SPACE-TIME MULTIUSER DETECTION IN MULTIPATH CDMA CHANNELS *

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ABSTRACT

The problem of multiuser detection in multipath CDMA channels with receiver antenna array is considered. The optimal space-time multiuser receiver structure is first outlined, followed by linear space-time multiuser detection methods based on iterative interference cancellation. Single-user-based space-time processing methods are also considered and are compared with the multiuser approach. It is seen that the proposed multiuser space-time processing techniques offer substantial performance gains over the single-user-based methods, especially in a near-far situation.

1. INTRODUCTION

Wireless communications for mobile telephony and data transmission is currently undergoing very rapid development. Many wireless systems operate as multiple-access systems, in which channel bandwidth is shared by many users on a random-access basis. One type of multiple-access technique that has become very popular in recent years is direct-sequence code-division multiple-access (DS-CDMA). Two major factors that limit the performance of DS-CDMA systems are the multiple-access interference (MAI) and the multipath channel distortion. Many advanced signal processing techniques have been proposed for combating interference and multipath channel distortion, and these techniques fall largely into two categories: *multiuser detection* [7] and *space-time processing* [5]. In this paper we consider the problem of combined space-time processing and multiuser detection.

2. SIGNAL MODEL

Consider a DS/CDMA mobile radio network with K users, employing normalized spreading waveforms $s_1, s_2, \cdots, s_K$, and transmitting sequences of BPSK symbols through their respective multipath channels. The transmitted baseband signal due to the k-th user is given by

$$x_k(t) \quad = \quad A_k \sum_{i=0}^{M-1} b_k(i)\, s_k(t - iT), \qquad k = 1, \cdots, K, \tag{1}$$

where M is the number of data symbols per user per frame; T is the symbol interval; $b_k(i) \in \{+1, -1\}$ is the i-th transmitted symbol by the k-th user; and A_k and $s_k(t)$ denote respectively the amplitude and normalized signaling waveform of the k-th user. It is assumed that $s_k(t)$ is supported only on the interval $[0, T]$ and has unit energy. It is also assumed that each user transmits independent equiprobable symbols and the symbol sequences from different users are independent.

At the receiver an antenna array of P elements is employed. Assuming that each transmitter is equipped with a single antenna, then the baseband multipath channel between the k-th user's transmitter and the base station receiver can be modeled as a single-input multiple-output channel with the following vector impulse response

$$\underline{h}_k(t) \quad = \quad \sum_{l=1}^{L} \underline{a}_{kl}\, g_{kl}\, \delta(t - \tau_{kl}), \tag{2}$$

where L is the number of paths in each user's channel, g_{kl} and τ_{kl} are respectively the complex gain and delay of the l-th path of the k-th user's signal, and $\underline{a}_{kl} = [a_{kl,1} \ \cdots \ a_{kl,P}]^T$ is the array response vector corresponding to the l-th path of the k-th user's signal. The total received signal at the receiver is then the superposition of the signals from the K users plus the additive ambient noise, given by

$$\underline{r}(t) \quad = \quad \sum_{k=1}^{K} x_k(t) \star \underline{h}_k(t) + \sigma\, \underline{n}(t) \tag{3}$$

where $\star$ denotes convolution; $\underline{n}(t) = [n_1(t) \ \cdots \ n_P(t)]^T$ is a vector of independent zero-mean complex white Gaussian noise processes, each with unit power spectral density; and σ^2 is the variance of the ambient noise at each antenna element.

3. OPTIMAL SPACE-TIME MULTIUSER DETECTION

For $1 \le k \le K, 0 \le i \le M-1$, define the space-time matched filter output

$$y_k(i) \triangleq \sum_{l=1}^{L} g_{kl}^*\, \underline{a}_{kl}^H \int_{-\infty}^{\infty} \underline{r}(t)\, s_k(t - iT - \tau_{kl})\, \mathrm{d}t. \tag{4}$$

Denote $\underline{b}(i) \triangleq [b_1(i) \cdots b_K(i)]^T$, and $\mathbf{b} \triangleq [\underline{b}(0)^T \cdots \underline{b}(M-1)^T]^T$. Denote further $\underline{y}(i) \triangleq [y_1(i) \cdots y_K(i)]^T$ and $\mathbf{y} \triangleq$

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$[\underline{y}(0)^T \cdots \underline{y}(M-1)^T]^T$. Assume that the multipath spread of any user signal is limited to at most Δ symbol intervals, where Δ is a positive integer, i.e., $\tau_{kl} \leq \Delta T$. Then it can be shown that the maximum likelihood sequence detection rule chooses $\mathbf{b}$ that maximize the following log-likelihood function

$$\Omega(\mathbf{b}) \quad = \quad 2\Re\left\{\mathbf{b}^T \mathbf{A}\, \mathbf{y}\right\} - \mathbf{b}^T \mathbf{A}\, \mathbf{H}\, \mathbf{A}\, \mathbf{b}. \tag{5}$$

In (5) $\mathbf{A} \stackrel{\triangle}{=} \underline{I}_M \otimes \mathsf{diag}(A_1, \cdots, A_K)$; and $\mathbf{H}$ is an $(MK \times MK)$ block Jacobi matrix of the form

$$\mathbf{H} \stackrel{\triangle}{=} \begin{bmatrix} \underline{H}^{[0]} & \underline{H}^{[1]} & \cdots & \underline{H}^{[\Delta]} & & & \\ \underline{H}^{[-1]} & \underline{H}^{[0]} & \underline{H}^{[1]} & \cdots & \underline{H}^{[\Delta]} & & \\ & & & & & & \\ & \underline{H}^{[-\Delta]} & \cdots & \underline{H}^{[0]} & \cdots & \underline{H}^{[\Delta]} & \\ & & & & & & \\ & & \underline{H}^{[-\Delta]} & \cdots & \underline{H}^{[-1]} & \underline{H}^{[0]} & \underline{H}^{[1]} \\ & & & \underline{H}^{[-\Delta]} & \cdots & \underline{H}^{[-1]} & \underline{H}^{[0]} \end{bmatrix}$$

where the entries of the $(K \times K)$ matrix $\underline{H}^{[j]}$ are determined by the crosscorrlations of the user signal waveform, the relative delays, the array responses and the multipath gains of the channels.

The Viterbi algorithm can be applied to compute the sequence $\hat{\mathbf{b}}$ that maximizes $\Omega(\mathbf{b})$, i.e., the maximum likelihood estimate of the transmitted multiuser symbol sequences. The computational complexity of the maximum likelihood sequence detector is on the order of $O\left(2^{(\Delta+1)K}\right)$. Note that in the absence of multipath (i.e., $L = 1$ and $\Delta = 1$), if the users are numbered according to their relative delays in an ascending order (i.e., $0 \leq \tau_{11} \leq \cdots \leq \tau_{K1} < T$), then the matrix $\underline{H}^{[-1]}$ becomes strictly upper triangular. In this case, the dimension of the state vector is reduced to $(K-1)$ and the computational complexity of the corresponding maximum likelihood sequence detection algorithm is $O\left(2^K\right)$ [3, 6]. However, in the presence of multipath, even if the multipath delays are still within one symbol interval (i.e., $\Delta = 1$), the matrix $\underline{H}^{[-1]}$ no longer has an upper triangular structure in general. Hence the dimension of the state vector in this case is $(2K-1)$ and the complexity of the Viterbi algorithm is $O\left(2^{2K}\right)$.

4. LINEAR SPACE-TIME MULTIUSER DETECTION

It can be shown that the sufficient statistic vector $\mathbf{y}$ can be expressed as

$$\mathbf{y} \quad = \quad \mathbf{H}\, \mathbf{A}\, \mathbf{b} + \sigma\, \mathbf{v}, \tag{6}$$

where $\mathbf{v} \sim \mathcal{N}_c\left(\mathbf{0}, \mathbf{H}\right)$. In linear multiuser detection, a linear transformation is applied to the sufficient statistic vector $\mathbf{y}$, followed by local decisions for each user. That is, the multiuser data bits are demodulated according to

$$\hat{\mathbf{b}} \quad = \quad \mathrm{sign}\left[\Re\left\{\mathbf{W}\, \mathbf{y}\right\}\right], \tag{7}$$

where $\mathbf{W}$ is an $(MK \times MK)$ complex matrix. Two popular linear multiuser detectors [7] are the linear decorrelating (i.e., zero-foring) detector, which chooses the weight matrix $\mathbf{W}$ to completely eliminate the interference (at the expense of enhancing the noise); and the linear MMSE detector, which chooses the weight matrix $\mathbf{W}$ to minimize the mean-square error (MSE) between the transmitted user signals and the outputs of the linear transformation, i.e., $E\left\{\|\mathbf{A}\, \mathbf{b} - \mathbf{W}\, \mathbf{y}\|^2\right\}$. The corresponding weight matrices for these two linear multiuser detectors are given respectively by

$$\mathbf{W}_d \quad = \quad \mathbf{H}^{-1}, \qquad [\text{ linear decorrelating detector }] \tag{8}$$

$$\mathbf{W}_m \quad = \quad \left(\mathbf{H} + \sigma^2 \mathbf{A}^{-2}\right)^{-1}. \ [\text{ linear MMSE detector }] \tag{9}$$

Since the frame length M is usually large, direct inversion of the above matrices is too costly for practical purposes. Next we consider an iterative interference cancellation method which converges to the linear multiuser detector output under certain conditions. This method performs serial interference cancellation on the sufficient statistic vector $\mathbf{y}$ and recursively refines the estimates of the multiuser signals $\{x_k(i) \stackrel{\triangle}{=} A_k b_k(i) : 1 \leq k \leq K, 0 \leq i < M\}$. Denote such an estimate at the m-th iteration as $x_k^m(i)$. The algorithm is listed in Table 1.

$x_k(i) \ = \ y_k(i), \quad k = 1, \cdots, K; \quad i = 0, \cdots, M-1$
for $\quad m = 1, 2, \cdots$ $\quad$ for $\quad i = 0, \cdots, M-1$ $\quad\quad$ for $\quad k = 1, \cdots, K$ $x_k^m(i) = \frac{1}{\gamma_k}\Big[y_k(i) - \sum_{j=-\Delta}^{-1} \sum_{k'=1}^{K} h_{k,k'}^{[j]} x_{k'}^m(i+j)$ $- \sum_{k'=1}^{k-1} h_{k,k'}^{[0]} x_{k'}^m(i) - \sum_{k'=k+1}^{K} h_{k,k'}^{[0]} x_{k'}^{m-1}(i)$ $- \sum_{j=1}^{\Delta} \sum_{k'=1}^{K} h_{k,k'}^{[j]}\, x_{k'}^{m-1}(i+j)\Big]$ $\quad\quad$ end $\quad$ end   end
linear decorrelating detector: $\quad \gamma_k \ = \ h_{kk}^{[0]}$ linear MMSE detector: $\quad \gamma_k \ = \ h_{kk}^{[0]} + \sigma^2 / A_k^2$

Table 1. Iterative Implementation of linear space-time multiuser detection using serial interference cancellation.

The computational complexity of the above algorithm *per user per bit* is $\left[\overline{m} M (2\Delta + 1) K^2\right] / KM = O\left(\overline{m} \Delta K\right)$ where $\overline{m}$ is the total number of iterations. By exploiting the Hermitian $(2\Delta+1)$-block Toeplitz structure of the matrix $\mathbf{H}$, the complexity *per user per bit* of inverting the matrices in (8) or (9) is $O\left(K^2 M \Delta\right)$ [3]. Since in practice the number of iterations is a small number, e.g., $\overline{m} \leq 5$, the above iterative method for linear multiuser detection achieves significant complexity reduction over the direct matrix inversion method. Denote $\underline{x}^m(i) \stackrel{\triangle}{=} [x_1^m(i) \ \cdots \ x_K^m(i)]^T$, and $\mathbf{x}^m \stackrel{\triangle}{=} \left[\underline{x}^m(0)^T \ \cdots \ \underline{x}^m(M-1)^T\right]^T$. Next we turn to the convergence properties of the above serial interference cancellation algorithm, which is characterized by the following result.

Proposition 1 [Convergence of iterative serial interference cancellation]
(1) If $\gamma_k = h_{kk}^{[0]}$, and if $\mathbf{H}$ is positive definite, then $\mathbf{x}^m \longrightarrow \mathbf{W}_d\, \mathbf{y}$, as $m \longrightarrow \infty$;
(2) If $\gamma_k = h_{kk}^{[0]} + \sigma^2 / A_k^2$, then $\mathbf{x}^m \longrightarrow \mathbf{W}_m\, \mathbf{y}$, as $m \longrightarrow \infty$.

4.1. Single User Space-Time Detection

In this section we consider the single-user-based linear space-time processing methods. These methods have been advocated in the recent literature as they lead to several space-time adaptive receiver structures [1, 2, 4, 8].

Denote $r_p(t)$ as the received signal at the p-th antenna element, i.e., the p-th element of the received vector signal $\underline{r}(t)$ in (3),

$$r_p(t) = \sum_{i=0}^{M-1} \sum_{k=1}^{K} A_k b_k(i) \sum_{l=1}^{L} a_{kl,p} g_{kl} s_k(t - iT - \tau_{kl}) + \sigma n_p(t).$$

Suppose that the user of interest is the k-th user. In the single-user approach, in order to demodulate the i-th symbol of the k-th user, that user's matched filter output corresponding to each path at each antenna element is first computed, i.e., for $l = 1, 2, \cdots, L; p = 1, 2, \cdots, P$,

$$z_{kl,p}(i) \quad \stackrel{\triangle}{=} \quad \int_{-\infty}^{\infty} r_p(t) s_k(t - iT - \tau_{kl}) \mathrm{d}t. \qquad (10)$$

Denote $\underline{\tilde{z}}_{kp}(i) \stackrel{\triangle}{=} [z_{11,p}(i) \quad \cdots \quad z_{1L,p}(i)]^T$, and $\mathbf{z}_k(i) = [\underline{\tilde{z}}_{k1}(i)^T \cdots \underline{\tilde{z}}_{kP}(i)^T]^T$. Then it can be shown that [1]

$$\mathbf{z}_k(i) \quad = \quad \sum_{j=-\Delta}^{\Delta} \mathbf{\Xi}_k^{[j]} \, \underline{A} \, \underline{b}(i + j) + \sigma \, \mathbf{n}_k(i). \qquad (11)$$

In the single-user-based space-time processing methods, the k-th user's i-th bit is demodulated according the following rule

$$\hat{b}_k(i) \quad = \quad \text{sign} \left[\Re \left\{ \mathbf{w}_k^H \, \mathbf{z}_k(i) \right\} \right], \qquad (12)$$

where $\mathbf{w}_k \in \mathcal{C}^{LP}$. In MMSE combining, the weight vector is chosen to minimize the mean-squared error between the k-th user's transmitted signal and the output of the linear combiner, i.e.,

$$\mathbf{w}_k = \arg \min_{\mathbf{w} \in \mathcal{C}^{LP}} E \left\{ \left\| b_k(i) - \mathbf{w}^H \, \mathbf{z}_k(i) \right\|^2 \right\} = \mathbf{\Sigma}_k^{-1} \, \mathbf{p}_k, \quad (13)$$

where $\mathbf{\Sigma}_k \stackrel{\triangle}{=} E\left\{\mathbf{z}_k(i)\mathbf{z}_k(i)^H\right\}$, $\mathbf{p}_k \stackrel{\triangle}{=} E\left\{\mathbf{z}_k(i)\, b_k(i)\right\} = \mathbf{\Xi}_k^{[0]} \, \mathbf{1}_k$. with $\mathbf{1}_k$ is a K-vector of all zeros except for the k-th entry, which is 1.

4.2. Combined Single-user/Multiuser Linear Detection

In this section, we consider space-time processing in a scenario where some users' channels are unknown. The basic strategy is to combine the single-user and multiuser approaches. Consider the received signal model (3). Assume that the users of interest are users $k = 1, \cdots, K_0 < K$, and the spreading waveforms as well as the channel parameters of these users are known to the receiver. Users $k = K_0 + 1, \cdots, K$ are unknown external interferers whose data are not to be demodulated. For each user of interest, the receiver first computes the (LP)-vectors of the matched-filter outputs, $\mathbf{z}_k(i)$, $1 \le k \le K_0$, $i = 0, \cdots, M-1$, [cf.(11)]. The space-time matched-filter outputs $y_k(i)$ [cf.(4)] are then computed by correlating $\mathbf{z}_k(i)$ with a space-time matched filter. Next the serial interference cancellation algorithm in Table 1 [Here the total number of users K is replaced by the total number of users of interest K_0] is applied to the data $\{y_k(i) : 1 \le k \le K_0; i = 0, \cdots, M-1\}$ to suppress the interference from the known users. This is equivalent to implementing a linear multiuser detector assuming only K_0 (instead of K) users present. As a result, only the known interferers' signals are suppressed at the detector output. Upon convergence, denote $\hat{x}_k(i)$ as the estimate of the user signals, i.e., $\hat{x}_k(i) \stackrel{\triangle}{=} \lim_{m \longrightarrow \infty} x_k^m(i)$, $1 \le k \le K_0$, $i = 0, \cdots, M-1$. Note that $\hat{x}_k(i)$ contains the desired user's signal, the unknown interferers' signals and the ambient noise. Using these estimates and based on the signal model (11), we next cancel the known interferers' signal from the vector $\mathbf{z}_k(i)$ to obtain

$$\hat{\mathbf{z}}_k(i) \stackrel{\triangle}{=} \mathbf{z}_k(i) - \sum_{j=-\Delta}^{\Delta} \sum_{\substack{k'=1 \\ (j,k') \neq (0,k)}}^{K_0} \mathbf{\Xi}_k^{[j]} \, \mathbf{e}_{k'} \, \hat{x}_{k'}(i + j). \qquad (14)$$

Finally a single-user combining weight $\mathbf{w}_k$ is applied to the vector $\hat{\mathbf{z}}_k(i)$ and the decision rule is given by

$$\hat{b}_k(i) \quad = \quad \text{sign} \left[\Re \left\{ \mathbf{w}_k^H \, \hat{\mathbf{z}}_k(i) \right\} \right]. \qquad (15)$$

If the combining weight is chosen to according to the MMSE criterion, then it is given by

$$\begin{aligned} \mathbf{w}_k \quad &= \quad \arg \min_{\mathbf{w} \in \mathcal{C}^{LP}} E \left\{ \left\| b_k(i) - \mathbf{w}_k^H \, \mathbf{z}_k(i) \right\|^2 \right\} \\ &= \quad \left[E \left\{ \hat{\mathbf{z}}_k(i) \, \hat{\mathbf{z}}_k(i)^H \right\} \right]^{-1} \cdot E \left\{ \hat{\mathbf{z}}_k(i) \, b_k(i) \right\} \quad (16) \end{aligned}$$

It is clear from the above discussion that in this combined approach, the interference due to the known users are suppressed by serial multiuser interference cancellation, whereas the the residual interference due to the unknown users is suppressed by the single-user MMSE combiner.

5. SIMULATION RESULTS

In this section, we assess the performance of the various multiuser and single-user space-time processing methods discussed in this paper by computer simulations. The simulated CDMA system consists of 8 users ($K = 8$) with a spreading gain 16 ($N = 16$). Each user's propagation channel consists of three paths ($L = 3$). The receiver employs a linear antenna array with three elements ($P = 3$) and half-wavelength spacing. Let the direction of arrival (DOA) of the k-th user's signal along the l-th path with respect to the antenna array be ϕ_{kl}, then the array response is given by

$$a_{kl,p} \quad = \quad \exp\left[j(p-1)\pi \sin\left(\phi_{kl}\right) \right], \qquad (17)$$

The spreading sequences, multipath delays and complex gains, and the DOA's of all user signals in the simulated system are randomly generated and kept fixed for all the

[1]The exact expressions of the matrices $\mathbf{\Xi}_k^{[j]}$ and the covariance matrix of $\mathbf{n}_k(i)$ are omitted here due to space limitation.

simulations. All users have equal transmitted power, i.e., $A_1 = \cdots = A_K$. However, the received signal powers are unequal due to the unequal strength of the multipath gain for each user. In the simulated system there is a near-far situation, e.g., user 3 is the weakest user and user 6 is the strongest.

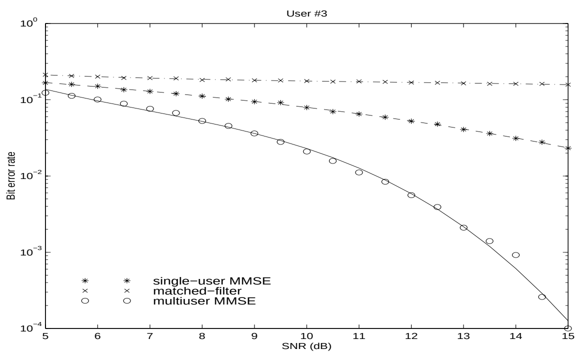

Figure 1. Comparison of bit error rate vs. signal-to-noise ratio (SNR) for three receivers: single-user space-time matched-filter, single-user space-time MMSE receiver and multiuser space-time MMSE receiver.

We first compare the performance of the multiuser linear space-time detection and that of the single-user linear space-time detection. The figure of merit for comparison is the probability of bit detection error. Figure 1 shows the performance of the weakest user 3. It is evident that the multiuser approach offers substantial performance improvement over the single-user methods.

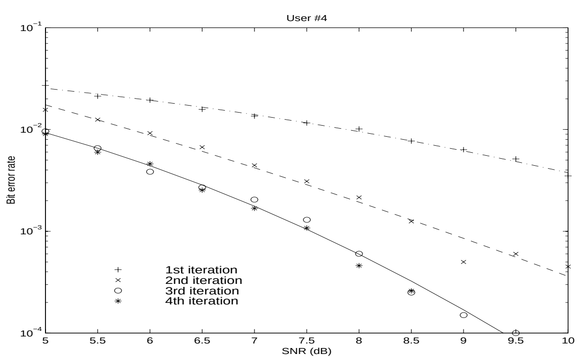

Figure 2. Bit error rate vs. SNR of the iterative interference cancellation method (first 4 iterations).

The second example serves to illustrate the convergence behavior of the iterative interference cancellation method. The bit error rate performance corresponding to the first 4 iterations for user 4 is shown in Figure 2. It is seen that the algorithm converges within 4-5 iterations. It is also seen that the biggest performance improvement occurs at the second iteration.

In the third example, it is assumed that users 7 and 8 are external interferers, and their signature waveforms and channel parameters are not known to the receiver. Therefore the combined multiuser/single-user space-time processing method discussed is employed at the receiver. Figure

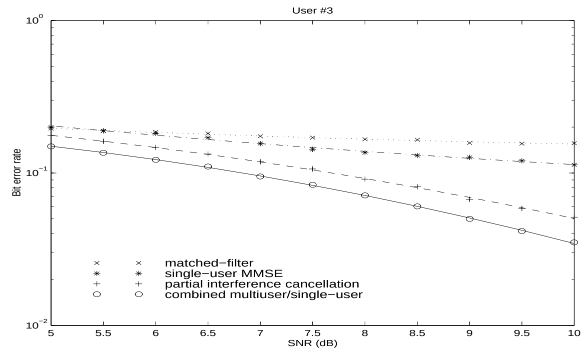

Figure 3. Bit error rate vs. SNR for four receivers in the presence of unknown interferers.

3 illustrates the bit error rate performance of user 3. Four methods are considered here: the single-user matched-filter, the single-user MMSE receiver, the partial interference cancellation followed by a matched-filter or a single-user MMSE receiver. It is seen that the combined multiuser/single-user space-time processing achieves the best performance among the four method.

6. CONCLUSIONS

In this paper, we have considered the problem of space-time multiuser detection in multipath CDMA channels with receiver array antennas. We have first derived the optimal space-time multiuser receiver structure, which produces the maximum likelihood estimate of the multiuser symbol sequences. We have then developed linear space-time multiuser processing methods based on iterative interference cancellation. The proposed multiuser space-time processing methods are seen to significantly outperforms their single-user-based counterparts, especially in a near-far situation.

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