BIAS ANALYSIS IN CONTINUOUS ADAPTATION SYSTEMS FOR HEARING AIDS

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ABSTRACT

This paper studies analytically the steady-state convergence behavior of adaptive algorithms that approximate the Wiener solution when operating in continuous adaptation to reduce acoustic feedback in hearing aids. A bias is found in the adaptive filter's estimate of the hearing-aid feedback path when the input signal is not white. Delays in the forward and cancellation paths are shown to reduce the magnitude of the bias. Equations for the bias transfer function are obtained. A discussion about properties of the bias when delays are placed in the forward and cancellation paths follows.

1. INTRODUCTION

A major complaint of hearing-aid users is acoustic feedback which is perceived as whistling or howling (at oscillation) or distortion (at sub-oscillatory intervals). This feedback occurs, typically at high gains, because of leakage from the receiver to the microphone (Fig. 1). Acoustic feedback suppression in hearing-aids is important since it can increase the maximum insertion gain of the aid. The acoustic path transfer function can vary significantly depending on the acoustic environment [1]. Hence, effective acoustic feedback cancellers must be adaptive.

This paper analyzes the performance of adaptive filtering algorithms [2] when correlated inputs are used in continuous adaptation systems for hearing aids. It will be shown that any algorithm that attempts to approximate the Wiener solution in continuous adaptation systems will obtain a biased estimate of the feedback path due to the cross-correlation between input and output signals in the hearing aid. Although previous works have addressed the necessity of decorrelation mechanisms for a better estimate of the feedback path by using delays in the cancellation and forward paths [3], [4], none of these works have proposed an analysis for the bias. This work proposes analytical expressions to predict the bias; properties of continuous adaptation systems based on these analytical expressions are then discussed.

2. THE BIAS IN THE WIENER ESTIMATE

A Wiener filter estimates a system transfer function $H(z)$ given the input signal to the system $s(n)$ and a corrupted system output signal $d(n)$. The corrupted signal is assumed to be the convolution between the system impulse response and the input signal $[s(n) * h(n)]$ added to a noisy component $r(n)$. It can be written as

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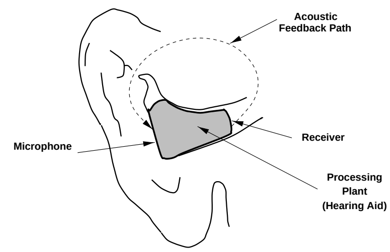

Figure 1: Acoustic feedback in a hearing aid coupled with a human ear.

$$d(n) = s(n) * h(n) + r(n). \tag{1}$$

In most cases, $r(n)$ is assumed to be white noise. The Wiener filter estimate [5] for $H(z)$ is a vector $\hat{\mathbf{h}}$ with the impulse response coefficients given by

$$\hat{\mathbf{h}} = \mathbf{R}^{-1}\mathbf{p}_{sd} \tag{2}$$

where

$$\mathbf{R} \triangleq E\{\mathbf{s}(n)\mathbf{s}^T(n)\} \tag{3}$$

$$\mathbf{p}_{sd} \triangleq E\{\mathbf{s}(n)d(n)\} \tag{4}$$

$$\mathbf{s}(n) \triangleq [s(n)\; s(n-1)\; \cdots\; s(n-N)]^T \tag{5}$$

and N is the order of the Wiener estimate.

Now consider the case where the interference signal $r(n)$ is not white and is derived from the input signal $s(n)$ by a linear system. It is easy to prove that (2) will not lead to a proper estimate of the impulse response of $H(z)$. Suppose that $r(n)$ is derived from

$$r(n) = s(n) * q(n) \tag{6}$$

where $q(n)$ is the impulse response of an unknown system that derives the noisy signal $r(n)$ from the input signal $s(n)$. It is possible to show that the Wiener estimate impulse response vector $\hat{\mathbf{h}}$ deviates from the impulse response of $H(z)$ by

$$\boldsymbol{\varepsilon} = \mathbf{R}^{-1}\mathbf{p}_{sr} \tag{7}$$

where $\boldsymbol{\varepsilon}$ is defined as

$$\varepsilon = \hat{\mathbf{h}} - \mathbf{h} \tag{8}$$

and $\mathbf{h}$ is a vector with the impulse response of $H(z)$. It should be noted that the entries of the vector ε contain a Wiener estimate for the impulse response $q(n)$.

3. APPLICATION TO A FEEDBACK REDUCTION SYSTEM

In this section, expressions for the bias are derived for the cases of delays in the forward or cancellation paths. Superscripts f and c will be used to indicate delays in the forward and cancellation paths, respectivelly. It is assumed that the forward path transfer function is $G(z) = G_c z^{-1}$ [4]. The used feedback path was obtained from measurements [1] and is shown in Fig. 2. All the vectors are assumed to have the Wiener Filter length ($N+1$ taps - N^{th}-order estimate) except where indicated with subscripts.

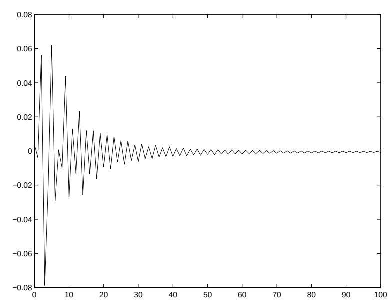

Figure 2: Impulse response of a hearing-aid feedback path for a human subject.

In the analysis, the bias in the feedback reduction system will be defined as

$$\varepsilon = \mathbf{w} - \mathbf{f} \tag{9}$$

where $\mathbf{w}$ is a vector with the Wiener estimate for a hearing aid plant based on the signals $x(n)$ and $d(n)$ and $\mathbf{f}$ is a vector with the feedback path impulse response.

3.1. The bias with delays in the forward path

In the hearing aid plant shown in Fig. 3, it can be seen that the reference signal $d(n)$ is

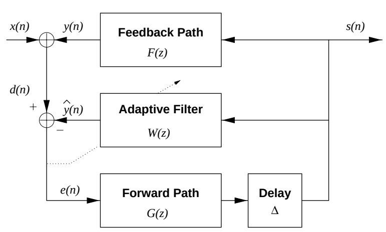

Figure 3: Model of a hearing aid plant with a delay in the forward path.

$$d(n) = y(n) + x(n) \tag{10}$$

where $y(n)$ is the output of the feedback path. By using (7) and (10), it follows that the bias in the Wiener estimate of the feedback path $F(z)$ is

$$\varepsilon^f = \mathbf{R}^{-1}\mathbf{p}_1^f \tag{11}$$

where

$$\mathbf{p}_1^f = E\{\mathbf{s}(n)x(n)\}. \tag{12}$$

Supposing that the bias vector is sufficiently small, it can be assumed that

$$s(n) \approx G_c x(n - \Delta - 1). \tag{13}$$

Using the above approximation in (11), it is possible to obtain

$$\varepsilon^f \approx \frac{1}{G_c}\mathbf{R}_{xx}^{-1}\mathbf{p}_{x,\Delta+1} \tag{14}$$

where

$$\mathbf{p}_{x,\Delta+1} = E\{\mathbf{x}(n-\Delta-1)x(n)\} \tag{15}$$

Simulations with the NLMS algorithm [2] using speech-shaped noise as input, a normalized convergence factor of $\mu' = 10^{-4}$, a total of 5 million points, an impulse response with the first 20 samples of Fig. 2 were run to determine the value of $\|\varepsilon^f\|$ for various values of delays in the forward path. The results with gain $G_c = 2$ shown in Fig. 4. which illustrates that the predicted and simulated results are in good agreement. Simulations with different gains ranging between $G_c = 3$ to 12 were also run, and there was good agreement between simulated and predicted results.

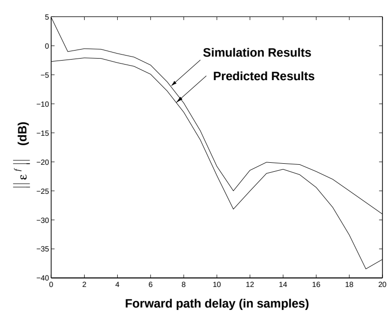

Figure 4: $\|\varepsilon^f\|$ as a function of forward path delay

3.2. The bias with delays in the cancellation path

In the case of the hearing aid plant shown in Fig. 5, the Wiener filter will be able to estimate the last $N+1$ terms of the impulse response, since the first Δ samples of the impulse response will be approximated by zeroes.
It is also assumed that $F(z)$ can be divided into two components

$$F_1(z) = \mathbf{f}_{1,\Delta}^T \mathbf{T}_\Delta(z^{-1}) \tag{16}$$

$$F_2(z) = \mathbf{f}_2^T \mathbf{T}(z^{-1}) \tag{17}$$

where $\mathbf{f}_{1,\Delta}$ is a vector with the first Δ samples of the impulse response, $\mathbf{f}_2$ is a vector with the remaining $N+1$ samples, and $\mathbf{T}_\Delta(z)$ and $\mathbf{T}(z)$ are defined as

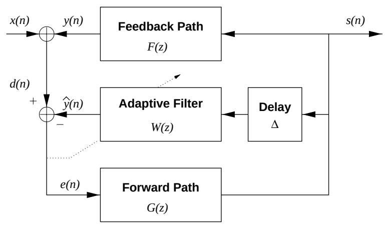

Figure 5: Model of a hearing aid plant with a delay in the cancellation path

$$\mathbf{T}_\Delta(z) \triangleq [1\; z\; \cdots\; z^{\Delta}]^T \tag{18}$$
$$\mathbf{T}(z) \triangleq [1\; z\; \cdots\; z^{N}]^T. \tag{19}$$

Consequently, the reference signal can thus be written as

$$d(n) = y_1(n) + y_2(n) + x(n) \tag{20}$$

where

$$y_1(n) \triangleq \mathbf{f}_{1,\Delta}^T \mathbf{s}_\Delta(n) \tag{21}$$
$$y_2(n) \triangleq \mathbf{f}_2^T \mathbf{s}(n). \tag{22}$$

The components $y_1(n)$ and $y_2(n)$ are illustrated in Fig. 6.

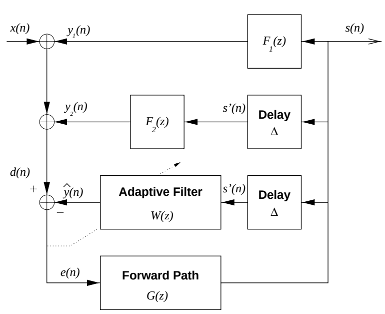

Figure 6: Model of a hearing aid plant with a delay in the cancellation path and with the feedback path divided into two components.

It is clear that the bias will contain two components in this case; one, due to the transfer function between $x(n)$ and $s(n)$, and a second bias term due to the output of $F_1(z)$. The bias can then be written as

$$\boldsymbol{\varepsilon}_2^c = \mathbf{R}^{-1}\mathbf{p}_1^c + \mathbf{R}^{-1}\mathbf{p}_2^c \tag{23}$$

where

$$\mathbf{p}_1^c \triangleq E\{\mathbf{s}(n-\Delta)x(n)\} = E\{\mathbf{s}(n)x(n+\Delta)\} \tag{24}$$
$$\mathbf{p}_2^c \triangleq E\{\mathbf{s}(n-\Delta)y_1(n)\} = E\{\mathbf{s}(n)\mathbf{s}_\Delta^T(n+\Delta)\}\mathbf{f}_{1,\Delta} \tag{25}$$

There is, however, another term in the bias vector due to approximating the first Δ samples of $F(z)$ with zeroes. This term can be written as

$$\boldsymbol{\varepsilon}_{1,\Delta}^c = \mathbf{0} - \mathbf{f}_{1,\Delta} = -\mathbf{f}_{1,\Delta}. \tag{26}$$

Thus, the total bias should be equal to

$$\boldsymbol{\varepsilon}^c = \begin{bmatrix} \boldsymbol{\varepsilon}_{1,\Delta}^c \\ \boldsymbol{\varepsilon}_2^c \end{bmatrix} = \begin{bmatrix} -\mathbf{f}_{1,\Delta} \\ \mathbf{R}^{-1}\mathbf{p}_1^c + \mathbf{R}^{-1}\mathbf{p}_2^c \end{bmatrix}. \tag{27}$$

If the bias vectors $\boldsymbol{\varepsilon}_2^c$ and $\boldsymbol{\varepsilon}_1^c$ are sufficiently small, the following approximation for $s(n)$ holds

$$s(n) \approx G_c x(n-1). \tag{28}$$

Substituting the above approximation into (23), it is possible to obtain the following approximation for $\boldsymbol{\varepsilon}_2^c$

$$\boldsymbol{\varepsilon}_2^c \approx \frac{1}{G_c}\mathbf{R}_{xx}^{-1}\mathbf{p}_{x,\Delta+1} + \mathbf{R}_{xx}^{-1}E\{\mathbf{x}(n)\mathbf{x}^T(n+\Delta)\}\mathbf{f}_1. \tag{29}$$

Thus, the approximation for total bias in this case is

$$\boldsymbol{\varepsilon}^c \approx \begin{bmatrix} -\mathbf{f}_{1,\Delta} \\ \mathbf{R}_{xx}^{-1}\left\{\frac{\mathbf{p}_{x,\Delta+1}}{G_c} + E[\mathbf{x}(n)\mathbf{x}^T(n+\Delta)]\mathbf{f}_1\right\} \end{bmatrix}. \tag{30}$$

Simulations with the NLMS algorithm [2] using the same parameters described in the previous section were run to determine the value of $\boldsymbol{\varepsilon}^c$ for various values of delays in the cancellation path. The results are shown in Fig. 7 which illustrate the accuracy of the predictions.

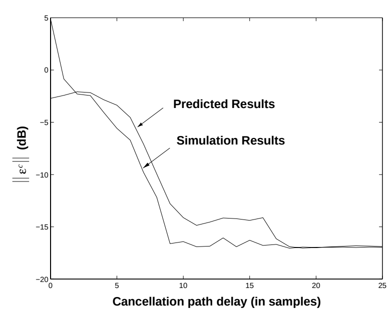

Figure 7: $\|\boldsymbol{\varepsilon}^c\|$ as a function of cancellation path delay.

4. PROPERTIES OF FEEDBACK REDUCTION SYSTEMS

4.1. Performance in Steady-State

Simulations were carried out to compare the performance of the NLMS adaptive filter [5] with and without delayed forward and cancellation paths. The inputs were white gaussian noise and speech-shaped noise [6] with a low-pass magnitude transfer function. The used convergence metric is defined by $\mathcal{C}(n)$

$$\mathcal{C}(n) = 20\log_{10}\|\mathbf{w}(n) - \mathbf{f}\|. \tag{31}$$

Figure 8 shows the convergence of the NLMS adaptive filter with convergence factor $\mu' = 0.1$, order $N = 100$ and $G_c = 2$. As expected, when a white noise input signal is used, the hearing-aid input and output signals are reasonably well decorrelated, thus reducing the adaptive filter bias. Figure 8 also shows that if the input is correlated noise and delays are included in the forward path the performance improves by 20 dB resulting in $\mathcal{C}(n) \leq -10$ dB in steady state. The used delay was 10 samples at 8KHz. When the same delay is used in the cancellation path, the adaptive filter

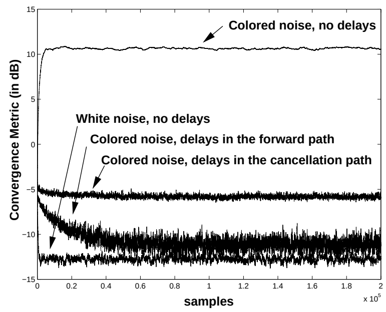

Figure 8: Adaptive filter convergence using either white or speech-shaped noises. The used delay of 10 samples was used in the forward and cancellation paths

also converges, but with a higher bias. This can be explained by the fact that delays in the cancellation path approximate the first Δ samples of the feedback impulse response with zeros, a bad approximation for the feedback path used here, thus causing a higher bias.

4.2. The Dependency on the Feedback Path Characteristics and on the Gain G_c

Equation (14) shows that the bias vector is dependent on the input signal characteristics, the forward path gain G_c and the number of delays in the forward path. Hence, there is no dependency on the feedback path impulse response for the delayed forward path case. On the other hand, by inspecting (30) it is clear that the bias is a function of the first Δ samples of the feedback path ($\mathbf{f}_{1,\Delta}$) when delays are placed in the cancellation path.

Equation (14) also shows that the magnitude of the bias vector is inversely propotional to the gain G_c, while equation (30) indicates that only one of the terms is inversely proportinal to G_c.

4.3. The System Transfer Function

In the case that the forward path delay is Δ, the overall transfer function between $S(z)$ and $X(z)$ can be written as below

$$H_{sx}(z) = \frac{S(z)}{X(z)} = \frac{G_c z^{-\Delta-1}}{1 + \sum_{i=0}^{N} G_c \varepsilon_i z^{-i-\Delta-1}} \quad (32)$$

where ε_i are the elements of the vector $\boldsymbol{\varepsilon}$. It is possible to obtain an approximation for $S(z)$ in the case that the coefficients ε_i are sufficiently small

$$S(z) \approx G_c X(z) z^{-\Delta-1} \left[1 - \sum_{i=0}^{N} G_c \varepsilon_i z^{-i-\Delta-1} \right] \quad (33)$$

The factors $G_c\varepsilon_i$ are given approximately by (14) that are the coefficients of a predictor with lag $\Delta + 1$. Thus, (33) is the multiplication of $X(z)$ with the inverse of the LPC transfer function [7]. The inverse of the LPC transfer function is known to remove the peaks associated with formants in a voiced segment of speech [7]. This explains the whitening of voiced signals observed by other investigators [3], [4] when continuous adaptation is performed.

5. CONCLUSIONS

Unlike most adaptive filtering applications [2], [5], the LMS algorithm and any algorithm that attempts to approximate a Wiener solution in continuous feedback reduction systems for hearing aids has a bias in its estimate when correlated inputs, such as speech, are used. This bias is due to non-zero cross-correlation of the input and output signals in the hearing aid plant. It was shown that it is possible to derive analytical approximations that lead to closed-form equations for the bias when delays are placed in the forward or cancellation paths.

The derived equations indicate that the bias decreases with increasing values of the forward gain G_c. The analysis showed that the bias in the adaptive filter's estimate is a function of the feedback path characteristics only in the case where a delay is placed in the cancellation path. On the other hand, when a delay is placed in the forward path, the bias is independent of the feedback path. In addition, the derived equations showed that when a delay is placed in the cancellation path, the minimum bias will be limited by the first Δ samples of the feedback path impulse response. Such limit does not exist when a delay is placed in the forward path. This might explain why Estermann and Kaelin [4], who used a delayed forward path, obtained better results than other works [8], [3] which used a delayed cancellation path to decorrelate the input and output signals.

6. REFERENCES

[1] M. Siqueira, R. Speece, E. Petsalis, A. Alwan, S. Soli, and S. Gao, "Subband adaptive filtering applied to acoustic feedback reduction in hearing aids," *Proceedings of the Asilomar Conference on Signals, Systems, and Computers*, pp. 778–782, Nov. 1996.

[2] P. Diniz, *Adaptive Filtering*. Norwell, Massachussets: Kluwer Academic Publishers, 1997.

[3] D. Bustamante, T. Worrall, and M. Williamson, "Measurement of adaptive suppression of acoustic feedback in hearing aids," *Proc. 1989 IEEE ICASSP*, pp. 2017–2020, 1989.

[4] P. Estermann and A. Kaelin, "Feedback cancellation in hearing aids: Results from using frequency-domain adaptive filters," *Proc. 1994 IEEE ISCAS*, pp. 257–260, 1994.

[5] S. Haykin, *Adaptive Filter Theory*. Upper Saddle River, New Jersey: Prentice-Hall, 1996.

[6] D. Byrne and H. Dillon, "New procedure for selecting the gain and frequency response of a hearing aid," *Ear and Hearing 7, The National Acoustic Laboratories*, pp. 257–265, 1986.

[7] L. Rabiner and R. Schafer, *Digital Processing of Speech Signals*. Englewood Cliffs, New Jersey: Prentice-Hall, 1978.

[8] J. Maxwell and P. Zurek, "Reducing acoustic feedback in hearing aids," *IEEE Transactionson Speech and Audio Processing*, vol. 4, pp. 304–313, July 1995.

[9] M. Siqueira, A. Alwan, and R. Speece, "Steady-state analysis of continuous adaptation systems for hearing aids," *Proc. of the IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, 1997.