A NOVEL CHANNEL EQUALIZER FOR CHAOTIC DIGITAL COMMUNICATIONS SYSTEMS

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ABSTRACT

In recent years, a variety of communications systems based on chaos and nonlinear dynamics have been proposed. However, most of these algorithms fail to work under realistic channel conditions. This paper presents a channel equalization scheme for chaotic communication systems based on a family of archetypal chaotic maps. The symbolic-dynamic representation of these maps is exploited to allow a straightforward and efficient implementation. Equalizer filter coefficients are updated using appropriately modified versions of decision-directed and decision-feedback equalization algorithms with adaptation based on the NLMS algorithm.

1. INTRODUCTION

Chaos has received a great deal of attention in the past few years from a variety of researchers, including mathematicians, physicists, and engineers. Researchers in the area of signal processing have largely been interested in chaos for the development of nonlinear communications techniques. Such communications systems offer the promise of inherent security, resulting from the broadband and 'noise-like' appearance of chaotic signals, and efficiency, since systems could be allowed to operate in their natural nonlinear states. Even simple one-dimensional maps can produce random-like yet deterministic signals. A variety of approaches to chaotic communications have been proposed, including chaotic modulation, masking, and spread-spectrum [1,2,3,4]. Because we believe that the chaotic modulation techniques show the most promise, they are the focus of this research.

A generic chaotic communications system based on chaotic modulation is shown in Figure 1. In such a system the information bits to be transmitted must first be encoded in the signal waveform generated by the chaotic system using what is termed *symbolic dynamics*. Rather than using structured signals, such as rectangular pulses or sinusoids, to denote '0's and '1's, these communications systems embed the information in the time evolution, or dynamics, of the transmitted signal. Regions of the state space formed by the chaotic system's dynamics are designated to represent different symbols (i.e., sequences of '0's and '1's). The process of mapping the information bits to the state of the chaotic system is termed chaotic modulation. This assignment of information bits to state should not be arbitrary, and the greatest efficiency is achieved when the information transmission rate matches the topological entropy of the chaotic system [3]. Next, using an appropriate carrier, the chaotic sequence is transmitted through the channel. The effect of the channel is to distort the transmitted sequence and corrupt it with additive noise. The goal of the equalizer is to undo the distortions caused by the channel. While most modern equalizers rely on their knowledge of the transmitted signal's waveform, either in the form of a specific training signal or known signal structure such as a constant modulus, this information is not available to the equalizer of Figure 1. In this case the equalizer has information about the dynamics of the transmitted signal but not its waveform. Finally, the recovered chaotic sequence is passed through a chaotic demodulator to obtain an estimate of the transmitted bit sequence from the symbolic dynamics of the reconstructed signal.

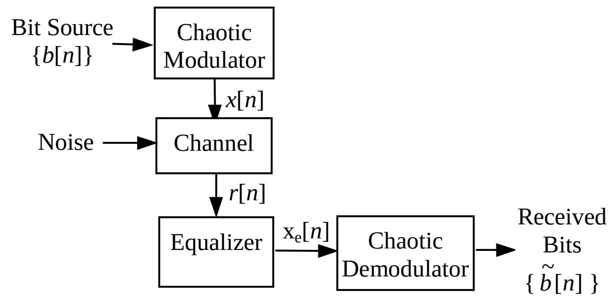

Figure 1. Block diagram for chaotic communication system.

Although there have been many algorithms proposed for using chaotic signals for communications purposes, there remain several basic issues that need to be addressed. First of all, almost all of these algorithms disregard channel effects or fail to work under realistic channel conditions. There has been some research into equalization algorithms, but these only compensate for very simple distortions such as a constant gain [5]. There is also a need to address the finite precision processing used in any digital communications system. Under fixed point arithmetic, chaotic systems are no longer chaotic and lose many of the properties that make them attractive for communications purposes [7]. Finally, lack of efficiency and speed is a severe limitation for many of the existing chaotic communications schemes.

In this paper, we present a framework to address all of these important problems. The proposed equalization algorithm is able to compensate for the effects of a fading dispersive channel with AWGN. Simple techniques are also proposed for chaotic modulation and demodulation at the maximum possible information rate. Additionally, alternative representations of the proposed chaotic systems provide a means for implementation in finite precision arithmetic. Chaotic communications systems are often said to be secure because the transmitted signal has a random appearance with little further justification. Results presented here will provide some insight into when such systems

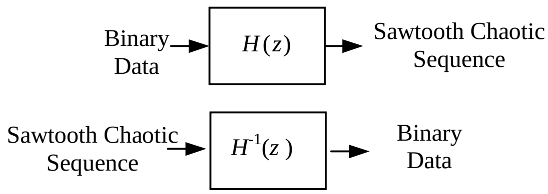

Figure 2. Representation for sawtooth chaotic modulation and demodulation through linear filtering.

are secure. Finally, it is shown that the proposed algorithms are fast, accurate, and efficient.

Section 2 provides background information on the class of chaotic systems that will be considered. Then, the chaotic communications systems and equalization algorithms are presented in Section 3. Simulations and results for the proposed algorithms are given in Section 4.

2. BACKGROUND

In general, discrete-time chaotic signals are represented as

$$x[n+1] = f(x[n]), \tag{1}$$

where $f(\cdot)$ is a nonlinear dynamical equation satisfying certain properties such as a sensitivity to initial conditions. Once the nonlinear dynamics, $f(\cdot)$, and an initial condition, $x[0]$, are specified, it is straightforward to generate a chaotic sequence. However, because of finite precision, for many chaotic maps, the sequence degenerates after a few iterations and is not chaotic. Two of the most popular one-dimensional chaotic maps, the sawtooth and tent maps, are examples of this problem. For our purposes, instead of generating the sequence directly, an alternative symbolic-dynamic representation will be used. This symbolic-dynamic representation not only is the key to efficient chaotic modulation and demodulation but also makes equalization possible. In the remainder of this section, the dynamical equations for these two maps and their corresponding symbolic-dynamic representations are given.

The sawtooth map is given by the following dynamical equation

$$x[n+1] = f(x[n]) = 2\cdot x[n] \bmod (1)$$

$$= \begin{cases} 2\cdot x[n] & if \;\; x[n] < 0.5 \\ 2\cdot x[n]-1 & if \;\; x[n] > 0.5 \end{cases} \tag{2}$$

Its symbolic-dynamic representation is found from the binary representation of the chaotic signal

$$x[n] = 0.b_n b_{n+1} b_{n+2} \cdots \qquad where \quad b_k = \{0,1\}$$

so that the sawtooth map is given by

$$x[n+1] = f(x[n]) = 0.b_{n+1} b_{n+2} b_{n+3} \cdots \tag{3}$$

More importantly, Drake and Williams [7] have shown that the sawtooth map's output is equivalent to the response of a linear filter to a binary sequence. The impulse response of this filter is given by

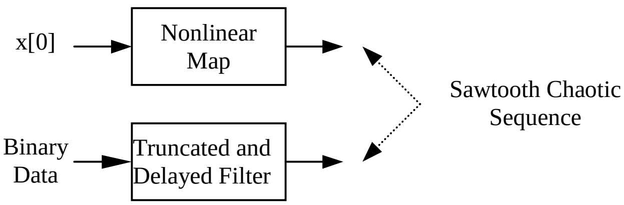

Figure 3. Two different implementations for generating indistinguishable chaotic sequences.

$$h[n] = 2^{n} u[-n-1], \tag{4}$$

with corresponding transfer function

$$H(z) = \frac{z/2}{1-z/2}, \quad |z| < 2 \tag{5}$$

Thus, the output of the filter is given by $x[n] = \sum_{k=-\infty}^{\infty} b[n-k]h[k]$,

where the initial condition, $x[0]$, completely determines the sequence b[n]

$$x[0] = 0.b_0 b_1 b_2 \ldots, \qquad and \qquad b[n+1] = b_n, \quad for \quad n > 0,$$

This linear filtering representation for the sawtooth map provides a means for the chaotic modulation shown in Figure. The input is the information bit sequence and the output is a chaotic sequence. This implementation is illustrated in Figure 2.

The inverse filter for this chaotic modulator is a simple two-tap FIR filter whose input is a sawtooth chaotic sequence and output is a binary sequence. The transfer function for this inverse filter is

$$H^{-1}(z) = -1 + 2z^{-1} \tag{6}$$

Thus, the chaotic demodulator of Figure 2 is just this inverse filter.

To be able to represent the sawtooth chaotic map in terms of linear filtering is especially important from a signal processing point of view. With this representation, not only is the chaotic system written in terms of an operation that is extremely familiar to the signal processing community (i.e., convolution), but it demonstrates that chaotic systems are not necessarily nonlinear and may have a completely equivalent linear representation. Other chaotic systems with linear representations are considered in [8].

Consequently, a binary bit sequence can be used as the input to the filter of Eq. (5) to get a chaotic data sequence. In practice this is not a possible because this filter is noncausal and has an infinite length. However, one need only to make reasonable modifications for it to be realizable, i.e. make simplifications such that the filter has finite-length and is causal. First, as the filter coefficients decay very rapidly, the filter can be truncated. Second, by shifting the resulting coefficients, we can make the filter causal. The final filter impulse response will then be

$$\tilde{h}(n) = \frac{2^{N-1}}{2^n} \qquad for \quad 0 \le n \le N, \qquad (7)$$

Truncation results in a loss of precision and shifting the coefficients results in delay, both of which are tolerable. Because of these modifications, the output will approximate a chaotic sequence within the limits of finite precision arithmetic. This structure is shown in Figure 3. The bit sequence can then be obtained from the chaotic signal by use of the inverse filter given previously. Thus, the chaotic modulation and demodulation problems are solved with very simple filtering operations for the sawtooth map. This forward and inverse filtering representation is the key for the proposed channel equalization algorithm.

A similar framework exists for another popular chaotic map, the tent map, given by the dynamical equation

$$x[n+1] = f(x[n]) = \begin{cases} 2 \cdot x[n] & x[n] < 0.5 \\ 2 - 2 \cdot x[n] & x[n] > 0.5 \end{cases} \qquad (8)$$

The tent map also has a binary representation given by

$$\begin{aligned} f(x[n]) &= f(0.b_n b_{n+1} b_{n+2} \cdots) \\ &= \begin{cases} 0.b_{n+1} b_{n+2} b_{n+3} \cdots & if \quad b_n = 0 \\ 0.b^*_{n+1} b^*_{n+2} b^*_{n+3} \cdots & if \quad b_n = 1 \end{cases} \end{aligned} \qquad (9)$$

where $b_k^* = \begin{cases} 1 & if\ b_k = 0 \\ 0 & if\ b_k = 1 \end{cases}$

Thus, this map may be implemented in a similar manner as the sawtooth map with a simple nonlinearity added to the modulator and demodulator to account for b_k^*.

Using these two popular one-dimensional chaotic maps, one can generate more chaotic maps through one-to-one transformations. For instance, the logistic map, which is another popular chaotic map, can also be produced with this approach. Additionally, by combining one-dimensional maps in different forms, one can obtain higher dimension chaotic maps [8]. Thus, one could easily map bit sequences to higher dimension chaotic sequences.

3. CHAOTIC COMMUNICATIONS SYSTEM AND EQUALIZER

At this point, it should be clear how chaotic modulation and demodulation are performed. To implement chaotic demodulation, i.e. to invert the symbolic dynamics, one needs ideally to have the original transmitted chaotic sequence. As mentioned earlier, because of the channel effects, the received chaotic sequence will be distorted. Therefore, an equalizer is needed to recover the transmitted chaotic sequence such that the demodulator gives the correct bit sequence. This section looks at the equalizer component of the block diagram. For simplification, a discrete-time representation will be used and the sawtooth map is assumed.

The equalizer and chaotic demodulator are shown in detail in Figure 4. In this block diagram, $r[n]$ represents the output of the channel, which is a distorted chaotic sequence, and the equalizer is an adaptive FIR filter. The output of this equalizer, denoted by $x_e[n]$, is the restored transmitted chaotic sequence. Then, $x_e[n]$ is passed through the inverse filter, $H^{-1}[z]$, given in Section 2 and thresholded by

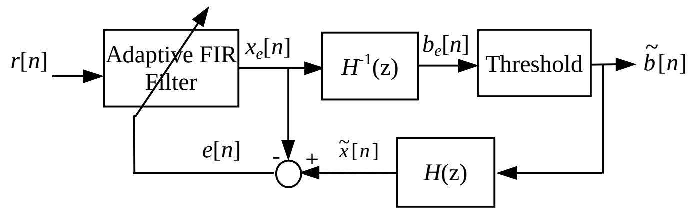

Figure 4. Channel equalizer for the sawtooth chaotic map.

$$\tilde{b}[n] = \begin{cases} 0 & if\ b_e[n] < 0.5 \\ 1 & if\ b_e[n] \ge 0.5 \end{cases} \qquad (10)$$

to demodulate the chaotic sequence.

To update the filter coefficients, the error signal, defined as the difference between the desired signal and the output of the equalizer, is needed. In this case, it is obtained by assuming that the received bit sequence, $\tilde{b}[n]$, resembles the transmitted bit sequence close enough so that it can be used as the input to the linear filter, $H(z)$. At the output of that filter, the chaotic sequence is regenerated (see $\tilde{x}[n]$ in Figure 4). Therefore, the necessary error sequence is calculated from the following equation

$$e[n] = \tilde{x}[n] - x_e[n] \qquad (11)$$

Once the error is obtained, well-known adaptive filter algorithms such as NLMS can be used to update the filter coefficients.

Though chaotic modulation and equalizer have only been explained for the sawtooth map, it is straightforward to extend these results for the tent map or any other map based on the symbolic representation given in Section 2. Also, instead of using decision directed equalizer, one could use a decision feedback equalizer. This equalizer would have an additional FIR filter on the feedback path whose input is the regenerated chaotic sequence $\tilde{x}[n]$.

4. SIMULATIONS AND RESULTS

To simulate the proposed algorithm, the channel was modeled to have the impulse response

$$c(n) = \begin{cases} \frac{1}{2} + \frac{1}{2}\cos[2\pi(n-2)/3] & \text{for } n = 1,2,3 \\ 0 & \text{otherwise} \end{cases} \qquad (12)$$

It was also assumed that the channel adds white Gaussian noise, $w[n]$, to the signal. Therefore, the signal at the output of the channel can be written as

$$r[n] = c[n] * x[n] + w[n] \qquad (13)$$

where $x[n]$ is the output of the chaotic modulator. Typical sawtooth and tent map chaotic sequences prior to channel

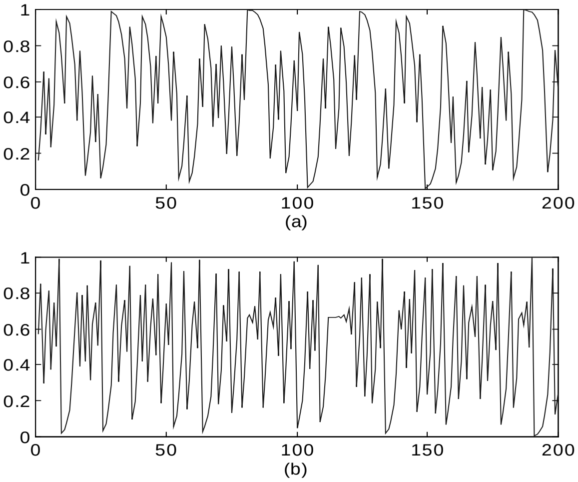

Figure 5. Typical chaotic sequences generated by (a) the sawtooth map and (b) the tent map.

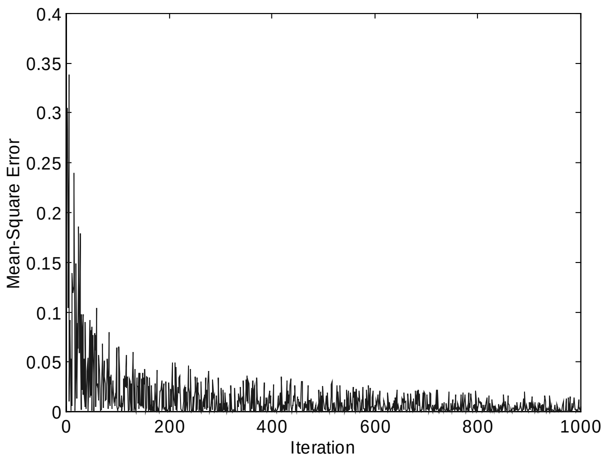

Figure 6. Mean-square error for the first 1000 iterations.

distortion are plotted in Figure 5 to illustrate the unstructured nature of these signals.

To update the equalizer filter coefficients, the NLMS algorithm was used. Adaptation starts with a training sequence, i.e., a known sequence of information bits, to get an initial estimate of the filter coefficients. For these simulations 20^{th} order equalizers were used with a step size of 0.1. The mean-square error is plotted for the first 1000 iterations in Figure 6 with a noise variance of 0.001. Algorithm convergence speed depends on step size.

We compare the performance of the proposed equalizer with a standard equalizer, i.e., one that knows the training sequence but is unaware that chaotic modulation has been used. In Figure 7, we plot bit error rate versus signal-to-noise ratio for both types of equalizers and for both tent map and sawtooth map modulations. From that graph, it is clear that the sawtooth map modulator is not secure as a standard equalizer of sufficient order works as well as the sawtooth map equalizer. On the other hand, for tent map modulation with its additional nonlinearity, the standard equalizer fails completely.

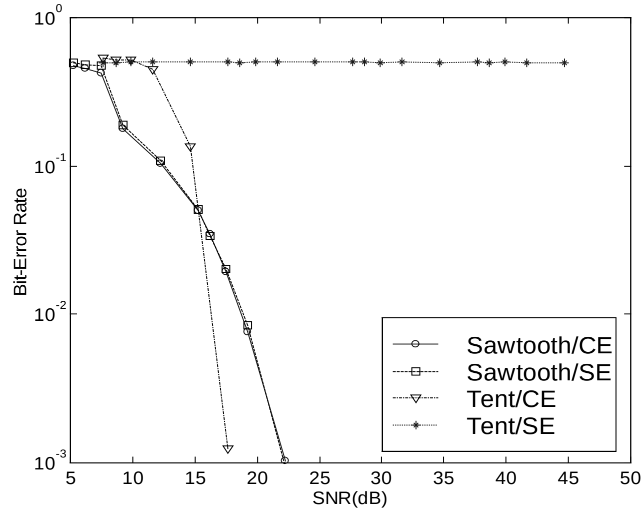

Figure 7. Bit-error rate versus SNR (dB) for chaotic equalizers (CE) and the standard equalizer (SE).

5. CONCLUSIONS

In this paper, we have proposed a chaotic communications system and channel equalizer using one-dimensional chaotic maps. The symbolic-dynamic representation of these maps was shown to be the key that makes possible chaotic modulation, demodulation, and equalization. By exploiting this alternative symbolic-dynamic representation, finite precision processing is possible. Moreover, the proposed algorithm is fast and efficient. Finally, the extension to higher dimension chaotic systems is straightforward.

6. REFERENCES

[1] L.M. Pecora, and T.L. Carroll, "Synchronization in chaotic systems," *Physical Review Letters*, vol. 64, no. 8, pp. 821-824, Feb. 1990.

[2] A.V. Oppenheim, G.W. Wornell, S.H. Isabelle, and K.M. Cuomo, "Signal processing in the context of chaotic signals," *IEEE Int. Conf. on Acous., Speech and Sig. Proc.*, May 1992.

[3] S. Hayes, "Communication with chaos," *Physical Review Letters*, vol. 70, no. 20, pp. 3031-3034, May 1993.

[4] G. Heidari-Bateni, and C.D. McGillem, "A chaotic communication direct-sequence spread-spectrum communication system," *IEEE Trans. on Communication*, vol. 42, no. 2/3/4, pp. 1524-1527, Feb. 1994.

[5] K.M. Cuomo, A.V. Oppenheim, and R.J. Barron, "Channel equalization for self-synchronizing chaotic systems," *IEEE Int. Conf. on Acous., Speech and Sig. Proc.*, May 1996.

[6] H. Peitgen, H. Jurgens, and D. Saupe, *Chaos and Fractals: New Frontiers of Science*, Springer-Verlag 1992.

[7] D. F. Drake, and D. B. Williams, "Pseudo-Chaos for Direct-Sequence Spread-Spectrum Communication," *Proc. of SPIE*, vol. 2612, pp. 104-114, 1995.

[8] D.F. Drake, *Information's Role in the Estimation of Chaotic Signals*, PhD thesis, Georgia Institute of Technology, 1998.