BLIND KNOWLEDGE BASED ALGORITHMS BASED ON SECOND ORDER STATISTICS

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ABSTRACT

Most second order Single Input Multiple Output (SIMO) identification algorithms identify the global impulse channel response, convolution of an emission filter and a propagation channel. This paper makes an explicit use of this channel structure in a second order algorithm. We present several structured methods exploiting more or less prior informations on the emission filter. Proofs of convergence are provided, and simulations show that some knowledge based algorithms greatly improve over classical blind algorithms, even in the case where the knowledge is partial.

1. INTRODUCTION

Several recent works [1]-[3] have shown that non minimum phase impulse responses can be blindly estimated using second order statistics only. These methods are based on a single input and multiple outputs (SIMO) system model. Main algorithms are the least square [1], the subspace [2] and the linear prediction [3] methods.

These algorithms identify the global impulse response i.e. the combination of the unknown propagation channel and the transmitter/receiver filters. In classical digital transmission systems the emission filter is in most cases known by the receiver. Thus, it has been proposed recently to make use of this knowledge in order to estimate only the unknown channel, with the intent to improve the performance. These methods are named structured or knowledge based or multipath channel identification methods [6]-[8]. The purpose of this paper is to generalize this approach, and to introduce the channel structure into algorithms, in order to take advantage of (even partial) knowledge on the emission filter.

The proposed algorithms are based on subspace techniques and use both spatial and temporal diversity. The use of both diversities is seen to be necessary in order to avoid the most frequent cases of non identifiability [4]-[5]. It is shown below that oversampling is necessary for obtaining an accurate system model, while the spatial diversity is necessary for obtaining more robustness than plain fractionally sampling SOS based algorithms in a band-limited context.

Section 3 provides a decomposition of the global response in terms of the emission filter and propagation channel responses with no approximation. This factorization provides the structure used by the algorithms proposed in section 4. The first algorithm assuming a total knowledge of the emission filter is essentially similar to the multisensor algorithm proposed by Ding [8]. Then we present a fully blind structured subspace method. Finally we assume that we have information on the emission filter shape, for instance a square-root Nyquist pulse (a classical situation in many HF systems). The algorithm only estimates its excess bandwidth. Simulations in section 5 present performance of these methods. In particular, they show that the second proposed method, in which one makes use of the sole shape of the filter performs almost as well as the knowledge based method making full use of the filter.

2. THE SITUATION OF INTEREST

Let $\{s_k\}$ denote the symbols emitted by the digital source with symbol duration T. The standard baseband representation of the received signal is $\tilde{y}(t) = \sum_{k=-\infty}^{\infty} s_k \tilde{h}(t-kT) + \tilde{b}(t)$. The global impulse response $\tilde{h}(t)$ encompasses the effects of a time-varying unknown propagation channel $\tilde{c}(t)$ and the composite response of transmitter filter, receiver filter and modulation/demodulation (which are assumed to be linear), denoted as $\tilde{n}(t)$. $\tilde{b}(t)$ is a stationary noise independent of the channel input.

The signal is received on q sensors and fractionally sampled with sampling interval $\Delta = T/p$, thus forming p virtual channels on each sensor. Hence, after sampling the received signal, we come up to the following discrete model: $y_i^j(mT) = \sum_{k=-\infty}^{\infty} \tilde{h}_i(t_0 + mT - kT + (j-1)\frac{T}{p}) s_k + b_i^j(mT)$ where $i = 0 \ldots q-1$ indicates the sensor number and $j = 0 \ldots p-1$ the sampling epoch.

The problem, now, is to derive a discrete model of this signal in which the effects of the physical channel would be separated from those of the system (filters, modulation,...). The solution heavily relies on the following assumption:

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H 1 *$\tilde{n}(t)$ is a low-pass band-limited filter with bandwidth*

$$B = \frac{1+\alpha}{T} \quad 0 \leq \alpha \leq 1 \tag{1}$$

In practice, $\tilde{n}(t)$ may be a (square-root) raised-cosine pulse with roll-off factor α.

3. MODELIZATION

This section intends to provide a discrete-time version (z-transform) of $\tilde{h}(t)$ involving the z-transforms of $\tilde{n}(t)$ and $\tilde{c}(t)$. Would all quantities be frequency limited, the solution would be straightforward. Here, the difficulty is that the classical model for the propagation channel is a sum of several discrete paths, $\tilde{c}(t) = \sum_{p=1}^{P} \alpha_p \delta(t-\tau_p)$, then $\tilde{c}(t)$, which is band-illimited. Thus, the true channel cannot be recovered from its samples. The solution involves the use of the band-limitedness of the composite filter $\tilde{n} * \tilde{c}(t)$ to define a z-transform to the propagation channel and a factorization relationship.

The corresponding factorization was used in previous papers on partial knowledge information systems [6]-[8]. However, we show that using this approach, the channel response is not uniquely defined, and that the factorization is not an approximation for band limited filters.

3.1. Notations

Assume that the oversampling factor p is greater than two. Let $\tilde{g}(t)$ a continuous-time filter, and $g_{\frac{T}{p}}(t)$ the filter reconstructed from its samples at $\frac{T}{p}$. We denote respectively $\tilde{G}(f)$ and $G_{\frac{T}{p}}(e^{j2\pi f})$ their Fourier transforms. Moreover, we denote $G_{\frac{T}{p}}(z)$ the z-transform of the filter sampled with period $\frac{T}{p}$ and $G^k(z)$ the k^{th} polyphase component.

$$\begin{aligned} G_{\frac{T}{p}}(z) &= \textstyle\sum_{k=-\infty}^{\infty} \tilde{g}(t_0 + k\frac{T}{p}) z^{-k} \\ G^k(z) &= \textstyle\sum_{i=-\infty}^{\infty} \tilde{g}(t_0 + iT + k\frac{T}{p}) z^{-i} \quad (k = 0..p-1) \end{aligned}$$

3.2. Factorization and polyphase decomposition

Under H.1, by sampling $\tilde{h}(t) = \tilde{n} * \tilde{c}(t)$ with period $\frac{T}{p}$ one do not introduce aliasing. Thus,

$$\begin{aligned} N_{\frac{T}{p}}(e^{j2\pi f})\tilde{\Pi}_\alpha(f) &= \tilde{N}(f) \\ H_{\frac{T}{p}}(e^{j2\pi f})\tilde{\Pi}_\alpha(f) &= \tilde{H}(f) = \tilde{N}(f)\tilde{C}(f) \end{aligned}$$

where $\tilde{\Pi}_\alpha(f)$ is an ideal filter, limiting the bandwidth to that of $\tilde{n}(t)$ i.e. it is zero for frequencies larger than $\frac{1+\alpha}{T}$ and constant below. Let $\tilde{P}(f)$ be a continuous filter verifying

$$\tilde{P}(f) = 1 \quad |f| \leq \frac{1+\alpha}{2T} \tag{2}$$

$$\tilde{P}(f) = 0 \quad |f| \geq \frac{p-1}{T} + \frac{1-\alpha}{2T} \tag{3}$$

$\tilde{C}_P(f) = \tilde{P}(f)\tilde{C}(f)$ is a band-limited filter defined in such a way that $C_{\frac{T}{p}P}(e^{j2\pi f})\tilde{\Pi}_\alpha(f) = \tilde{C}(f)\tilde{\Pi}_\alpha(f)$. $\tilde{P}(f)$ allows to limit the bandwidth of $\tilde{C}(f)$. By doing so, one is able to define the z-transform of $\tilde{c}(t)$ while keeping the shape of $\tilde{C}(f)$ inside the band B of the emission filter. Eq. 3 ensures there is no aliasing inside the band B. Note that $\tilde{P}(f)$ is not uniquely defined, neither $C_{\frac{T}{p}P}(e^{j2\pi f})$. This is due to the assumption that the spectrum of the shaping filters is strictly zero outside its bandwidth, and indicates that one can estimate the channel only in a bandwidth where it is observed. Thus,

$$H_{\frac{T}{p}}(e^{j2\pi f})\tilde{\Pi}_\alpha(f) = N_{\frac{T}{p}}(e^{j2\pi f})\tilde{\Pi}_\alpha(f) C_{\frac{T}{p}P}(e^{j2\pi f}) \tag{4}$$

$$\tilde{h}(t) = \textstyle\sum_{l=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \tilde{n}(l\frac{T}{p})\tilde{c}_P(k\frac{T}{p})\tilde{\pi}_\alpha(t - l\frac{T}{2} - k\frac{T}{p})$$

Hence, with $H_{\frac{T}{p}}(z) = \sum_{m=-\infty}^{\infty} \tilde{h}(m\frac{T}{p}) z^{-m}$ we obtain the relation below between the corresponding z-transforms of the impulse responses of the filters, sampled at rate $\frac{T}{p}$.

$$H_{\frac{T}{p}}(z) = N_{\frac{T}{p}}(z) C_{P\frac{T}{p}}(z) \tag{5}$$

In order to simplify the notations, from now we will omit the reference to the filter $\tilde{P}(f)$ when considering the propagation channel z-transforms.

Second order blind identification algorithms work on stationary discrete signals i.e. sampled at symbol rate. Hence, we introduce the polyphase decomposition:

$$H_{\frac{T}{p}}(z) = \sum_{k=0}^{p-1} H^k(z^p) z^{-k} \tag{6}$$

When applying similar polyphase decompositions to all filters in eq.(5) and identifying terms of same power in z, it is easily seen that

$$H^k(z) = \sum_{m=0}^{p-1} N^{(k-m)}(z) C^m(z) \qquad k = 0 \ldots (p-1)$$

4. ALGORITHMS

By taking q sensors, each one oversampled by a factor p, we obtain $p.q$ virtual channels, each one characterized by its discrete valued response. Define

$$\begin{aligned} \mathbf{H}(z) &= [H_0^0(z)\, H_0^1(z) \ldots H_0^{p-1}(z) \ldots H_{q-1}^{p-1}(z)]^T \\ \mathbf{C}(z) &= [C_0^0(z)\, C_0^1(z) \ldots C_0^{p-1}(z) \ldots C_{q-1}^{p-1}(z)]^T \\ \mathbf{N}(z) &= [N^0(z)\, N^1(z) \ldots N^{p-1}(z)]^T \end{aligned}$$

where $H_i^j(z)$ refers to the (i,j) virtual channel: j^{th} sampling moment on the i^{th} sensor. Note that the transmitter/receiver filter $\tilde{n}(t)$ does not depend on the sensor.

Classical SIMO second order based algorithms allows to identify a FIR filter of order L precisely known verifying the identifiability condition $\mathbf{H}(z) \neq 0 \forall z$. Thus, $\mathbf{H}(z)$, $\mathbf{C}(z)$ and $\mathbf{N}(z)$ being IIR filter because of their band-limited property, they are strictly unidenfiable by methods relying only on second order statistics. Denote respectively L_c, L_n, $L_h = L_c + L_n$ the orders of the "significant" part of $\mathbf{C}(z)$, $\mathbf{N}(z)$, $\mathbf{H}(z)$. We keep the same notations to represent filters limited to the significant part. From now we only deal with FIR filters, and we assume that the truncated part is sufficiently small, so that the factorization relation (5) still holds.

4.1. Classical subspace method

Let $\mathbf{h} = [h_0^0(0) \dots h_0^1(L_h) \dots h_0^{p-1}(0) \dots h_{q-1}^{p-1}(L_h)]^T$ be the vector constructed from $\mathbf{H}(z)$ coefficients. We recall that the quadratic estimation criterion of the subspace method is $q(\mathbf{h}) = \mathbf{h}^H \mathbf{Q} \mathbf{h}$ where $\mathbf{Q}$ is entirely defined by the null subspace of the covariance matrix of the receive signal.

It had been shown [2] that under a constraint such that $||\mathbf{h}|| \neq 0$, if the quadratic form is constructed from the true covariance matrix, the true impulse response is the unique vector up to a multiplicative constant verifying $q(\mathbf{h}) = 0$.

4.2. Knowledge based subspace method

The subspace method is now derived under assumption that the shaping filter is known. i.e. we are searching for an estimate of the sole propagation channel.

The model explained in section 3 allows us to factorize $\mathbf{h}$ in function of $\mathbf{c}$ and a matrix depending only on $\mathbf{N}_{\frac{T}{p}}(z)$ coefficients. First, we present a lemma about a convolution property.(Proof is omitted)

Lemma 1 *Let $\mathbf{V}(z)$ and $\mathbf{U}(z)$ be M-vector impulse responses and $X(z)$ a scalar impulse response of order respectively L_v, L_u and L_x. Let $\mathbf{v}$ and $\mathbf{u}$ be the vectors constructed by stacking the coefficients of $\mathbf{V}(z)$ and $\mathbf{U}(z)$.*

Assume $\mathbf{V}(z) = X(z)\mathbf{U}(z)$ then $\mathbf{v} = (\mathbf{I}_M \otimes \mathcal{T}_{L_u}(X)^T)\mathbf{u}$ where $\otimes$ is the Kronecker product, $\mathcal{T}_{L_u}(X)$ is the classic Toeplitz filtering matrix of dimension $(L_u+1) \times (L_x + L_u + 1)$ associated to $X(z)$ and $\mathbf{I}_M$ is the identity matrix $(M \times M)$.

We recall the decomposition obtained in section 3

$$H_i^j(z) = \sum_{m=0}^{p-1} N^{(j-m)}(z) C_i^m(z) \qquad \begin{smallmatrix} 0 \leq j \leq p-1 \\ 0 \leq i \leq q-1 \end{smallmatrix} \tag{7}$$

By laying down $C_i^t(z) = 0$ if $t \notin [0..p-1]$ and $\mathbf{C}^m(z) = [C_0^{-m}(z) \dots C_0^{(p-1)-m}(z) \dots\dots C_{(q-1)}^{(p-1)-m}(z)]^T$, the equation can be rewritten in a vector form as

$$\mathbf{H}(z) = \sum_{m=-(p-1)}^{p-1} N^m(z) \mathbf{C}^m(z) \tag{8}$$

Then, by applying the previous lemma to each component of eq. 8, we obtain the following expression

$$\mathbf{h} = \sum_{m=-(p-1)}^{p-1} (\mathbf{I}_{p.q} \otimes \mathcal{T}_{L_c}(N^m)^T) \mathbf{c}^m$$

Note that $\mathbf{C}^0(z) = \mathbf{C}(z)$ and that non zero polynomial components of the vector $\mathbf{C}^m(z)$ are all components of $\mathbf{C}^0(z)$. Thus, $\mathbf{c}^m = (\mathbf{I}_q \otimes \mathbf{P}_p^m \otimes \mathbf{I}_{L_c+1})\mathbf{c}$ where $\mathbf{P}_p^m$ are null matrixes with ones on the m^{th} diagonal.

Hence, matrix manipulations give the factorization:

$$\mathbf{h} = \mathcal{N}\mathbf{c} \tag{9}$$

where $\mathcal{N} = \sum_{m=-(p-1)}^{p-1} (\mathbf{I}_q \otimes \mathbf{P}_p^m \otimes \mathcal{T}_{L_c}(N^m)^T)$

The quadratic form may then be rewritten as

$$q(\mathbf{c}) = \mathbf{c}^H \mathcal{N}^H \mathbf{Q} \mathcal{N} \mathbf{c} \tag{10}$$

It is easily seen that if $\mathbf{H}(z) \neq 0 \forall z$ by minimizing $q(\mathbf{c})$ in a way that $||\mathbf{c}|| \neq 0$, we estimate $\mathbf{c}$ up to a multiplicative constant. The theorem 1 below provides an other identifiability sufficient condition depending only of the propagation channel. First, we need to introduce a lemma.

We denote $\mathbf{C}_{\frac{T}{p}}(z) = [C_{\frac{T}{p},0}(z) \dots C_{\frac{T}{p},q-1}(z)]^T$ the $\frac{T}{p}$ z-transform of the multisensors propagation channel.

Lemma 2 *Let $\tilde{\mathbf{H}}(z)$ the irreducible factor of $\mathbf{H}(z)$.*
If $\mathbf{C}_{\frac{T}{p}}(z) \neq 0 \forall z$, $\mathbf{H}(z) = r(z)\tilde{\mathbf{H}}(z)$ iff $\mathbf{N}(z) = r(z)\tilde{\mathbf{N}}(z)$.

Theorem 1 *In the noiseless case, if $\mathbf{C}_{\frac{T}{p}}(z)$ has no common zero and if $N_{\frac{T}{p}}(z)$ is not the null polynomial, $q(\hat{\mathbf{c}}) = 0$ iff $\hat{\mathbf{c}} = \lambda \mathbf{c}$.*

Thus, if we know the emission filter $\mathbf{N}(z)$ and the propagation channel $\mathbf{C}(z)$ has no common roots, then we can identify it even if the global filter $\mathbf{H}(z)$ has common roots. This is due to the fact that, would $\mathbf{H}(z)$ have common roots these would also be roots of $\mathbf{N}(z)$.

4.3. Blind structured subspace method

Eq.(7) can be processed to obtain $\mathbf{h} = \mathcal{C}\mathbf{n}$ with

$$\mathcal{C} = \sum_{m=-(p-1)}^{p-1} [(\mathbf{P}_p^m \otimes \mathcal{T}_{L_n}(C_0^m)) \dots (\mathbf{P}_p^m \otimes \mathcal{T}_{L_n}(C_{q-1}^m))]$$

It is thus easily recognized that the problem is symmetric: would the channel be known, the method allows to identify the emission filter. This section thus proposes an iterative algorithm estiming blindly both filters. The fact that the channel be multi-sensors whereas the emission filter is the same on each sensor ensures that the algorithm converges to the right solution in the noiseless case.

Algorithm: iteration $m+1$

- $\hat{\mathbf{c}}_{m+1} = \text{argmin}_{||c||=1} \mathbf{c} \hat{\mathcal{N}}_m \mathbf{Q} \hat{\mathcal{N}}_m^T \mathbf{c}$
- $\hat{\mathbf{n}}_{m+1} = \text{argmin}_{||n||=1} \mathbf{n} \hat{\mathcal{C}}_m \mathbf{Q} \hat{\mathcal{C}}_m^T \mathbf{n}$

This algorithm minimizes the conjoint criterion

$$q(\mathbf{n}, \mathbf{c}) = \mathbf{c}^H \mathcal{N}^H \mathbf{Q} \mathcal{N} \mathbf{c} \quad (11)$$

with the constraints $||\mathbf{n}|| = 1$ and $||\mathbf{c}|| = 1$.

The theorem 2 provides sufficient conditions for criterion 11 to have the true filters as unique global minimum.

Theorem 2 *In the noiseless case, if* $\mathbf{C}_{\frac{T}{p}}(z) \neq 0 \ \forall z$ *and* $\mathbf{N}(z) \neq 0 \ \forall z$ *then* $q(\hat{\mathbf{n}}, \hat{\mathbf{c}}) = 0$ *iff* $\exists (\mu, \nu) \in \mathbf{C}$ *such as* $\hat{\mathbf{n}} = \mu \mathbf{n}$ *and* $\hat{\mathbf{c}} = \nu \mathbf{c}$.

4.4. Emission filter shape knowledge based structured subspace method

This last method uses a prior knowledge of the emission filter shape, for instance a square roots raised-cosine pulse and estimates the roll-off factor α.

Algorithm: iteration $m+1$

- $\hat{\mathbf{c}}_{m+1} = \text{argmin}_{||c||=1} \mathbf{c}^H \mathcal{N}(\hat{\alpha}_m) \mathbf{Q} \mathcal{N}(\hat{\alpha}_m)^H \mathbf{c}$
- $\hat{\alpha}_{m+1} = \text{argmin}_{\alpha \in [0,1]} \mathbf{n}(\alpha)^H \hat{\mathcal{C}}_m \mathbf{Q} \hat{\mathcal{C}}_m^H \mathbf{n}(\alpha)$

The criterion is not a quadratic function of the roll-off factor. However with only one unknown parameter some algorithms can find the global minimum. Identifiability conditions given in theorem 2 are still sufficient.

5. SIMULATIONS

This section compares performances of the four algorithms presented in this paper.

The emitted symbols are i.i.d.8PSK. Emission filter is a square-root raised-cosine pulse with roll-off 0.25. Propagation splits the channel into P paths, characterized by an angle of incidence θ_p, a delay τ_p and an attenuation factor a_p. The signal is received on two sensors. We use the linear equalizer matrix $\Phi = (\mathcal{T}_W(\hat{\mathbf{H}})^H \mathcal{T}_W(\hat{\mathbf{H}}))^{-1} \mathcal{T}_W(\hat{\mathbf{H}})^H$. The simulations give the symbol error rate between the emitted symbols and the equalized signal. In each trial, N symbols are used for the identification.

First, simulations show as in [6]-[8] that the knowledge of the emission filter allows to significantly outperform classical blind algorithms. But, moreover we can see that knowing the sole shape of the filter we obtain almost the same performance. These (total or partial) knowledge based methods perfor much better than fully blind algorith, all these being almost equivalent, structured or not.

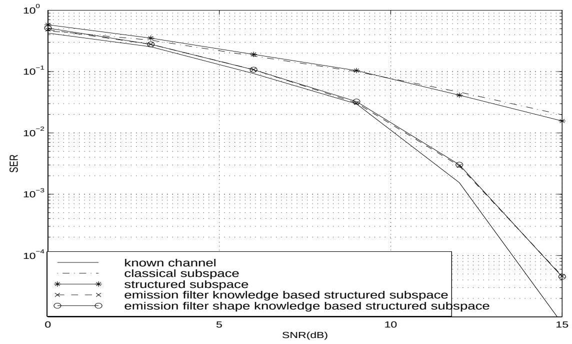

Figure 1: $\tau = (0, 0.6)T$, $\mathbf{a} = (1, 0.5)$, $\theta = (0, 20)°$, $N = 200$

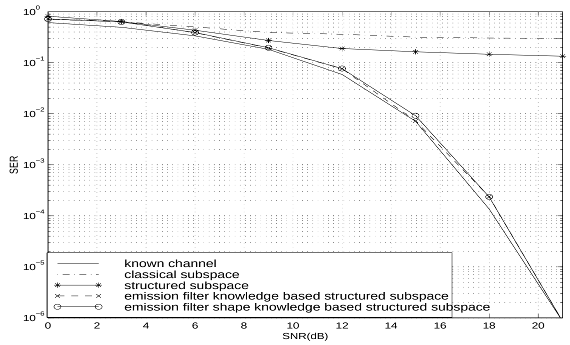

Figure 2: $\tau = (0, 0.25, 1.19, 2.4)T$, $\mathbf{a} = (1, 0.2, 0.4, 0.9)$, $\theta = (50, -20, -80, 10)$, $N = 500$

6. REFERENCES

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