HYBRID SEMI-BLIND MULTI-USER DETECTORS: SUBSPACE TRACKING METHODS

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ABSTRACT

We consider the problem of multi-user detection for CDMA systems where the codes of some users are known while others are unknown, called semi-blind detectors. An example is at the base station of a cellular communication systems with interference from both in-cell users, with known codes, and out-of-cell users, with unknown codes. In this paper we develop a hybrid semi-blind detector, which is partly decorrelating, partly MMSE. Subspace tracking methods are developed for on-line implementation of the detector. The performance of the detectors is compared to that of the purely blind MMSE detector and the non-blind MMSE detector, and the semi-blind detector is seen to have a considerable better performance.

1. Introduction

Multi-user detection is a method to improve detection performance and capacity of multiple access spread spectrum (or CDMA) systems. Multi-user detection was introduced by Verdú in [1], where the optimal multi-user detector was derived. The optimal detector has an exponential complexity in the number of users, and less complex (linear) multi-user detectors were therefore derived in an number of papers, in particular the decorrelating detector [2] and the minimum mean square error (MMSE) detector [3].

The early works on multi-user detection assumed that the codes of all users were known at the receiver, and made a simultaneous detection of all users (therefrom the name multi-user detection or joint detection). If the detection for example is at the base station of a mobile communication system, this seems realistic, as the base station needs to perform detection for all users. On the other hand, it is unrealistic that a mobile station should know the codes of all other users in a cell, and therefore it is desirable to consider multi-user detectors that need to know only the code of the desired user and does not use a training sequence, blind multi-user detection. The blind MMSE detector was introduced in a number of papers [5], and recently it was also shown that the decorrelating detector can be implemented blindly [8,9].

Although a base station knows all codes of the users within a cell, it typically will not know the codes of interfering users from surrounding cells. Even if a base stations could obtain this knowledge from surrounding base stations, it would be a waste of resources if it were to also perform detection for these users just to cancel interference (including synchronization etc.). This is a serious problem to multi-user detection, since typically 1/3 of the interference could be from other cells, intercell interference [6]. Thus, also at the base station blind detectors could be relevant. On the other hand, blind detectors do not use neither the fact that the codes of in-cell users are known at the base station, nor that these other users also have to be detected.

This has led us to consider multi-user detectors that can cancel interference from both known and unknown users, while utilizing the information about known users and the fact that detection has to be done for all known users. A blind multi-user detector basically in some way has to estimate the codes of interfering users, and by using the known codes the estimation accuracy can be improved. On the other hand, since several users have to be detected jointly, it is also advantageous, considering computational complexity, if some of the processing can be common to all users. In this paper we will develop detectors that satisfy these two criteria.

In a previous paper [10], the decorrelating semi-blind detector was derived and implemented using SVD. In the current paper we will extend this to a detector that is decorrelating among the known users, and MMSE with respect to the unknown users. Furthermore, we develop a subspace tracking method for the detector, which is more efficient than using SVD. We will use the subspace approach of Wang and Poor [9], and as in [9] we will only consider the synchronous case, since this gives a better understanding of the underlying geometry of the problem. Since most applications of the detectors would probably be in asynchronous systems, the detectors will be generalized to the asynchronous case in a later paper.

2. System model

Consider a synchronous spread spectrum communications system with the users transmitting through an additive white Gaussian noise channel. The user population consists of K users with known codes, and $\tilde{K}$ users with unknown codes. As we consider synchronous systems, it is sufficient to consider a single symbol interval $[0,T]$, where the received signal can be written as

$$r(t)=\sum_{k=1}^{K} b_k A_k s_k(t)+\sum_{k=1}^{\tilde{K}} \tilde{b}_k \tilde{A}_k \tilde{s}_k(t)+n(t), \quad t\in[0,T]$$

where s_k is the normalized code waveform of the k'th *known* user with support in $[0,T]$, $\tilde{s}_k$ the waveform of the k'th *unknown* user, $b_k, \tilde{b}_k$ are the transmitted bits (± 1), $A_k, \tilde{A}_k$ the amplitudes and n white Gaussian noise. We assume that the codes are given by chip codes, $s_k(t)=\sum_{i=0}^{M-1} c_k^i \psi(t-iT/M)$, where $c_k^i \in \{-1,+1\}$ and ψ is the chip waveform, and similarly for the unknown users. A sufficient statistic for the received signal is therefore the output of a chip rate sampled chip matched filter, and we can write this statistic on vector form as

$$\mathbf{r}=\sum_{k=1}^{K} b_k A_k \mathbf{s}_k+\sum_{k=1}^{\tilde{K}} \tilde{b}_k \tilde{A}_k \tilde{\mathbf{s}}_k+\mathbf{n}=\mathbf{SAb}+\tilde{\mathbf{S}}\tilde{\mathbf{A}}\tilde{\mathbf{b}}+\mathbf{n}$$

where $\mathbf{S}=\left[\mathbf{s}_1,\mathbf{s}_2,...,\mathbf{s}_K\right]$, $\tilde{\mathbf{S}}=\left[\tilde{\mathbf{s}}_1,\tilde{\mathbf{s}}_2,...,\tilde{\mathbf{s}}_{\tilde{K}}\right]$. The correlation matrix of **r** is given by

$$\mathbf{R}=E[\mathbf{r}\mathbf{r}^T]=\mathbf{SA}^2\mathbf{S}^T+\tilde{\mathbf{S}}\tilde{\mathbf{A}}^2\tilde{\mathbf{S}}^T+\sigma^2\mathbf{I}$$

We will assume that all codes, of both known and unknown users, are linearly independent, so that $\begin{bmatrix}\mathbf{S} & \tilde{\mathbf{S}}\end{bmatrix}$ has full rank.

3. Detector Structures

A general linear detector for user i is given by

$$\hat{b}_i=\operatorname{sgn}\left(\mathbf{w}_i^T\mathbf{r}\right)$$

For jointly detection of all users, the detector can be written

$$\hat{\mathbf{b}}=\operatorname{sgn}\left(\mathbf{W}^T\mathbf{r}\right)$$

where $\mathbf{W}=\begin{bmatrix}\mathbf{w}_1 & \mathbf{w}_2 & \cdots & \mathbf{w}_K\end{bmatrix}$. As argued in [7], any interesting detector must have $\mathbf{w}_i\in\operatorname{range}\left(\begin{bmatrix}\mathbf{S} & \tilde{\mathbf{S}}\end{bmatrix}\right)$, since any component of $\mathbf{w}_i$ outside this subspace will only increase noise without reducing interference.

4. Semi-blind decorrelating detectors

We will derive two different semi-blind detectors. Define

$$\mathbf{S}_1=[\mathbf{s}_2\ \mathbf{s}_3\ldots\mathbf{s}_K] \tag{1}$$

and let

$$\mathbf{P}_1=\mathbf{S}_1\left(\mathbf{S}_1^T\mathbf{S}_1\right)^{-1}\mathbf{S}_1^T \tag{2}$$

be the projection unto the subspace spanned by $\mathbf{S}_1$, with $\mathbf{P}_1^\perp=\mathbf{I}-\mathbf{P}_1$ the projection unto the orthogonal subspace.

In our first approach to semi-blind decorrelating detectors, we use a mixed projection/orthogonalization approach. The idea is to first project **r** on the subspace orthogonal to $\operatorname{span}(\mathbf{S}_1)$, with $\mathbf{S}_1$ given by (**1**). Thereby all interference from the *known* users has been removed. The orthogonalization approach is then used in $\operatorname{span}(\mathbf{S}_1)^\perp$. The calculations can be done as follows. Let $\mathbf{P}_1$ be defined by (**2**), and define $\tilde{\mathbf{S}}_1=\mathbf{P}_1^\perp\begin{bmatrix}\tilde{\mathbf{S}} & \mathbf{s}_1\end{bmatrix}$, $\tilde{\mathbf{A}}_1=\begin{bmatrix}\tilde{\mathbf{A}} & A_1\end{bmatrix}$ and

$$\tilde{\mathbf{W}}_1=\tilde{\mathbf{S}}_1\tilde{\mathbf{A}}_1\left(\tilde{\mathbf{A}}_1\tilde{\mathbf{S}}_1^T\tilde{\mathbf{S}}_1\tilde{\mathbf{A}}_1\right)^{-2}\tilde{\mathbf{A}}_1\tilde{\mathbf{S}}_1^T$$

Then we have

Theorem 1: The decorrelating detector is given by

$$\mathbf{w}_1=\frac{1}{\mathbf{s}_1^T\mathbf{P}_1^\perp\tilde{\mathbf{W}}_1\mathbf{s}_1}\mathbf{P}_1^\perp\tilde{\mathbf{W}}_1\mathbf{s}_1 \tag{3}$$

Furthermore, the eigenvalue decomposition of $\tilde{\mathbf{R}}_1=\mathbf{P}_1^\perp\mathbf{R}\mathbf{P}_1^\perp$ can be written as

$$\tilde{\mathbf{R}}_1=\begin{bmatrix}\mathbf{U}_s & \mathbf{U}_n & \mathbf{U}_o\end{bmatrix}\begin{bmatrix}\Lambda_s & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \sigma^2\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{U}_s^T\\ \mathbf{U}_n^T\\ \mathbf{U}_o^T\end{bmatrix}$$

where $\Lambda_s=\operatorname{diag}(\lambda_1,\lambda_2,...,\lambda_{\tilde{K}+1})$ with $\lambda_i>\sigma^2$. Finally

$$\tilde{\mathbf{W}}_1=\mathbf{U}_s\left(\Lambda_s-\sigma^2\mathbf{I}_{\tilde{K}+1}\right)^{-1}\mathbf{U}_s^T$$

In our setting, $\tilde{\mathbf{S}}$ is unknown. The theorem, however, outlines a method for estimating $\tilde{\mathbf{W}}_1$, and the method can therefore be implemented without knowledge of the unknown users' codes.

The problem of the above method is that $\tilde{\mathbf{W}}_1$ depends is specific to user 1. Thus, if several users are to be detected in parallel, an SVD/subspace tracking has to be done for each user.

We will therefore develop a method that requires only one SVD common to all users. This method will be based on an orthogonalization approach. First we define the projection on $\operatorname{span}(\mathbf{S})$,

$$\mathbf{P}=\mathbf{S}\left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T$$

and $\mathbf{P}^\perp=\mathbf{I}-\mathbf{P}$. The eigenvalue decomposition of $\mathbf{P}^\perp\mathbf{R}\mathbf{P}^\perp$ is then given by

$$\mathbf{P}^\perp\mathbf{R}\mathbf{P}^\perp=\begin{bmatrix}\tilde{\mathbf{U}}_s & \tilde{\mathbf{U}}_n & \tilde{\mathbf{U}}_o\end{bmatrix}\begin{bmatrix}\tilde{\Lambda}_s & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \sigma^2\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{U}}_s^T\\ \tilde{\mathbf{U}}_n^T\\ \tilde{\mathbf{U}}_o^T\end{bmatrix}$$

where $\tilde{\Lambda}_s=\operatorname{diag}(\tilde{\lambda}_1,\tilde{\lambda}_2,...,\tilde{\lambda}_{\tilde{K}})$ with $\tilde{\lambda}_i>\sigma^2$, and $\tilde{\mathbf{U}}_o$ has K rows.

Theorem 2: The decorrelating detector is given by

$$\hat{\mathbf{b}}=\operatorname{sgn}\left(\left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T\left(\mathbf{I}-\mathbf{R}\tilde{\mathbf{U}}_s\left(\tilde{\Lambda}_s-\sigma^2\mathbf{I}\right)^{-1}\tilde{\mathbf{U}}_s^T\right)\mathbf{r}\right) \tag{4}$$

Proof: see [11].

Notice that all quantities in the theorem can be estimated. Notice also, that although **R** appears explicitly, **R** itself does not have to be estimated. We can use that

$$\begin{aligned}\mathbf{S}^T\hat{\mathbf{R}}(n)\hat{\tilde{\mathbf{U}}}_s(n)&=\mathbf{S}^T\left(\frac{1}{n}\sum_{i=1}^{n}\mathbf{r}_i\mathbf{r}_i^T\right)\hat{\tilde{\mathbf{U}}}_s(n)\\ &=\frac{1}{n}\left(\sum_{i=1}^{n}\left(\mathbf{S}^T\mathbf{r}_i\right)\left(\hat{\tilde{\mathbf{U}}}_s^T(n)\mathbf{r}_i\right)^T\right)\end{aligned} \tag{5}$$

where $\hat{\mathbf{R}}(n)$ etc. denote the estimated quantities over n samples. The matrix on the left side is only a $K\times\tilde{K}$ matrix, instead of the M^2 matrix **R**.

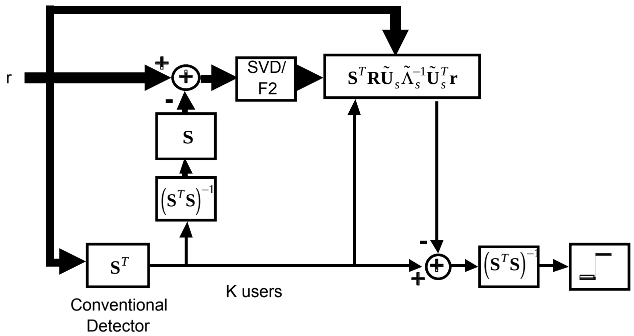

Figure 1: Hybrid semi-blind detector.

5. Hybrid Semi-Blind Detectors

We will in this section consider detectors that are combinations of the decorrelating detector and the MMSE detector. Specifically, we will study detectors that are decorrelating among the known users and MMSE with respect to the unknown users. We will call this class of detectors hybrid semi-blind detectors. The idea is similar to ideas used in array processing: to direct nulls in the direction of known interferers, and find the MMSE solution for the remaining interference, and has in this context also been studied for multi-user detection.

As for the decorrelating detector we will give two solutions for the hybrid detector: one that makes a subspace calculation for each user, and one that makes a common subspace tracking. For the former case we get the following theorem 3. We use the same notation as for theorem 1.

Theorem 3: The hybrid semi-blind detector is given by

$$\mathbf{w}_1 = \frac{1}{\mathbf{s}_1^T\mathbf{U}_s\Lambda^{-1}\mathbf{U}_s^T\mathbf{s}_1}\mathbf{U}_s\Lambda^{-1}\mathbf{U}_s^T\mathbf{s}_1$$

where $\mathbf{U}_s$ and Λ_s is given from the eigenvalue decompositon of $\tilde{\mathbf{R}}_1 = \mathbf{P}_1^{\perp}\mathbf{R}\mathbf{P}_1^{\perp}$:

$$\tilde{\mathbf{R}}_1 = \begin{bmatrix}\mathbf{U}_s & \mathbf{U}_n & \mathbf{U}_o\end{bmatrix}\begin{bmatrix}\Lambda_s & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \sigma^2\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{U}_s^T\\ \mathbf{U}_n^T\\ \mathbf{U}_o^T\end{bmatrix}$$

where $\Lambda_s = \mathrm{diag}(\lambda_1, \lambda_2, ..., \lambda_{\tilde{K}+1})$ with $\lambda_i > \sigma^2$.

For the case of common subspace tracking we get, with the same notation as for theorem 2

Theorem 4: The hybrid semi-blind detector is given by

$$\hat{\mathbf{b}} = \mathrm{sgn}\left(\left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T\left(\mathbf{I} - \mathbf{R}\tilde{\mathbf{U}}_s\tilde{\Lambda}_s^{-1}\tilde{\mathbf{U}}_s^T\right)\mathbf{r}\right) \qquad (6)$$

Proof: see [11].

In general, the MMSE detector has superior performance to the decorrelating detector [7], and it would therefore be natural to study pure MMSE semi-blind detectors. However, the hybrid approach has a number of advantages: when the multi-user detector for the known users is implemented non-adaptively, the MMSE detector requires estimates of user powers, while the decorrelating detector does not require any estimated information (except timing information in the asynchrouneous case).

6. Subspace Tracking

A host of different subspace tracking methods, with varying convergence speed and complexity, esist, and most of them can be adapted to the current problem. To illustrate the principles we have chosen to use the F2 algorithm [12]. Define

$$\mathbf{W}(n) = \left[\sqrt{\beta(n)}\hat{\tilde{\mathbf{U}}}_s(n-1)\hat{\tilde{\Lambda}}(n-1) \quad \sqrt{\alpha(n)}\mathbf{P}^{\perp}\mathbf{r}_n\right]$$

Let the SVD of $\mathbf{W}(n)$ truncated to the largest $\tilde{K}$ singular values be $\mathbf{X}\Gamma\mathbf{Y}^H$. The matrices $\hat{\tilde{\mathbf{U}}}_s(n-1)$ and $\hat{\tilde{\Lambda}}_s(n-1)$ can then be updated as follows:

$$\hat{\tilde{\mathbf{U}}}_s(n) := \mathbf{X}$$

$$\hat{\tilde{\Lambda}}(n) := \Gamma$$

When using subspace tracking for the semi-blind detectors (6) and (4), the main problem is that equation (5) is not directly suitable for recursive implementation, since $\hat{\mathbf{U}}_s(n)$ should be applied to all previous samples. For a recursive implementation, either $\mathbf{R}$ must be calculated (recursively) or a recursive update formula for (5) must be found. One possibility is to replace $\hat{\mathbf{U}}_s(n)$ with $\hat{\mathbf{U}}_s(i)$ inside the summation, but we have found this does not give too good convergence (but sufficient for a slow converging tracking method as PASTd). Another approximation can be found as follows. Define

$$\mathbf{T}(n) = \mathbf{S}^T\hat{\mathbf{R}}(n)\hat{\tilde{\mathbf{U}}}_s(n)$$

then this can be updated approximately as follows:

$$\mathbf{T}(n+1) \approx \mathbf{T}(n)\hat{\tilde{\mathbf{U}}}_s^T(n)\mathbf{U}_s(n+1) + \mathbf{S}^T\mathbf{r}_{n+1}\left(\hat{\tilde{\mathbf{U}}}_s^T(n+1)\mathbf{r}_{n+1}\right)^T \qquad (7)$$

The justification of this approximation is geometrical. $\hat{\tilde{\mathbf{U}}}_s(n)\mathbf{T}^T(n)$ represents the orthogonal projection of $\hat{\mathbf{R}}(n)\mathbf{S}^T$ onto the subspace spanned by $\hat{\tilde{\mathbf{U}}}_s(n)$. To find the projection of $\hat{\mathbf{R}}(n)\mathbf{S}^T$ onto the subspace spanned by $\hat{\tilde{\mathbf{U}}}_s(n+1)$ we should reproject $\hat{\mathbf{R}}(n)\mathbf{S}^T$. Instead we project the previous projection. If the subspace does not change too much from n to n+1, the approximation is only slight.

The calculation of $\hat{\tilde{\mathbf{U}}}_s^T(n)\hat{\tilde{\mathbf{U}}}_s(n+1)$ is complex in itself. Fortunately, this can be obtained as a intermediate result in some subspace tracking algorithms, for example F2. Notice that since $\hat{\tilde{\mathbf{U}}}_s^T(n)\hat{\tilde{\mathbf{U}}}_s(n) = \mathbf{I}$, we have

$$\Gamma \mathbf{Y}^H \hat{\tilde{\Lambda}}(n-1)^{-1} = \hat{\tilde{\mathbf{U}}}(n)^H \mathbf{W}(n) \hat{\tilde{\Lambda}}(n-1)^{-1}$$
$$= \left[\sqrt{\beta(n)} \hat{\tilde{\mathbf{U}}}(n)^H \hat{\tilde{\mathbf{U}}}_s(n-1) \quad \sqrt{\alpha(n)} \hat{\tilde{\mathbf{U}}}(n)^H \mathbf{P}^{\perp} \mathbf{r}_n \hat{\tilde{\Lambda}}(n-1)^{-1} \right]$$

Thus, $\hat{\tilde{\mathbf{U}}}_s^T(n)\hat{\tilde{\mathbf{U}}}_s(n+1)$ can be found from the first $\tilde{K}$ columns of $\Gamma \mathbf{Y}^H \hat{\tilde{\Lambda}}(n-1)^{-1}$

7. Simulation results

We consider a system with K=7 users with known codes, and 4 users with unknown codes. The users are assigned purely random codes of length M=31. An ensemble of 100 different random code assignments is generated, and the average signal to inference and noise ratio (SINR) is calculated over all code choices and users.

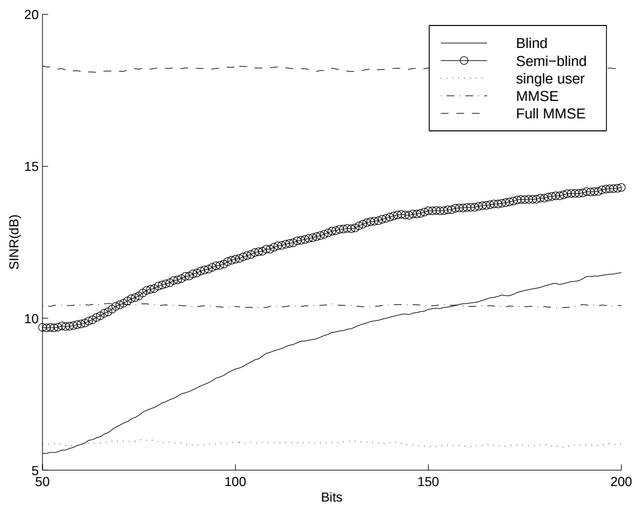

Figure 2: Peformance with 7 known and 4 unknown users.

We consider 5 different detectors, all implemented using the F2 subspace tracking

Single user: the traditional single user matched filter detector.

Full MMSE: the hypothetical MMSE detector that knows all $K+\tilde{K}$ codes, giving a performance bound for any blind/semi-blind linear detector.

MMSE: the MMSE detector for the known users, disregarding the unknown users.

Blind: the blind MMSE detector of [9].

Semi-blind: the detector given by (6).

The number of unknown users $\tilde{K}$ is, unrealistically, assumed to be known. We will not address the estimatiuon of $\tilde{K}$ here.

In Figure 2 we have plotted the convergence of the average SINR. The SNR is 20 dB and all users (known and unknown) have the same power. It is seen that the blind detector of [9] needs considerably more iterations to converge to a given level of performance than the semi-blind detectors in all situations. The performance using F2 is indistinguishable from the performance using SVD, in spite of the approximation we have committed.

8. Conclusion

In this paper we have developed hybrid semi-blind detectors. They are distinguished by the fact that they can cancel interference from both known and unknown users, like blind detectors, while simultaneously using the knowledge of known users. Simulations have shown that the semi-blind detectors have notably better performance than the pure blind detectors, and furthermore they have a considerably lower computational complexity.

We have also developed a subspace tracking algorithm based on the F2 algorithm. The performance of this is almost identical to SVD.

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