ANALYSIS OF SPECTRAL AND WAVELET-BASED MEASURES USED TO ASSESS CARDIAC PATHOLOGY

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ABSTRACT

In recent studies of heart rate variability in humans [2,5,6], it has been demonstrated that the scale-dependent wavelet transform (WT) standard deviation [$\sigma_{\text{wav}}(m)$] of the interbeat intervals can be used to discriminate patients with certain forms of cardiac dysfunction from normal subjects. This paper forges an explicit link between this measure and a corresponding spectral measure, which is also shown to provide discrimination between the two classes of data. The statistics of the estimator for $\sigma_{\text{wav}}(m)$ are obtained in the analytically simplest case, including expressions for its bias and variance. Numerical simulations are provided to support the theoretical analysis. We compare the bias, variance, and frequency resolution of WT and spectral measures, and conclude that the former appears more suited to our particular circumstances.

1. INTRODUCTION

Heart rate variability (HRV) is a commonly used term to describe variations of human heart interbeat intervals measured using non-invasive electrocardiographic methods. HRV analysis has been considered by many researchers, and has been shown to be a useful clinical tool, though its use as an *early* predictor of underlying pathology is still under investigation [4]. In recent studies, multiresolution wavelet analysis of HRV was shown to discriminate between healthy patients and those with some forms of underlying cardiac pathology [2,5,6]. The discriminating measure used was the standard deviation [$\sigma_{\text{wav}}(m)$] of the dyadic Discrete Wavelet Transform (DWT) of the sequence of R-R heartbeat intervals. In particular, it was found that the value of $\sigma_{\text{wav}}(m)$ over a small range of scales m differed for the two classes of data considered (normal and pathological). However, for reliable clinical use, we must clearly understand the statistical properties of this discriminating measure. In this paper, we consider the statistical properties of $\sigma_{\text{wav}}(m)$, and explicitly demonstrate its links to power spectral density (PSD) measures of the same data. We assess their relative utility by evaluating the bias, variance, and frequency resolution of estimators for both measures. Analysis of WT variance is of general interest, as it has been widely used in a variety of fields. Measures of WT variance have been used in assessing long-term correlation [1,2,7].

2. THEORY

2.1 Spectral and DWT Variance Measures

In [5], the statistic used was $\sigma_{\text{wav}}(m)$, the standard deviation of the DWT of the discrete-index sequence of RR intervals. The DWT was defined in the following way:

$$W[m,n] = 2^{-m/2}\sum_{i=0}^{M-1}\tau[i]\psi\left(2^{-m}i-n\right) \tag{1}$$

where the scale variable m and the translation variable n are non-negative integers, $\tau[i]$ is the discrete-index sequence of RR intervals, ψ is the wavelet basis function, and M represents the total number of intervals analyzed. The quantity of interest in this paper is the variance of $W[m,n]$ as a function of m, which we will define as $D(m)$. WT variance measures can be interchangeably expressed either in *variance* or *standard-deviation* format (*e.g.*, $\sigma_{\text{wav}}(m)$ as in [5]), which are equivalent.

We proceed by demonstrating the equivalence between $D(m)$ and PSD measures of the same data. The analysis and interpretation is clearer in the continuous domain, but the results translate easily to the discrete domain by interpreting the DWT as a discretized version of a continuous wavelet transform (CWT) of a signal $\tau(t)$ defined as:

$$CWT^{\tau}(a,b) = \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty}\tau(t)\psi^{*}\left(\frac{t-b}{a}\right)dt \tag{2}$$

where a and b are scale and translation parameters respectively, ψ is a wavelet basis function, and * is complex conjugation. The CWT has the salutatory effect of removing non-stationarities, so that at any fixed scale CWT^{τ} is a stationary sequence. Since the expected value of CWT^{τ} is 0, the variance of CWT^{τ} at scale a is:

$$D(a) = E\left[\left|CWT^{\tau}(a,b)\right|^{2}\right] \Rightarrow$$
$$D(a) = E\left[\frac{1}{\sqrt{a}}\int\tau(t)\psi^{*}\left(\frac{t-b}{a}\right)dt\frac{1}{\sqrt{a}}\int\tau^{*}(u)\psi\left(\frac{u-b}{a}\right)du\right]. \tag{3}$$

For a wide-sense stationary signal $\tau(t)$, this leads to:

$$D(a) = \frac{1}{a}\iint R(t-u)\psi^{*}\left(\frac{t-b}{a}\right)\psi\left(\frac{u-b}{a}\right)dt\,du, \tag{4}$$

where $R(s)$ is the autocorrelation function of $\tau(t)$. Routine algebraic manipulation leads to

$$D(a) = a\int_{-\infty}^{\infty}R(ay)CWT^{\psi}(1,y)\,dy, \tag{5}$$

or alternatively:

$$D(a) = a \int_{\omega=-\infty}^{\infty} S(\omega)\left[\int_{y=-\infty}^{\infty} CWT^{\psi}(1,y)\exp(j\omega a y)\,dy \right] d\omega \qquad (6)$$

where $CWT^{\psi}(1,y)$ is the wavelet transform of the wavelet itself (termed the wavelet kernel), and $S(\omega)$ is the PSD of the signal.

Equation (6) shows that the WT variance is directly related to the PSD through an integral transform. This has a simple interpretation: the term in square brackets is a bandpass filter that only passes components of the PSD in a bandwidth surrounding the frequency ω_a corresponding to scale *a*. We will return to this viewpoint later in discussing the frequency resolution of $D(a)$. For a discrete-index sequence, the sampling "time" can be arbitrarily set to 1 so that a frequency ω_a corresponds to $1/a$. Accordingly, any discrimination seen in a $D(a)$-based statistic such as $\sigma_{\text{wav}}(m)$ should also be accessible through PSD-based measures. In Section 3, this is illustrated in the particular context of HRV analysis by applying both PSD and $D(a)$ measures to the same data sets. Since the discrimination properties of these measures are decided by the accuracy of our estimates, we proceed to consider the statistics of both $D(a)$ and PSD estimators.

2.2 Properties of Spectral and *D*(*a*) Estimators.

In Section 2.1, we showed that PSD and DWT variance measures are intimately linked, and that in theory either can be used to provide discrimination between data sets. Here, we consider the statistics of the estimators used to determine $D(a)$ and PSD measures. In order to present an analytically tractable case, we consider estimating both measures for a data set of *M* samples of white Gaussian noise with zero mean and variance σ^2. The sampling rate is arbitrarily chosen as unity. The theoretical value for the PSD of this signal is σ^2. We will evaluate the discrete dyadic version of $D(a)$, *i.e.* $D(m) = \sigma^2_{\text{wav}}(m)$.

The bias and variance of non-parametric PSD estimators is well studied; for an overview see Reference [3]. For comparison with DWT variance estimators, we assume that $S(\omega)$ is calculated using an averaged periodogram technique, with *K* non-overlapping rectangular data windows of M/K datapoints each. This provides a biased estimate of $S(\omega)$, with a variance equal to σ^4/K, and a frequency resolution of approximately $2K/M$. The bias cannot be removed, since it arises from the integration of nearby values of $S(\omega)$ which are unknown, but can be minimized by using longer data segments (*i.e.*, reducing *K*). However, this reduces the number of available segments for averaging, leading to the well-known bias-variance tradeoff in PSD estimation.

Results for the bias and variance of DWT variance estimators are not well studied, however, and we will consider estimators of DWT variance obtained from the definition of Eq. (1), with $\tau[i]$ assumed to be the same WGN sequence described above. Without loss of generality, we assume that *M* is an integer power of 2. We also make the simplifying assumption that $\psi[i]$ is obtained by sampling an underlying Haar wavelet basis compactly supported on [0,1/2], so that the discrete-index sequence for scale m=1 is [1,-1]. In addition, we restrict ourselves to integer values of *n*. The number of valid samples of $W[m,n]$ (*i.e.*, with no edge effects) changes with scale. For scale m=1, we have $M/2$ DWT coefficients, and for general *m*, we have $M/2^m$ samples in the range $n = [0,(M/2^m)-1]$. The expected value of $D(m)$ is σ^2 for all values of *m*. We will compare this with the expected value from a suitable sample variance estimator, such as:

$$\hat{D}(m) = \frac{2^m}{M-2^m} \sum_{n=0}^{2^{-m}M-1} \left[2^{-m/2} \sum_{i=0}^{M-1} \tau[i]\psi\left(2^{-m}i - n\right) \right]^2, \qquad (7)$$

where we have used the fact that $\mathrm{E}\left[W[m,n]\right] = 0$, where E represents the expectation operator. The caret over $D(m)$ indicates that it is an estimator. We are interested in the bias and variance of this estimator. Consider first its expected value which turns out to be

$$\mathrm{E}\left[\hat{D}(m)\right] = \frac{\sigma^2}{M-2^m} \left[\sum_{n=0}^{(2^{-m}M-1)} \sum_{i=0}^{M-1} \psi^2\left(2^{-m}i - n\right) \right], \qquad (8)$$

using $\mathrm{E}[\tau[i]\,\tau[j]] = \sigma^2\delta(i-j)$. For the specific case of the Haar wavelet, where ψ^2 only takes the values 1 or 0, the quantity inside the square brackets evaluates to *M*. Therefore the expected value of our sample estimator is

$$\mathrm{E}\left[\hat{D}(m)\right] = \frac{M\sigma^2}{M-2^m}. \qquad (9)$$

While this is an asymptotically unbiased estimator of the required quantity, for finite *M* the sample variance will show considerable bias at large values of *m*. If we assumed *a priori* a signal with white noise properties, we could deterministically remove the bias. For more general classes of signal, it may not be possible to analytically determine the bias. Qualitatively, we may expect bias effects to increase at large scales regardless of the signal properties.

Calculating the *variance* of the estimator proceeds as follows. We are interested in calculating $\mathrm{E}\left[\left(\hat{D}(m) - \mathrm{E}\left[\hat{D}(m)\right]\right)^2\right]$. The new quantity of interest required is $\mathrm{E}\left[\hat{D}^2(m)\right]$, which is:

$$\mathrm{E}\left[\hat{D}^2(m)\right] = \mathrm{E}\left[\left(\frac{2^m}{M-2^m}\right)^2 \sum_n 2^{-m} \sum_i \sum_j \tau[i]\tau[j]\ \psi\left(2^{-m}i - n\right)\psi\left(2^{-m}j - n\right) \times \sum_p 2^{-m} \sum_k \sum_l \tau[k]\tau[l]\,\psi\left(2^{-m}k - p\right)\psi\left(2^{-m}l - p\right) \right], \qquad (10)$$

where the summation limits remain as before, but have been omitted for clarity. The expectation operator can be brought inside the summation to yield

$$\mathrm{E}\left[\hat{D}^2(m)\right] = \left(\frac{1}{M-2^m}\right)^2 \sum_n \sum_p \sum_i \sum_j \sum_k \sum_l \mathrm{E}\left[\tau[i]\tau[j]\tau[k]\tau[l]\right] \times \psi\left(2^{-m}i - n\right)\psi\left(2^{-m}j - n\right)\psi\left(2^{-m}k - p\right)\psi\left(2^{-m}l - p\right). \qquad (11)$$

There are four different combinations of i, j, k, and l which provide a non-zero contribution to this summation:

$$\mathrm{E}\big[\tau[i]\tau[j]\tau[k]\tau[l]\big] = \begin{cases} 3\sigma^4 \text{ for } i = j = k = l \\ \sigma^4 \text{ for } i = j,\ k = l,\ i \neq k \\ \sigma^4 \text{ for } i = k,\ j = l,\ i \neq j \\ \sigma^4 \text{ for } i = l,\ j = k,\ i \neq j \end{cases} \tag{12}$$

Evaluation of the summation of Eq. (11) using Eq. (12) results in:

$$E\big[\hat{D}^2(m)\big] = \frac{(M^2 + 2^{m+1}M)\sigma^4}{\left(M - 2^m\right)^2}, \tag{13}$$

which finally yields the result

$$\mathrm{var}[\hat{D}(m)] = \frac{M\,2^{m+1}\sigma^4}{(M - 2^m)^2} \text{ or } \mathrm{std}[\hat{D}(m)] = \frac{\sqrt{M}\,2^{(m+1)/2}\sigma^2}{M - 2^m}. \tag{14}$$

This provides the intuitively expected result of increasing variance with scale m. Note also that since the estimator is asymptotically unbiased, and its variance goes to 0 as $M \to \infty$, Eq. (7) defines a *consistent* estimator of DWT variance.

3. RESULTS

3.1 Comparison of Spectral and DWT Measures for HRV Discrimination

To confirm the validity of Eq. (6), we calculated estimates of both the PSD and the wavelet-transform variance $D(m)$ for a set of RR intervals. This data is derived from Holter monitor recordings drawn from the Beth Israel (Boston, MA) congestive heart-failure database. They comprise 12 records from normal patients and 15 records from severe congestive heart-failure patients. Complete details of the analyzed data sets can be found in Reference [8]. Figure 1(a) gives the PSD estimates for the 27 records, with normals shown as dashed, and abnormals as solid curves. These were calculated using the averaged periodogram technique with a rectangular window of length 512. A clear region of separation exists for frequencies in the range [0.03125–0.0625]. Note that the units for the x-axis are given in cycles/index rather than in Hz, since we are analyzing a *discrete-index* sequence. Spectral measures of *discrete-time* sampled HRV signals have been extensively used, as well as measures involving number of beats, where clear physiological interpretation of temporal frequencies is possible [4,8].

In Figure 1(b), we show the DWT variance estimates. There is a clear region of separation for scales m = 4 and 5. Note that these scales correspond to the frequencies 0.0625 and 0.03125 illustrated in Fig 1(a). These figures confirm the conclusions set forth in Section 2.1. Note that in both figures, there is considerable spread in the calculated curves; this is due both to inherent parametric differences between data sets *and* statistical fluctuations of the estimators. For maximum clinical utility, we must minimize the variability arising from the second factor.

3.2 Simulations of the Bias and Variance of *D*(*m*)

To confirm the analytical results obtained in Eqs. (9) and (14), we conducted numerical simulations using data sets of uncorrelated random Gaussian noise of zero mean and unit variance. A data length of M=2048 was chosen, and 500 trials were conducted in which the DWT variance was estimated using Eq. (7). Figure 2 shows the results of these simulations. These numerical results confirm the accuracy of the analysis.

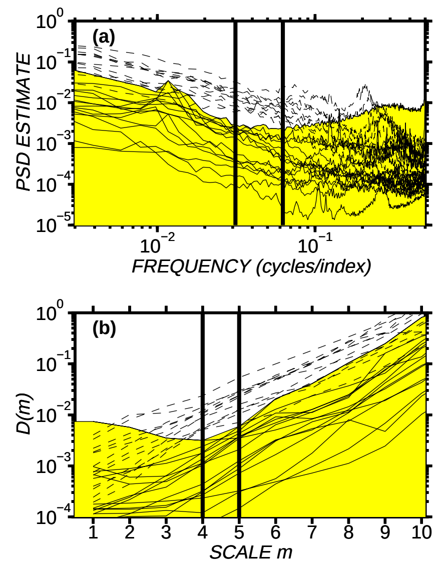

Figure 1. (a) Power spectral density versus frequency for the data sets used in [5]. The shaded region is limited by the maximum value of the PSD for all *abnormal* data sets (solid curves). Separation of the two classes of data can be seen over frequencies 0.03125–0.0625=1/32–1/16 (region bordered by vertical lines). (b) Wavelet transform variance $D(m) = \sigma^2_{\mathrm{wav}}(m)$, versus scale m for the same data sets. The shaded region is limited by the maximum value of $D(m)$ for all *abnormal* data sets (solid curves). Separation of the two classes of data occurs at scales 4 and 5, corresponding to the same frequency range observed in (a).

4. DISCUSSION AND CONCLUSIONS

We have shown that DWT variance measures give equivalent information to spectral measures. We have considered the bias and variance associated with estimators of these measures, but have not yet addressed their relative frequency (or scale) resolution. The frequency resolution of non-parametric PSD estimates is dictated by the width of the windows used in constructing an averaged periodogram. The frequency resolution of $D(m)$ can be determined as follows [1]. The quantity in square brackets in Eq. (6) can be usefully recast as $H(a\omega)$, where $H(\omega)$ is the bandpass filter generated by taking the Fourier transform of the wavelet kernel. This bandpass filter has an associated center frequency ω_0 and Q-factor. For example, the Haar wavelet basis with unit sampling has ω_0=0.5, and Q=0.5. Wavelets of higher regularity will typically have a higher Q. Eq. (6) tells us that the estimated $D(a)$ depends only on spectral components passed by $H(a\omega)$, so that the frequency resolution of $D(a)$ can be defined as the bandwidth of the corresponding $H(a\omega)$, *i.e.*, $a\,\omega_0/Q$. For the specific example analyzed here of a dyadic Haar-basis DWT, the frequency resolution at scale m is $1/2^m$. Combined with Eq. (15), this provides the pleasing result that $D(m)$ has a constant frequency-resolution/variance product. This is equivalent to the tradeoff in spectral estimation between those two quantities. Considering their close links, moreover, it is not surprising that the product is approximately equal for both measures:

$$\begin{aligned} D(m):\ \Delta f\Delta D &= \frac{1}{2^m}\frac{M\,2^{m+1}\sigma^4}{(M-2^m)^2} = \frac{2M\sigma^4}{(M-2^m)^2} \\ \text{SPECTRUM}:\ \Delta f\Delta S &= \frac{2K}{M}\frac{\sigma^4}{K} = \frac{2\sigma^4}{M}, \end{aligned} \qquad (16)$$

where Δf is frequency resolution, and ΔD and ΔS are the variances of $D(m)$ and the PSD estimate, respectively. For $D(m)$, we have good frequency resolution and poor estimator variance at low frequencies, whereas the PSD provides constant frequency resolution and estimator variance at all values, once the number of segments for averaging has been chosen.

This provides insight into choosing between $D(m)$ and the PSD for our particular analysis. Since the ability to discriminate data sets depends upon values of $D(m)$ or the PSD over a small set of scales/frequencies, we must select our analysis to have sufficient frequency resolution in that range. For example, we could set Δf at the center frequency of the discriminating band to equal some fraction of the lowest frequency discriminating range. This frequency resolution can be easily chosen for both measures (by selecting K for spectral estimation, or by arbitrarily assigning the support of the wavelet as scale m=1).

However, the specific data sets we are dealing with exhibit strong long-term correlation effects. Therefore, the power in the signal is concentrated at low frequencies, which is also the frequency range where discrimination is possible. In evaluating measures at low frequency, we should minimize the contribution of confounding energy at surrounding frequencies. Accordingly, the measure with good frequency resolution at low frequencies [$D(m)$] should prove more suitable for our particular application.

It is worth considering whether the performance of wavelet transform variance measures can be improved by evaluation either on a non-dyadic grid (finer scale spacing), or at non-integer values of n (finer time spacing). Evaluation on a non-dyadic scale provides us with interpolated values at scales in between powers of 2. Provided data discrimination is observed in a range which includes at least one power of 2, there appears to be little benefit in moving to a non-dyadic grid. Introducing samples of $W[m,n]$ at non-integer values is analogous to using overlapping windows in PSD estimation. Nearby values of $W[m,n]$ are highly correlated, and will have minimal effect on the overall performance of the estimator.

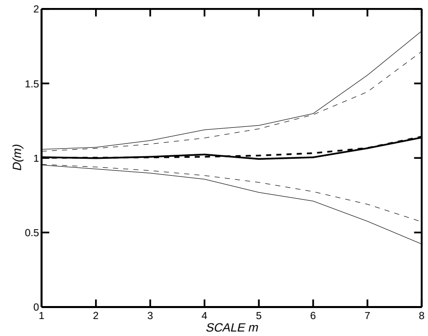

Figure 2. Comparison of predicted bias and variance of the DWT variance estimator with numerical simulation, for zero-mean, unit-variance WGN. Data sets of length M=2048 were chosen, and 500 estimates of $D(m)$ were obtained. The solid curves give the mean value of $D(m)$, in addition to the ±1 standard deviation for numerical estimates of $D(m)$. The dashed curves provide the corresponding analytical values.

The analysis presented here is considerably more difficult for wavelet bases other than the Haar basis. However, it seems reasonable to assume that the overall results will persist, though higher Q wavelets may give additional benefit. This will be investigated in future work.

5. REFERENCES

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