# A NEW DIRECTIONAL FILTER BANK FOR IMAGE ANALYSIS AND CLASSIFICATION

Sang-il Park, Mark J. T. Smith and Russell M. Mersereau

Center for Signal and Image Processing School of Electrical and Computer Engineering Georgia Institute of Technology Atlanta, Georgia, USA spark@ece.gatech.edu

# ABSTRACT

A new directional filter bank for image analysis and classification is proposed. This paper introduces an improved structure in order to visualize subband outputs of the directional filter banks, while retaining the attractive properties of the original directional filter banks such as 1-D separable filtering, perfect reconstruction, and maximal decimation.

Using this structure, any arbitrary  $2^n$  band directional filter bank can be implemented by cascading simple directional filter bank blocks, unlike the original structure that needs a parallel structure for visualizing subband outputs. Also, in order to have nondistorted phase information in the subbands for visualization, both FIR and IIR filter prototypes that can be implemented efficiently are provided for linear phase filtering. This paper shows the approach proposed here can be applied to image analysis and classification.

## 1. INTRODUCTION

Directional information in a given image is important in many image processing applications[1]. It can be extracted by various methods such as a simple fan filter, Radon transform[2], steerable filter bank[3], or directional filter bank[1]. Among these methods, the main advantages of a directional filter bank are the following:

- It can be implemented by 1-D separable filtering. Directional filter banks can extract 2-D directional information, as in Figure 1 (a), into 2<sup>n</sup> subbands, as shown in Figure 1 (b) for n = 3. This is possible using the polyphase structure[4], in Figure 3 (b). This polyphase structure can make the implementation of directional filter banks very efficient not only because of its structure by itself [4], but also because it makes 1-D separable filtering possible[1].
- We can acheive maximally-decimated filter banks with perfect reconstruction. The original image can be reconstructed perfectly from the decomposed subband outputs. In [5], a fingerprint image is successfully enhanced based on this perfect reconstruction property, using both analysis and synthesis systems of directional filter banks. Also, as seen in Section 4, the property of maximal decimation can be used in applications of image classification.



Figure 1: Frequency map for 8 band directional filter bank. (a) Directional frequencies of input to be decomposed. (b) Decomposed frequency maps of the 8 subbands.

In spite of their successful directional frequency decomposition with efficient structures, the main drawback of directional filter banks has been that the decomposed subband outputs visually distort the original input image as in Fig 2 (b).



Figure 2: Directionally decomposed images. (a) The original cameraman image. (b) Its decomposed subband images based on [1].

The reason for this is seen in the frequency mappings in Figure 1. Comparing the differently shaded regions in Figure 1 (a) and (b), we see that the low frequency area is severely misplaced in its subband as in Figure 1 (b). In [6], another structure for implementing a directional filter bank was proposed so that the resulting subband images would have more intuitive visual information. It replaced the initial tree-structured directional filter banks with parallel-structured ones, where each of the subband filters needed

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to be designed separately. It thus sacrificed computational efficiency.

This paper presents a new directional filter bank structure that has the following features:

- Maximally-decimated, and perfectly-reconstructing filters with 1-D separable filtering, as in the original implementation.
- Visualizable directional subbands. Each of the subbands has intuitive directional information as in Figure 8.
- Tree-structured filter banks. A 2<sup>n</sup> band directional filter bank can be implemented by cascading basic components, as in Figure 3, which are the only components that need to be designed.
- Linear phase filtering: In order to preserve the phase information of a given input image during filterings, both linear phase FIR and linear phase IIR filtering are considered and implemented.

In Section 2, the new structure for directional filter banks will be presented. In Section 3, all of the design parameters to implement this directional filter banks are provided. The next section, Section 4, briefly illustrates some possible new applications for directional filter banks.

# 2. STRUCTURE

In the remainder of this paper, we concentrate on only the analysis system of the directional filter banks, since the synthesis system can be understood as the inverse of the analysis system.



Figure 3: Overall Structure of a new directional filter banks. (a) First block for first 2 stages. (b) Its polyphase Structure. (c) Second block for the rest of stages.

Unlike typical tree-structured filter banks, where only one 2band filter bank block needs to be cascaded, tree-structured directional filter banks need to use two: One, as in Figure 3 (a), is used for first 2 stages, and the other, as in Figure 3 (c), is for the remaining stages. Both of structures in Figure 3 (a) and (c) can be implemented in polyphase form as in Figure 3 (b). For the structure in Figure 3 (a), a diamond filter  $H_0(\omega)$ , as in Figure 4 (a), is used, while an hour-glass filter,  $H_{m0}(\omega)$ , as in Figure 4 (b), can be used as in Figure 3 (a) if the modulator is omitted. Both cases are conceptually identical. Once the filter approach for the first two stages is decided upon, the remaining stages use the corresponding filters as in Figure 4, e.g., parallelogram filters,  $A_0(\omega)$  to  $D_0(\omega)$ , will be used for the remaining stages assuming that Figure 3 (a) is used in the first two stages.



Figure 4: 5 kinds of passbands for directional filter banks. (a) For modulated structure. (b) For non-modulated structure.

Using parallelogram filters in addition to diamond filters results in an additional design process. Also notice that the diamond filtering can be implemented by 1-D separable filtering using polyphase structure. If a resampling matrix,  $R_i$ , is inserted before each of the remaining stages, then the parallelogram filters can be replaced with another diamond filter. Therefore inserting an  $R_i$  matrix makes it possible not only to remove an additional filter design process, but also to implement the filter using separable filtering in a polyphase structure.

The motivation for another resampling matrix, the so-called *backsampling*,  $B_i$  follows.



Figure 5: Structure derivation (a) The Noble identity. (b) Combining all samplers. (c) Cascading the next stages. (d) Overall proposed structure.

Using the Noble identity[4] as in Figure 5 (a), the overall sampling in the upper path of the filter bank block in Figure 3 (c) can be reconfigured as in Figure 5 (b), combining the switched  $R_i$ , the quincunx downsampling  $Q_i$ , and  $B_i$ . Note that the first two stages of the directional filter banks do not use any resampling matrix but only use the quincunx matrix twice. The overall sampling corresponds to *x* and *y* axis decimation, i.e.  $Q_i * Q_i = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$ . Therefore,

at each of the remaining stages, as long as the overall sampling matrix,  $D_i$ , as in Figure 5 (c), is simply horizontal or vertical decimation, i.e.,  $D_i = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$  or  $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ , then directional filtering at each stage can be conceptually simplified to just *filtering* and *downsampling* so that each subband output has the visual appearance of the input images can have visual information of a given input image as with typical filter banks. Also, notice that, for the whole analysis system as in Figure 5 (d), the overall sampling matrix,  $D_i^{Total}$  should correspond to either vertical or horizontal decimation for the visualized subband images at the last stage.

This consideration results in appending the backsampling matrix  $B_i$  after the quincunx matrix at the remaining stages as in Figure 3 (c) in order to make the overall sampling  $D_i$  just downsampling to visualize the subband outputs. But, inserting the backsampling matrix  $B_i$  at each of subbands imposes additional computation. In order to minimize the additional computation due to appending  $B_i$  at each stage, all of the  $B_i$  backsampling matrix,  $B_i^{Tn}$ , and appended at the end of the analysis system as in Figure 5 (d).  $B_i^{Tn}$ , for the  $2^n$  analysis system (n > 2), adjusts the sampling over the whole system after the 2nd stage to be simple downsampling such as  $\begin{bmatrix} 2^m & 0\\ 0 & 1 \end{bmatrix}$  or  $\begin{bmatrix} 1 & 0\\ 0 & 2^m \end{bmatrix}$ , where m = n - 2. Therefore, in terms of computational expense, the directional

Therefore, in terms of computational expense, the directional filter banks proposed here has exact same amount of computation as in the initial structure[1] plus one resampling matrix  $B_i^{Tn}$ . A closed form for  $B_i^{Tn}$  is provided in the next section in addition to the other design parameters.

#### 3. DESIGN PARAMETERS

## 3.1. Filter Prototype

One of the main purposes for this paper is to introduce an application, where the analysis system of the directional filter is used without the synthesis system. Here, for visualizing subband outputs without phase distortions, linear phase subband filtering is more important than perfect reconstruction. We use a 2nd-order linear-phase IIR filter[7] that not only guarantees perfect reconstruction but also can be implemented faster than by using an FIR filter with similar characteristics. The linear phase IIR filter coefficients used here are  $\omega_1 = 4.126081$ ,  $\omega_2 = 4.488383$ ,  $\alpha = 0.711746$ , and K = 0.295035 for the following transfer function:

$$H(z) = K \frac{(1+z^{-1})I(\omega_1)I(\omega_1)}{(1+\alpha^2 z^{-2})(1+\frac{1}{-2}z^{-2})}, \text{ where } (1)$$

$$I(w) = 1 - (2\cos\omega)z^{-1} + z^{-2}$$
(2)

Another interested choice for the filter is a linear-phase oddlength FIR filter. For a 2N + 1 tap linear phase FIR filter, in addition to its whole sample delay property, one of its polyphase filters always has the form of a simple delay, which saves computation by a factor of 2, as in Figure 6.

# 3.2. Expanding Tree Structure

Up to the second stage, the tree can be expanded easily by cascading blocks as in Figure 3 (a). After the second stage, all 4 subbands are processed by an expanding tree formed by cascading blocks as in Figure 3 (c). This process is governed by the following rules:

1. Define a *path type* as  $T_i^n$  at the  $n^{\text{th}}$  stage (n > 2), i = path type index (i = 1, 2, 3, and 4) that determines all of



Figure 6: Polyphase structure using a linear-phase odd-length FIR filter.

sampling matrices needed at the  $n^{\text{th}}$  stage (n > 2) as in Figure 3 (c).

- 2. At n = 2, Initially  $T_i^2$  is assigned to subband in consecutive order.
- 3. Then at the  $n^{\text{th}}$  stage, where  $n \ge 2$ , the path types for the succeeding stages are determined as follows:



Figure 7: Expanding tree for directional filter banks.

Using these rules, at the  $n^{\text{th}}$  stage, each of subbands can be labeled easily so that all of the sampling matrices can be determined based on  $T_i^n$ , which is made possible by carefully chosen the design parameters as follows.

#### 3.3. Quincunx Matrix

At first, there are totally 4 kinds of quincunx matrices that can be used in directional filter banks as follows:

$q_1$	=	$\begin{bmatrix} 1\\ -1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$q_2$	=	$\begin{bmatrix} 1\\ 1 \end{bmatrix}$	$\begin{bmatrix} -1\\1 \end{bmatrix}$
$q_3$	=	$\begin{bmatrix} -1\\1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$q_4$	=	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

For first two stages, any quincunx matrix can be used. A typical choice is  $q_1$ . Note that, unlike  $q_3$  or  $q_4$ ,  $q_1 * q_1 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}^2$ , which is a clockwise-rotation matrix of  $\begin{bmatrix} 2 & 0 \\ -2 & 0 \end{bmatrix}$  so that a counterclockwise rotation needs to be additionally added after the first two stages if  $q_1$  is chosen. For the remaining stages, in order to make a simple rule for an expanding tree as in Figure 7, a pair of quincunx matrices need to be used as follows:

$$\mathbf{Q}_i = \begin{cases} q_2 & \text{if } i \text{ is } 1 \text{ or } 4\\ q_1 & \text{if } i \text{ is } 2 \text{ or } 3 \end{cases}$$

# 3.4. Resampling Matrix

The definition of resampling matrix is a matrix whose determinant should be 1 with integer matrix entries so that its inverse matrix is also a resampling matrix. Among the infinitely many resampling matrices, in order to replace the parallelogram filters with the diamond filter, the resampling matrices that define the change of variables[8] are the following:

$$r_1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \qquad r_2 = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$
$$r_3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad r_4 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

At any stage after the second stage,  $R_i = r_i$ .

#### 3.5. Backsampling Matrix

As mentioned earlier in Section 2,  $B_i^{Tn}$  makes the overall sampling for the analysis system of any  $2^n$  (n > 2) directional filter banks either simple vertical or horizontal sampling, which is the following:

$$\mathbf{B}_{i}^{Tn} = \left[ \left[ \mathbf{B}^{T(n-1)} \right]^{-1} \mathbf{R}_{i} * \mathbf{Q}_{i} \right]^{-1} * \mathbf{D}_{i}, \text{ where}$$
$$\mathbf{D}_{i} = \begin{cases} \left[ \begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix} \right] & \text{if } i = 1 \text{ or } 2 \\ \left[ \begin{smallmatrix} 2 & 0 \\ 0 & 1 \end{smallmatrix} \right] & \text{if } i = 3 \text{ or } 4 \end{cases}$$
$$\mathbf{B}_{i}^{T3} = \mathbf{R}_{5-i}, \text{ where } i = 1, 2, 3, 4$$

# 4. APPLICATIONS

The subband outputs for the cameraman image as in Figure 2 (a) are given in Figure 8. As expected, each of subbands has its directional information. For instance, in Figure 8 (b), one of the tripod's legs are directionally visualized with the cameramen's body contour that has similar direction. But, in Figure 8 (d), most of directional information that is captured in Figure 8 (b) is missing, while another tripod's leg is captured. Also notice that the contour of cameraman's right hand that has slightly different direction from the tripod's left leg is captured in (i), unlike in (b), as expected.

This approach has a potential in any application where directional edge detection can solve given problems, also in other applications where rotational invariance is important such as common automatic target detection[9].

# 5. CONCLUDING REMARKS

In this paper, the basic idea of directional filter banks are briefly reviewed. a new structure for directional filter banks is proposed based on the idea in [1] and [6]. But using the new structure proposed here, the directional filter banks can combine the advantages of both earlier implementations: a tree-structure and easily visualized subbands. It can be expanded to any  $2^n$  subbands by cascading the basic 2-band filter blocks as in Figure 3. Also its subband images can have easily visible directional information, which can be used for image classification successfully as in Section 4. In Section 3, all of designing parameters for directional filter banks are provided.



Figure 8: Another set of subband images produced by the treestructured directional filter bank described in this paper. (a) to (h) corresponds to directional decomposition of bands 1 to 8 in Figure 1

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