CANONICAL SPACE-TIME PROCESSING IN CDMA SYSTEMS

Eko N. Onggosanusi, Akbar M. Sayeed, and Barry D. Van Veen

Department of Electrical and Computer Engineering
University of Wisconsin–Madison
Madison, WI 53706
{onggo,akbar,vanveen}@ece.wisc.edu

ABSTRACT

We propose a canonical space-time receiver structure for wireless communications based on front-end processing with respect to a fixed basis that is independent of the true channel parameters. The basis is dictated by a canonical characterization of channel propagation dynamics in terms of discrete multipath delays, Doppler shifts, and directions of arrival that capture the essential degrees of freedom in the received signal. In addition to dramatically reducing the number of channel parameters to be estimated, performance analysis demonstrates that canonical space-time receivers can deliver optimal performance at substantially reduced complexity compared to existing designs.

1. INTRODUCTION

The use of antenna arrays for enhancing the capacity and quality of multiuser wireless communication systems has spurred significant interest in space-time signal processing techniques [3]. A key consideration in space-time receiver design is the complex time-varying multipath propagation environment. The front-end processing in most existing receivers is based on matched filtering corresponding to all the dominant multipaths and corresponding direction of arrivals (DOAs). In addition to suffering from high computational complexity in a dense multipath environment, such receivers rely heavily on accurate estimation of delay and DOA parameters of dominant scatterers.

In this paper we propose a canonical space-time receiver structure that is based on processing with respect to a *fixed* underlying basis. The basis is derived from a canonical representation of the channel propagation dynamics in terms of discrete multipath delays, Doppler shifts, and DOAs. These bases capture the essential degrees of freedom in the received signal. In addition to obviating the need for estimating arbitrary delays and DOAs, performance analysis demonstrates that the proposed receivers deliver optimal performance at substantially lower complexity compared to existing designs, especially in dense multipath environments.

The next section describes the canonical channel representation that underlies the proposed receivers. Section 3 describes the canonical receiver structure for coherent processing, and Section 4 provides performance analysis. Examples illustrating the performance are presented in Section 5.

2. CHANNEL MODEL

Consider the mobile to base station propagation over the wireless channel. We assume that the base station employs a sensor array of aperture D, and initially we consider a continuous aperture to develop the channel model, as illustrated in Figure 1. Let S_a denote the angular spread of the scatterers involved in signal propagation. We study the effect of the channel on a single symbol. The signal received at location α within the array is described as

$$s_\alpha(t) = \int_{-S_a/2}^{S_a/2} e^{-j\frac{2\pi \sin(\phi)\alpha}{\lambda}} x_\phi(t) d\phi \ , \quad -\frac{D}{2} < \alpha \leq \frac{D}{2}, \quad (1)$$

where $x_\phi(t)$ denotes the signal arriving from the direction ϕ, and λ denotes the carrier wavelength. The signal $x_\phi(t)$ is related to the transmitted spread-spectrum symbol waveform $q(t)$ via the angle-dependent multipath-Doppler spreading function $H_\phi(\zeta,\tau)$:

$$x_\phi(t) = \int_0^{T_m} \int_{-B_d}^{B_d} H_\phi(\zeta,\tau) q(t-\tau) e^{j2\pi\zeta t} d\zeta d\tau, \quad (2)$$

where T_m and B_d denote the multipath and Doppler spreads, respectively, encountered during propagation. We note that T_m and B_d denote the maximum possible values of the spreads. In general, the actual spreads will vary with ϕ, and that dependence is captured by the variation of $H_\phi(\zeta,\tau)$ as a function of ϕ.

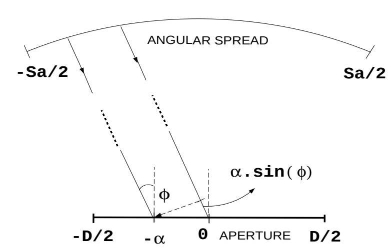

Figure 1: Signal reception geometry.

The basic idea behind the canonical characterization of $s_\alpha(t)$ is as follows. Due to the finite symbol duration T and essentially finite bandwidth B of $q(t)$, the received signal $x_\phi(t)$ exhibits only a finite number of *temporal* degrees of freedom that are captured by uniformly spaced discrete multipath delays and Doppler shifts [2]:

$$x_\phi(t) \approx \frac{1}{TB}\sum_{l=0}^{L}\sum_{k=-K}^{K}\widetilde{H}_\phi\left(\frac{k}{T},\frac{l}{B}\right) q\left(t-\frac{l}{B}\right)e^{j\frac{2\pi kt}{T}}, \quad (3)$$

where $\widetilde{H}_\phi(\zeta,\tau)$ is a (ζ,τ)-smoothed version of $H_\phi(\zeta,\tau)$ arising due to the finite duration and bandwidth of $q(t)$. By the same token, in a spatial context, the finite array aperture dictates that the received signal $s_\alpha(t)$ exhibits only a finite number of *spatial* degrees of freedom that are captured by certain discrete DOAs ϕ_l. The following canonical space-time characterization of the received signal $s_\alpha(t)$ identifies these essential spatio-temporal degrees of freedom in $s_\alpha(t)$.

Canonical Representation

The signal $s_\alpha(t)$, $0 < t \leq T$, $-D/2 < \alpha \leq D/2$, admits the following canonical representation

$$s_\alpha(t) \approx \frac{1}{DTB}\sum_{l=0}^{L}\sum_{k=-K}^{K}\sum_{p=-\widetilde{P}}^{\widetilde{P}}\widehat{H}_{\frac{p}{D}}\left(\frac{k}{T},\frac{l}{B}\right)q_\alpha^{klp}(t), \quad (4)$$

in terms of the space-time basis waveforms

$$q_\alpha^{klp}(t) = q\left(t-\frac{l}{B}\right)e^{j\frac{2\pi kt}{T}}e^{-j\frac{2\pi p\alpha}{D}}. \quad (5)$$

The coefficients in the representation are uniformly spaced samples of the smoothed spreading function

$$\begin{aligned}\widehat{H}_\phi(\zeta,\tau) &= DTB\int_{-\widetilde{S}_a}^{\widetilde{S}_a}\int_{-B_d}^{B_d}\int_0^{T_m}\overline{H}_{\phi'}(\zeta',\tau')e^{-j\pi(\zeta-\zeta')T} \\ &\quad \mathrm{sinc}((\phi-\phi')D)\mathrm{sinc}((\zeta-\zeta')T)\mathrm{sinc}((\tau-\tau')B) \\ &\quad d\zeta' d\tau' d\phi', \quad (6)\end{aligned}$$

where

$$\overline{H}_\phi(\zeta,\tau) = \frac{1}{\sqrt{1/\lambda^2-\phi^2}}H_{\sin^{-1}(\phi\lambda)}(\zeta,\tau). \quad (7)$$

The number of terms in (4) is determined by $L = \lceil T_m B\rceil$, $K = \lceil TB_d\rceil$ and $\widetilde{P} = \lceil D\widetilde{S}_a\rceil$ where

$$\widetilde{S}_a = \sin(S_a/2)/\lambda. \quad (8)$$

We note that under certain conditions (discussed later) the waveforms $q_\alpha^{klp}(t)$ constitute an orthogonal basis.

We now develop the canonical signal model for a uniform linear array consisting of M sensors with spacing d, corresponding to a uniform sampling of α. The M-dimensional vector $\mathbf{s}(t)$ corresponding to $s_\alpha(t)$ in (1) becomes

$$\int_{-S_a/2}^{S_a/2}\int_0^{T_m}\int_{-B_d}^{B_d}H_\phi(\zeta,\tau)\mathbf{a}(\phi)q(t-\tau)e^{j2\pi\zeta t}d\zeta d\tau d\phi \quad (9)$$

where

$$\mathbf{a}(\phi) = [1, e^{-j2\pi\frac{d\sin(\phi)}{\lambda}}, \cdots, e^{-j2\pi(M-1)\frac{d\sin(\phi)}{\lambda}}]^T/\sqrt{M}. \quad (10)$$

The space-time basis functions corresponding to (5) are

$$\begin{aligned}\mathbf{u}_{klp}(t) &= \mathbf{a}(\theta_p)q(t-\frac{l}{B})e^{j\frac{2\pi kt}{T}} \\ &\quad p=1,\cdots,P\leq M; \quad l=0,\cdots,L. \quad (11)\end{aligned}$$

The angles $\{\theta_p\}$ corresponding to the canonical spatial sampling in (4) and (5) are chosen so that $\{\mathbf{a}(\theta_p)\}$ form a complete basis for M-dimensional space spanned by the array response vectors corresponding to the desired signal. Note that (4) and (5) correspond to a uniform sampling in $\sin(\phi)$ with a maximum spacing of $1/D$, where $D = (M-1)d$. In particular, for $d = \lambda/2$, the following relationship governs the choice of θ_p

$$\sin(\theta_{p+1})-\sin(\theta_p) = \frac{\lambda}{Md} = 2/M, \quad p=1,\cdots,M-1. \quad (12)$$

which yield $\{\mathbf{a}(\theta_p)\}$ that span $\mathbf{C}^M$. For a given angular spread S_a of the desired signal, a subset of $\{\theta_p\}$ of size $P \approx \sin(S_a/2)(M-1)+1$ can be chosen to span the desired signal subspace. In terms of the basis functions $\{\mathbf{u}_{klp}(t)\}$, $\mathbf{s}(t)$ admits the canonical representation

$$\mathbf{s}(t) \approx \sum_{l=0}^{L}\sum_{k=-K}^{K}\sum_{p=1}^{P}\widehat{H}_{klp}\mathbf{u}_{klp}(t), \quad 0 < t \leq T. \quad (13)$$

This forms the basis of the canonical space-time receiver structures proposed in this paper.

For a direct sequence CDMA system, the spreading waveform $q(t)$ takes the form

$$q(t) = \sum_{i=0}^{N-1}c_i v(t-iT_c)/\sqrt{T}, \quad 0 \leq t < T \quad (14)$$

where $\{c_i\}$ is the spreading sequence of length N and $v(t)$ is the chip waveform of duration T_c. A rectangular chip waveform is assumed in the results presented here. The bandwidth B of the waveform is inversely proportional to T_c and its precise definition affects the structure of the resulting basis waveforms. We will consider bandwidth definitions of the form $B \approx \mathcal{O}/T_c$ where $\mathcal{O}$ corresponds to an oversampling factor, typically 2, 4 or 8. When $\mathcal{O} = 1$, choosing $\{\theta_k\}$ as in (12) gives a set of approximately orthonormal[1] set of basis functions $\{\mathbf{u}_{klp}(t)\}$,[2] albeit at the expense of a loss of accuracy in the representation (13) in the case of arbitrary multipath delays. The accuracy of (13) can be improved by increasing the oversampling factor $\mathcal{O}$, although at the expense of losing orthogonality of the basis functions $\{\mathbf{u}_{klp}(t)\}$ [4, 2].

3. COHERENT RECEIVER STRUCTURE

Here we address single-user binary antipodal communication with coherent reception. We consider a discrete multipath slow-fading environment with delay spread $T_m \ll T$ so that intersymbol interference is negligible and symbol-by-symbol detection suffices. Furthermore, we assume sufficiently slow fading so that Doppler effects are negligible.

[1] With respect to the inner product $\langle\mathbf{x},\mathbf{y}\rangle \doteq \int_0^T \mathbf{y}^H(t)\mathbf{x}(t)dt$.

[2] Due to the correlation properties of the spreading sequence.

The number of dominant multipath components is denoted by L_T. The fading coefficient β_l for the l-th path is assumed to be approximately constant within a symbol and uncorrelated with fading on other paths. The M-dimensional complex baseband signal within one symbol duration at the receiver is given by

$$\mathbf{r}(t) = \mathbf{s}(t)b + \mathbf{n}(t), \quad 0 \le t < T \tag{15}$$

where b is the data symbol. The noise vector $\mathbf{n}(t)$ is assumed to be complex Gaussian with zero mean and $E[\mathbf{n}(t)\mathbf{n}^H(t')] = \mathcal{N}_0\delta(t-t')\mathbf{I}_M$ where $\mathbf{I}_M$ is a $M \times M$ identity matrix. The signal $\mathbf{s}(t)$ can be written as

$$\mathbf{s}(t) = \sum_{l=1}^{L_T} \beta_l \mathbf{a}(\phi_l) q(t-\tau_l) \tag{16}$$

where $\phi_l \in [-S_a/2, S_a/2]$ and $\tau_l \in [0, T_m]$ are DOA and path delay corresponding to the l-th path, and $\mathbf{a}(\phi)$ denotes the array response for DOA ϕ.

Conventional coherent space-time receivers, such as those proposed in [3], are based on the test statistic

$$Z = Re\left\{\sum_{l=1}^{L_T} \hat{\beta}_l \langle \mathbf{r}(t), \mathbf{a}(\hat{\phi}_l) q(t-\hat{\tau}_l)\rangle\right\} \tag{17}$$

which require estimates of the DOA, delay, and fading of each multipath component. The detected symbol is given by $sgn(Z)$. This receiver performs matched-filtering to all the multipath components, resulting in high complexity in a dense multipath environment. Furthermore, the performance depends on the quality of the parameter estimates.

The canonical representation (13) provides a new framework for space-time processing that eliminates the need for DOA and delay estimates.[3] This representation suggests a canonical space-time receiver structure defined by the basis functions in (11). The canonical space-time receiver maps the received signal $\mathbf{r}(t)$ onto the basis functions to form the test statistic:

$$Z = Re\left\{\sum_{l=0}^{L}\sum_{p=1}^{P} \hat{H}_{lp}\langle \mathbf{r}, \mathbf{u}_{lp}\rangle\right\} \tag{18}$$

where $\{\hat{H}_{lp}\}$ are estimates of the canonical channel coefficients. In this paper, we assume perfect $\{\hat{H}_{lp}\}$ estimates that are obtained by projecting a noise-free pilot signal onto the canonical subspace.

We note that an increase in the multipath density does not affect canonical receiver performance as long as the basis functions span the signal space. Furthermore, the canonical receiver can easily adjust the number of basis functions to accommodate changes in the angular spread S_a and delay spread T_m. For even modestly dense multipath environments, the complexity of the canonical receiver is substantially less than that of the conventional receiver since fewer channel parameter estimates are required and fewer matched filters need to be implemented.

[3] All that is needed is synchronization to a global delay and DOA to "align" the basis, which is required in all receivers.

4. PERFORMANCE ANALYSIS

The performance of both receivers is compared based on the symbol-error probability (P_e) assuming perfect estimates of all multipath parameters $\{\hat{\beta}_l, \hat{\phi}_l, \hat{\tau}_l\}$ for the conventional receiver, and $\{\hat{H}_{lp}\}$ for the canonical receiver. Define

$$\begin{aligned}\mathbf{A}_T &= [\mathbf{a}(\phi_1), \cdots, \mathbf{a}(\phi_{L_T})], \\ \mathbf{A}_R &= ones(1, L+1) \otimes [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_P)]\end{aligned} \tag{19}$$

and

$$\begin{aligned}\mathbf{Q}_T(t) &= diag\{q(t-\tau_1), \cdots, q(t-\tau_{L_T})\}, \\ \mathbf{Q}_R(t) &= diag\{q(t), q\left(t-\frac{\mathcal{O}}{T_c}\right), \cdots, \\ &\quad q\left(t-\frac{L\mathcal{O}}{T_c}\right)\} \otimes \mathbf{I}_P\end{aligned} \tag{20}$$

where $\otimes$ denotes Kronecker product, $ones(I,J)$ a I $\times$ J matrix with 1 for all the entries, and $diag\{.\}$ forms a diagonal matrix of the elements inside the bracket. Also define

$$\begin{aligned}\mathbf{R}_w &= \int_0^T \mathbf{Q}_R(t)\mathbf{A}_R^H\mathbf{A}_R\mathbf{Q}_R(t)dt \quad , \\ \mathbf{R}_s &= \int_0^T \mathbf{Q}_R(t)\mathbf{A}_R^H\mathbf{A}_T\mathbf{Q}_T(t)dt\end{aligned} \tag{21}$$

Then, the symbol test statistics of the canonical receiver can be written as:

$$Z = Re\{\mathbf{h}^H\mathbf{R}_s\beta b + \mathbf{h}^H\mathbf{w}\} \tag{22}$$

where

$$\beta = [\beta_1, \cdots, \beta_{L_T}]^T, \tag{23}$$

$$\mathbf{h} = [\hat{H}_{00}, \hat{H}_{01}, \cdots, \hat{H}_{LP}]^T, \tag{24}$$

$$\mathbf{w} = \int_0^T \mathbf{Q}_R(t)\mathbf{A}_R^H\mathbf{n}(t)dt \tag{25}$$

$$E[\mathbf{w}\mathbf{w}^H] = \mathcal{N}_0\mathbf{R}_w$$

It can be shown that

$$\mathbf{h} = \mathbf{R}_w^{\dagger}\mathbf{R}_s\beta \tag{26}$$

where ($\dagger$) denotes Moore-Penrose pseudoinverse. Hence (22) can be written as

$$Z = \beta^H\mathbf{R}_s^H\mathbf{R}_w^{\dagger}\mathbf{R}_s\beta + Re\{\beta^H\mathbf{R}_s^H\mathbf{R}_w^{\dagger}\beta\} \tag{27}$$

Assuming equally likely symbols and β is complex Gaussian with zero mean and $E[\beta\beta^H] = \mathcal{E}\mathbf{I}_{L_T}/(\mathcal{N}_0 L_T)$, [4] where $\mathcal{E}$ is the total transmitted energy, P_e is given by [1]:

$$P_e = \frac{1}{2}\sum_{l=1}^{L'}\prod_{i=1, i\neq l}^{L'}\frac{\lambda_l}{\lambda_l-\lambda_i}\left(1-\sqrt{\frac{\rho\lambda_l}{\rho\lambda_l+1}}\right). \tag{28}$$

where $\rho = \mathcal{E}/(\mathcal{N}_0 L_T)$ and $\{\lambda_l\}$ are the non-zero eigenvalues of matrix $\mathbf{\Phi} \doteq \mathbf{R}_s^H\mathbf{R}_w^{\dagger}\mathbf{R}_s$. Notice that for $\mathcal{O}=1$, $\mathbf{R}_w = \mathbf{I_{L_T}}$ since (11) yields an orthogonal basis.

The P_e of the conventional receiver is analyzed in an analogous manner since its test statistic Z is a special case of (27) where $\mathbf{R}_s = \mathbf{R}_w = \int_0^T \mathbf{Q}_T(t)\mathbf{A}_T^H\mathbf{A}_T\mathbf{Q}_T(t)dt$.

[4] Uncorrelated scatterer model [1].

5. EXAMPLES

Two scenarios are consider to contrast the canonical and conventional receivers. As noted in Section 3, the canonical receiver only requires estimates of the canonical channel coefficients, while the conventional receiver requires estimates of *all* multipath DOAs, time delays, and fading parameters. For comparison purposes, we assume all parameters required are estimated perfectly. Although unrealistic, this assumption provides an upper bound on performance.

An eight element uniform linear array is assumed with half-wavelength spacing. A length-31 M sequence serves as the spreading code, and P_e as a function of $SNR\ (= \mathcal{E}/\mathcal{N}_0)$ is used as the performance measure.

Example 1. space-only processing. A dense multipath environment with angular spread S_a of $\pi/5$ is assumed with zero delay spread ($T_m = 0$). There are a total of 21 multipath arrivals with DOAs uniformly distributed on $[-\pi/10, \pi/10]$. The canonical receiver is based on up to eight beams with directions chosen according to (12) to obtain $\theta_k \in \{\pm 0.34\pi, \pm 0.21\pi, \pm 0.12\pi, \pm 0.04\pi\}$. Figure 2 depicts the performance of the conventional and several canonical receivers based on different numbers of beam directions. The receiver with "2 beams" uses the directions $\{\pm 0.04\pi\}$, "4 beams" uses $\{\pm 0.12\pi, \pm 0.04\pi\}$, "6 beams" uses $\{\pm 0.21\pi, \pm 0.12\pi, \pm 0.04\pi\}$, while "8 beams" uses all eight θ_k. The 2 beam canonical receiver experiences a 7 dB SNR loss at $P_e = 10^{-4}$ since the beams with directions $\{\pm 0.04\pi\}$ do not span the space corresponding to the angular spread of the multipath, ($|\theta| < \pi/5$). However, as suggested by the canonical signal model, 4 beams are sufficient to represent the given angle spread, as evident from the nearly identical performance of the canonical receiver with 4, 6 or 8 beams and the conventional receiver which is exactly matched to the DOAs. Note that the conventional receiver requires estimates of 21×2 DOA and fading parameters and forms 21 beams, whereas canonical receiver requires estimates of at most 8 channel parameters and forms at most 8 beams.

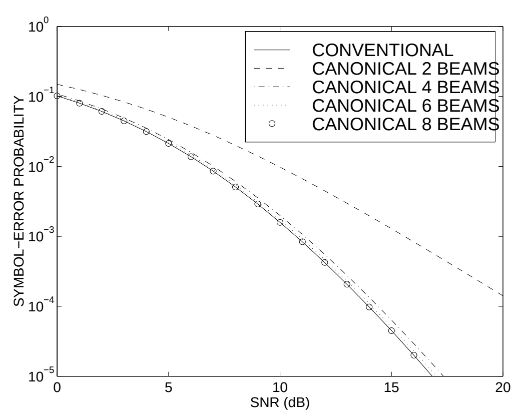

Figure 2: Space-only processing: conventional vs. canonical with different number of beams.

Example 2. Space-time processing. We repeat Example 1 with $T_m = 0.9375T_c$, using a total of 21×16 multipath components distributed evenly within $[-\pi/10, \pi/10] \times [0, 0.9375T_c]$. The canonical representation samples at DOAs $\{\pm 0.12\pi, \pm 0.04\pi\}$ and delays $\{0, T_c/\mathcal{O}, 2T_c/\mathcal{O}, \ldots, T_c\}$ where $\mathcal{O} = 1, 2, 4, 8$ is the oversampling factor. Figure 3 depicts the performance of the conventional and canonical receivers with different oversampling factors. At $P_e = 10^{-4}$ the canonical receivers display an SNR performance loss between 3 dB ($\mathcal{O} = 1$) and 0.5 dB ($\mathcal{O} = 8$). The performance loss in canonical receiver is a consequence of the band-limited approximation in (3) and resulting error in the approximation for the received signal. This error decreases as $\mathcal{O}$ is increased [4] and the performance loss decreases. Note that the conventional receiver requires estimates of $21 \times 16 \times 3$ delay, DOA, and fading parameters and implementation of 21×16 space-time filters. In contrast, the canonical receiver with $\mathcal{O} = 8$ only requires estimates of 4×9 channel coefficients $\hat{H}_{lp}$ and implementation of 4×9 space-time filters.

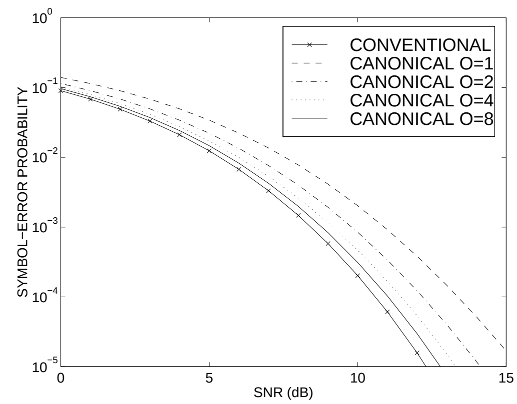

Figure 3: Space-time processing: conventional vs. canonical with different oversampling factor $\mathcal{O}$.

6. DISCUSSION

In practice all receivers have limited operational bandwidth. This limits the accuracy of closely-spaced delay estimates in a dense multipath environment and also limits the benefits of oversampling. Even if joint angle-delay estimation frameworks [3] are employed, a large number of observations and complex algorithms are necessary to obtain accurate parameter estimates for the conventional receiver. The canonical receivers have reduced complexity and are likely to be far more robust to channel estimation errors associated with limited data and the presence of noise.

7. REFERENCES

[1] J.G. Proakis, *Digital Communications,* 3rd ed., McGraw-Hill, 1995.

[2] A.M. Sayeed and B. Aazhang, "Joint multipath-Doppler diversity in mobile wireless communications," to appear in the *IEEE Trans. Commun.*

[3] A. Paulraj, " Space-Time Processing for Wireless Communications," *IEEE SP Magazine*, Nov. 1997.

[4] E.N. Onggosanusi, A.M. Sayeed and B.D. Van Veen, "Canonical Space-Time Processing in Wireless Communications", *in preparation.*