number of rows while the second subscript is the number of columns. The argument in parentheses denotes the subscript of the first entry. Considering the sampled data is cyclostationary with period T, we manipulate data with L samples as a set. When N is sufficiently large,

$$\frac{1}{N}[(\mathbf{X}_{N,m}(0))^H \mathbf{X}_{N,m}(0)] = \mathbf{HH}^H \tag{8}$$

$$\frac{1}{N}[(\mathbf{X}_{N,m}(L))^H \mathbf{X}_{N,m}(0)] = \mathbf{HJH}^H \tag{9}$$

In order to perform QR factorization of $\mathbf{X}_{N,m}(0)$ and $\mathbf{X}_{N,m}(L)$ at the same time, we construct the following matrix,

$$\mathbf{X}_{N,L+m}(0) = [\ \mathbf{X}_{N,m}(0) \ \vdots\ \mathbf{X}_{N,L}(m)\ ]. \tag{10}$$

From (10) we see $\mathbf{X}_{N,m}(0)$ has rank d with $d \le m$. Performing QR factorization with column pivoting to $\mathbf{X}_{N,m}(0)$ and apply the same factorization to $\mathbf{X}_{N,L}(m)$, we have [5]

$$\mathbf{Q}^H \mathbf{X}_{N,L+m}(0) \begin{bmatrix} \mathbf{\Pi} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_L \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{12} & \vdots & \mathbf{R}_{13} \\ \mathbf{0} & \mathbf{0} & \vdots & \mathbf{R}_{23} \end{bmatrix} \tag{11}$$

where $\mathbf{R}_{11}$ is upper triangular and nonsingular with dimension $d \times d$.

Theorem 2 [5]: Let

$$\mathbf{R}_1 = [\ \mathbf{R}_{11} \quad \mathbf{R}_{12}\ ]\mathbf{\Pi}^H, \quad \mathbf{R}_2 = [\ \mathbf{R}_{1s} \quad \mathbf{R}_{13}], \tag{12}$$

where $\mathbf{R}_{1s}$ is a submatrix consisting of the last $m - L$ columns of $\mathbf{R}_1$. Then

$$\mathbf{HH}^H = \frac{1}{N}\mathbf{R}_1^H \mathbf{R}_1, \ \ \mathbf{HJH}^H = \frac{1}{N}\mathbf{R}_2^H \mathbf{R}_1. \tag{13}$$

3. RECURSIVE ULV-BASED ALGORITHM

3.1. Updating Rank-Revealing ULV Decomposition

Based on Theorem 2, we developed a block QR factorization based algorithm for channel identification and equalization in [5], which has computation complexity of $O(Nm^2 + md^2)$. Using rank-revealing ULV decomposition, we will develop a recursive algorithm with computation complexity $O(dm^2)$, and even $O(m^2)$. We will also show that the recursive algorithm can track channel variations quickly and efficiently. Hence it is more suitable for real time implementation.

Theorem 4: Given $\mathbf{X}_{N,L+m}(0)$, there exists an $N \times N$ orthonormal matrix $\mathbf{U}$, an $m \times m$ orthonormal matrix $\mathbf{V}$ such that

$$\mathbf{U}^H \mathbf{X}_{N,L+m}(0) \begin{bmatrix} \mathbf{V} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_L \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{0} & \vdots & \mathbf{T}_{13} \\ \mathbf{0} & \mathbf{0} & \vdots & \mathbf{T}_{23} \end{bmatrix}, \tag{14}$$

where $\mathbf{T}_{11}$ is $d \times d$ lower triangular, $\mathbf{T}_{13}$ is $d \times L$.

Proof: Denoted as rank-revealing ULV decomposition in [3],[4], there exist orthonormal matrices $\mathbf{U}$ and $\mathbf{V}$ such that

$$\mathbf{U}^H \mathbf{X}_{N,m}(0)\mathbf{V} = \begin{bmatrix} \mathbf{T}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \triangleq \mathbf{T}. \tag{15}$$

Let

$$\mathbf{U}^H \mathbf{X}_{N,L}(m) = [\ \mathbf{T}_{13}^H \quad \mathbf{T}_{23}^H\ ]^H, \tag{16}$$

we proved Theorem 4. □

We briefly summarize the implementation of ULV in our case. Details about the decomposition and rank determination can be found in [3], [4]. We assume the sampled signal contains noise. In order to track possible channel variations, we have to phase out old data gradually. Hence a factor $\gamma \le 1$ is used. Suppose at iteration i, the decomposition is

$$\mathbf{U}^{(i)H} \mathbf{X}_{i,L+m}(0) \begin{bmatrix} \mathbf{V}^{(i)} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_L \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{11}^{(i)} & \mathbf{0} & \vdots & \mathbf{T}_{13}^{(i)} \\ \mathbf{T}_{21}^{(i)} & \mathbf{T}_{22}^{(i)} & \vdots & \mathbf{T}_{23}^{(i)} \\ \mathbf{0} & \mathbf{0} & \vdots & \mathbf{T}_{33}^{(i)} \end{bmatrix}, \tag{17}$$

Suppose the signal dimension is d, then $\mathbf{T}_{11}^{(i)}$ is $d \times d$ lower triangular. $\mathbf{T}_{22}^{(i)}$ is also lower triangular. Since $\mathbf{T}_{21}^{(i)}, \mathbf{T}_{22}^{(i)}$ are in the noise subspace, their elements are much smaller than those in $\mathbf{T}_{11}^{(i)}$. Specifically, they are $\mathbf{0}$ in the noiseless case.

At iteration $i+1$, we have a new set of samples (L samples which correspond to a new symbol), and we need to compute the ULV decomposition of $\mathbf{X}_{i+1,L+m}(0)$. Let

$$\begin{aligned}\mathbf{x} &= [x_{iL} \ \cdots \ x_{iL+m-1} \vdots x_{iL+m} \ \cdots \ x_{(i+1)L+m-1}] \\ &= [\ \mathbf{x}_1 \ \vdots \ \mathbf{x}_2\ ], \end{aligned} \tag{18}$$

then,

$$\begin{bmatrix} \gamma\mathbf{X}_{i,L+m}(0) \\ \mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{U}^{(i)} & \\ & 1 \end{bmatrix} \begin{bmatrix} \gamma\mathbf{T}_{11}^{(i)} & \mathbf{0} & \vdots & \gamma\mathbf{T}_{13}^{(i)} \\ \gamma\mathbf{T}_{21}^{(i)} & \gamma\mathbf{T}_{22}^{(i)} & \vdots & \gamma\mathbf{T}_{23}^{(i)} \\ \mathbf{0} & \mathbf{0} & \vdots & \gamma\mathbf{T}_{33}^{(i)} \\ \mathbf{x}_1\mathbf{V}^{(i)} & & \vdots & \mathbf{x}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}^{(i)H} & \\ & \mathbf{I}_L \end{bmatrix}. \tag{19}$$

According to ULV decomposition, we perform a series of left and right Givens rotations to zero the last row,

$$\begin{bmatrix} \gamma\mathbf{T}_{11}^{(i)} & \mathbf{0} & \vdots & \gamma\mathbf{T}_{13}^{(i)} \\ \gamma\mathbf{T}_{21}^{(i)} & \gamma\mathbf{T}_{22}^{(i)} & \vdots & \gamma\mathbf{T}_{23}^{(i)} \\ \mathbf{0} & \mathbf{0} & \vdots & \gamma\mathbf{T}_{33}^{(i)} \\ \mathbf{x}_1\mathbf{V}^{(i)} & & \vdots & \mathbf{x}_2 \end{bmatrix} = \mathbf{U}_1^{(i)} \begin{bmatrix} \mathbf{T}_{11}^{(i+1)} & \mathbf{0} & \vdots & \mathbf{T}_{13}^{(i+1)} \\ \mathbf{T}_{21}^{(i+1)} & \mathbf{T}_{22}^{(i+1)} & \vdots & \mathbf{T}_{23}^{(i+1)} \\ \mathbf{0} & \mathbf{0} & \vdots & \mathbf{T}_{33}^{(i+1)} \\ \mathbf{0} & \mathbf{0} & \vdots & \mathbf{x}_3 \end{bmatrix} \mathbf{V}_1^{(i)}, \tag{20}$$

where $\mathbf{T}_{11}^{(i+1)}, \mathbf{T}_{22}^{(i+1)}$ are lower triangular. The dimension of $\mathbf{T}_{11}^{(i+1)}$ is either $d+1, d$, or $d-1$, which can be determined by some reliable condition estimator [3].

Finally the decomposition result at iteration $i+1$ is similar to equation (17) with

$$\begin{aligned} \mathbf{U}^{(i+1)} &= \begin{bmatrix} \mathbf{U}^{(i)} & \\ & 1 \end{bmatrix} \mathbf{U}_1^{(i)} \\ \mathbf{V}^{(i+1)} &= \begin{bmatrix} \mathbf{V}^{(i)} & \\ & \mathbf{I}_L \end{bmatrix} \mathbf{V}_1^{(i)H}. \end{aligned} \tag{21}$$

Note that in our recursive channel identification algorithm, the matrix $\mathbf{U}^{(i+1)}$ does not need to be computed.

The above operations involve mainly Givens rotations and condition estimation. The total computation is in the order of $O(m^2)$.

3.2. Evaluation of H

In each iteration $i+1$, let

$$\mathbf{R}_1 = [\ \mathbf{T}_{11}^{(i+1)} \ \ \mathbf{0}\ ]\mathbf{V}^{(i+1)H}, \mathbf{R}_2 = [\ \mathbf{R}_{1s}^{(i+1)} \ \ \mathbf{T}_{13}^{(i+1)}\ ], \tag{22}$$

where $\mathbf{R}_{1s}^{(i+1)}$ is a submatrix consists of the last $m-L$ columns of $\mathbf{R}_1^{(i+1)}$. Our objective now is to estimate $\mathbf{H}$ based on $\mathbf{R}_1$ and $\mathbf{R}_2$.

Equation (13) implies that there exists a $d \times d$ orthonormal matrix $\mathbf{P}$ such that

$$\mathbf{H} = \frac{1}{\sqrt{N}}\mathbf{R}_1^H\mathbf{P}. \tag{23}$$

Let

$$\mathbf{R} = (\mathbf{R}_1^H)^+\mathbf{R}_2^H, \tag{24}$$

where $(\cdot)^+$ denotes pseudoinverse, then from (23), (24), (13),

$$\mathbf{R} = \mathbf{P}\mathbf{J}\mathbf{P}^H, \tag{25}$$

which is in the same form as the result in [1] although our $\mathbf{R}$ here may be an orthogonal transformed version of $\mathbf{R}$ in [1]. A transformed version of $\mathbf{P}$ is computed by SVD in [1], which, however, does not satisfy the orthonormal condition of $\mathbf{P}$.

The exact solution to (25) may not exist. Therefore, it is an optimization problem to evaluate $\mathbf{P}$. Using the orthogonality of $\mathbf{P}$ we have the following theorem to evaluate (25):

Theorem 3 [5]: Let $\mathbf{P} \triangleq [\ \mathbf{p}_1, \ \mathbf{p}_2, \ \cdots, \ \mathbf{p}_d\ ]$, then equation (25) gives

$$\mathbf{p}_{k+1} = \mathbf{R}\mathbf{p}_k, \quad k = 1, 2, \cdots, d-1. \tag{26}$$

$\mathbf{p}_1$ satisfies the following equations:

$$\mathbf{p}_1\mathbf{p}_1^H = \mathbf{I} - \mathbf{R}\mathbf{R}^H \tag{27}$$

$$\mathbf{R}^d\mathbf{p}_1 = \mathbf{0}. \tag{28}$$

Equation (27) shows that the matrix $\mathbf{I}-\mathbf{R}\mathbf{R}^H$ consists of scaled rows and columns of $\mathbf{p}_1$. Thus ideally any row or column of $\mathbf{I}-\mathbf{R}\mathbf{R}^H$ is an estimate of $\mathbf{p}_1$ up to a constant multiplier. Considering that $\mathbf{I}-\mathbf{R}\mathbf{R}^H$ contains noise (computation error or additive noise), we can choose $\mathbf{p}_1$ as the mean vector of the columns of $\mathbf{I}-\mathbf{R}\mathbf{R}^H$. Simulation results demonstrate the effectiveness of this approach. A detailed discussion of estimating $\mathbf{H}$ can be found in [5].

3.3. Computational Complexity

Because we have to compute d column vectors of $\mathbf{H}^{(i)}$, each with computation of $O(m^2)$. So the total computation is of $O(dm^2)$. However, if the channel does not vary too fast, we can update only one column at each recursion. So after d recursions, $\mathbf{H}^{(i)}$ is updated. Under these simplifications, the total computation is in the order of $O(m^2)$.

If our goal is to estimate the equalized symbols only, then we do not have to find the entire matrix $\mathbf{H}$. In fact, only one row of $\mathbf{H}^+$ is needed. According to (23), only $\mathbf{p}_1$ needs to be estimated. Hence, the total computation is $O(m^2)$ in this case. This is not the case for [1] and [2], where the entire SVD (EVD) still needs to be performed.

4. SIMULATION

In this experiment, we investigate the performance of our algorithm for a time-varying channel. The channel is a two-ray multipath channel. The continuous-time channel $h_c(t)$

for $t \in [0, 4T)$ is described by $h_c(t) = a_0(t) r_c(t - \gamma_0, \beta) + a_1(t) r_c(t - \gamma_1, \beta)$ where r_c is the raised cosine function [6], $\beta = 0.35$, $\gamma_0 = 0.25, \gamma_1 = 1$. The discrete-time channel $h(n)$ is obtained by $h(n) = h_c(nT/5)$ for $n = 0, 1, \cdots, 19$. $a_0(t)$ and $a_1(t)$ change with time. For $n \leq 150$, we set $a_0(t) = e^{-j2\pi(0.15)}$, $a_1(t) = 0.8e^{-j2\pi(0.6)}$. For $n > 150$, we set $a_0(t) = e^{-j2\pi(0.45)}, a_1(t) = 0.4e^{-j2\pi(0.1)}$. Let $L = 5, m = 10, \gamma = 0.99$. The source symbols were drawn from a 16 QAM signal constellation with a uniform distribution. SNR= 30dB.

From Figure 1, we see that our algorithm is able to identify the time-varying channel. The tracking speed depends on γ. Using a smaller γ may achieve faster convergence, but residual NRMSE may be larger.

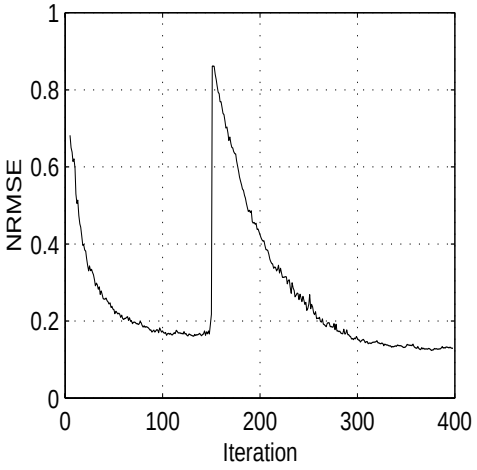

Figure 1: *NRMSE for time-varying channel. 100 Monte Carlo runs and 400 symbols in each run.*

In Fig. 2 we transmit 1000 symbols at SNR= 30dB and equalize by the identified channel matrix H at iteration $n = 396$ in Fig. 1. Clearly the channel is equalized.

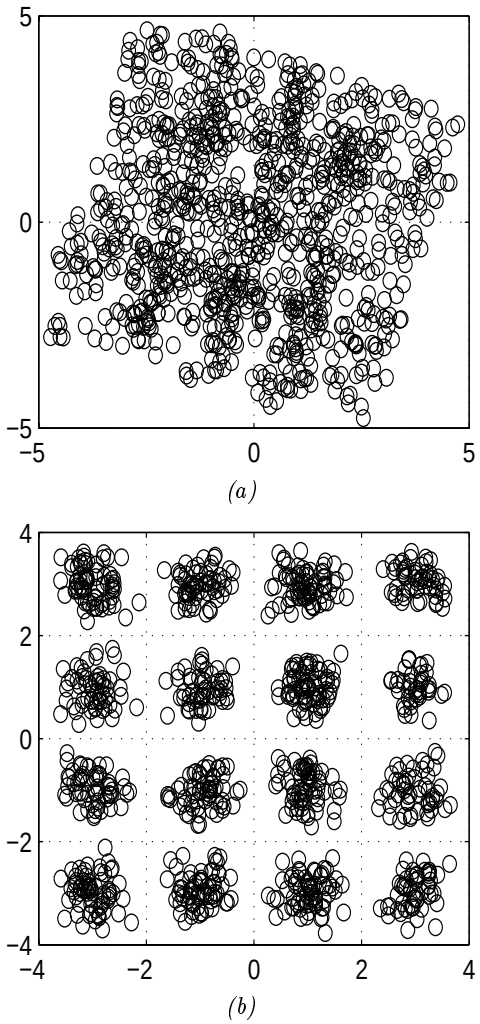

Figure 2: *(a): The unequalized channel output. (b): Equalized channel output.*

5. REFERENCES

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